**LUCRARE DE VERIFICARE LA MATEMATICI SPECIALE, FCIM, FR (Variantă în lucru)**

# „Funcția complexă de o variabilă complexă"

**Problema 1.**

1. $\ln\left(\sqrt{3}-i\right)$; 2. $1^{i}$; 3. $(1+i)^{\sqrt{2}}$; 4. $\ln\dfrac{1-i}{\sqrt{2}}$;

5. $\left(\dfrac{\sqrt{3}+i}{2}\right)^{1+i}$; 6. $\sin(1+i)$; 7. $e^{1+\frac{\pi}{2}i}$; 8. $\cos(2+i)$;

9. $(1-i)^{3-3i}$; 10. $\mathrm{Arc}\sin i$; 11. $\mathrm{Ln}(-1-i)$; 12. $(-1)^{\sqrt{5}}$

13. $\mathrm{Arccos}\left(\dfrac{\pi^3}{3}\right)$; 14. $\ln(1-i)$; 15. $\mathrm{Arcth}\ \pi i$;

16. $\ln(-\sqrt{3}-i)$; 17. $\mathrm{Arctg}(1-i)$; 18. $\mathrm{Arcsh}\ (1-i)$;

19. $\mathrm{Arc}\sin 3$; 20. $\mathrm{sh}\left(1+\dfrac{\pi}{2}i\right)$; 21. $\ln 1^{i}$;

22. $\mathrm{Arcch}\dfrac{\pi i}{2}$; 23. $\mathrm{Arccos}\ i$; 24. $\mathrm{Arctg}\dfrac{i}{3}$;

25. $\ln(1+i)$; 26. $(1+i)^{i}$; 27. $\ln i^{\frac{1}{i}}$;

28. $\mathrm{Ln}\left(\dfrac{\sqrt{3}}{2}-\dfrac{i}{2}\right)$; 29. $\mathrm{Atcctg}\left(\sqrt{3+i}\right)$; 30. $\mathrm{Arccos}(1+i)$.

**Problema 2.** Să se cerceteze analiticitatea funcției $f(z)=u(x,y)+iv(x,y)$, unde $z=x+iy$, și să se calculeze $f'(z_0)$, dacă

1. $u=3x^2y-y^3$, $v=3xy^2-x^3$, $z_0=-1+i$;

2. $f(z)=\dfrac{e^z}{z}$, $z_0=1+i$;

3. $u=e^x(x\cos x-y\sin y)$, $v=e^x(x\sin x+y\sin y)$, $z_0=-1+\pi i$;

4. $f(z)=(1+z)\cdot \mathrm{Im}z^2$, $z_0=-1$;

5. $u=x^3-3xy^2+x^2-y^2$, $v=3x^2y-y^3+2xy$, $z_0=\dfrac{2}{3}i$;

6. $f(z)=5e^{2z}$, $z_0=1-i$;

7) $u=e^{1+y}\cos x$, $v=-e^{1+y}\sin x$, $z_0=\dfrac{\pi}{2}+i$;

8. $f(z)=\left|z-1\right|^2$, $z_0=1$;

9. $u=2xy-2x$, $v=y^2-2y-x^2+2$, $z_0=1$;
10. $f(z)=sh3z$, $z_0=2+i$;
11. $u=x^3-3xy^2+3x$, $v=3x^2y-y^3+3y+1$, $z_0=-1-i$;
12. $f(z)=ich3z$, $z_0=i$;
13. $u=e^{1+3y}\cos 3x$, $v=-e^{1+3y}\sin 3x$, $z_0=\dfrac{\pi}{3}+i$;
14. $f(z)=2\sin 2z$, $z_0=\dfrac{\pi}{8}i$;
15. $u=x^2+2x-y^2$ $v=2xy+2y$, $z_0=i$;
16. $f(z)=(z+1)\cdot \operatorname{Re}z$, $z_0=-1$;

17. $u=e^{-1-y}\cos x$, $v=e^{-1-y}\sin x$, $z_0=\pi-i$;
18. $f(z)=z\cos z$, $z_0=\dfrac{\pi}{4}i$;
19. $u=e^{1-2x}\cos 2y$, $v=-e^{1-2x}\sin 2y$, $z_0=\dfrac{\pi}{4}i$;
20. $f(z)=2\operatorname{ch}2z$, $z_0=1+i$;
21. $u=e^{1+2y}\cos 2x$, $v=-e^{1+2y}\sin 2x$, $z_0=\dfrac{\pi}{6}$;
22. $f(z)=z\cdot e^z$, $z_0=1-i$;
23. $u=e^x\cos y+1$, $v=e^x\sin y+1$, $z_0=1+\dfrac{\pi}{4}i$;
24. $f(z)=z^3+2z+1$, $z_0=2+3i$;
25. $u=x$, $v=y$, $z_0=5+3i$;
26. $f(z)=z\bar{z}$, $z_0=0$;
27. $u=2e^x\cos y$, $v=2e^x\sin y$, $z_0=2-3i$;
28. $f(z)=z^2+4iz$, $z_0=2+i$;
29. $u=\dfrac{x}{x^2+y^2}$, $v=-\dfrac{y}{x^2+y^2}$, $z_0=2-i$;
30. $f(z)=2\sin z-z$, $z_0=\dfrac{\pi}{3}i$.

**Problema 3.** Să se calculeze integrala

1. $\int\limits_{AB}(1+i+4z)dz$, unde AB este segmentul de parabolă $y=x^2+1,\, x\in[0,2]$;

2. $\int\limits_{L}(z^3+z\bar{z})dz$, unde $\bar{z}=x-iy$, L este semicircumferința $|z|=2,\, 0\le \arg z\le \pi$;

3. $\int\limits_{L}\bar{z}e^{z}dz$, unde $\bar{z}=x-iy$ și L este segmentul de dreaptă, ce unește punctele $z_1=0,\, z_2=\pi-i\pi$;

4. $\int\limits_{1+i}^{3+i}(3z^2+z+5)dz$;  5. $\int\limits_{0}^{2i}z\cos 2z dz$;  6. $\int\limits_{i+1}^{i}(z+1)e^{iz}dz$;

7. $\int\limits_{L}z\operatorname{Re} z dz$, unde L este semicircumferința $|z-1|=1$, $\operatorname{Im} z\ge 0$;

8. $\int\limits_{L}\cos^2 z dz$, unde L este segmentul de dreaptă, ce unește punctele $z_1=\dfrac{\pi}{2},\, z_2=\pi+i$;

9. $\int\limits_{1}^{i}(3z^5+2z^4)dz$; 10. $\int\limits_{0}^{1+i}(z+i)e^{z}dz$; 11. $\int\limits_{0}^{2+i}\sin z\cos z dz$;

12. $\int\limits_{L}(z+i)\operatorname{sh} z\, dz$, unde L este circumferința $|z-i|=1$,

13. $\int\limits_{L}|z|\cdot z\, dz$, unde L este semicircumferința $|z|=2$, $\operatorname{Re} z\ge 0$.

14. $\int\limits_{L}|z|\cdot \operatorname{Im} z^2 dz$, unde L este semicircumferința $|z-2|=2$, $\operatorname{Im} z\ge 0$;

15. $\int_L (z + shz)dz$, unde L este circumferinţă $|z+i| = 1$.

16. $\int_L (\cos iz + chz)dz$, unde L este linia poligonală ce uneşte punctele $z_1 = 0$, $z_2 = 2$, $z_3 = 2+i$;

17. $\int_L z^{-2} dz$, unde L este segmentul de dreaptă ce uneşte punctele $z_1 = 1$, $z_2 = 2+2i$;

18. $\int_L (5z^3 + 4z + 3)dz$, unde L este segmentul de parabolă $y = 2x^2$, ce uneşte punctele $z_1 = 0$, $z_2 = 1+2i$;

19. $\int_L z \cdot \bar{z} dz$, unde $\bar{z} = x - iy$, iar L este circumferinţă $|z+1| = 2$.

20. $\int_0^{1-i} (z + i)e^{-z} dz$; 21. $\int_{-1}^{\frac{\pi}{i}i} \sin^2 z dz$; 22. $\int_{-1}^{\frac{\pi}{4}i} z \sin z \, dz$;

23. $\int_L (\cos iz + ze^{-z})dz$, unde L este arcul de circumferinţă $|z| = 2$, $\text{Im}z \geq 0$, $\text{Re}z \geq 0$;

24. $\int_L (\sin iz + z^2)dz$, unde L este linia poligonală, ce uneşte punctele $z_1 = 0$, $z_2 = 2$, $z_3 = 3+i$;

25. $\int_L (\cos z)^2 \sin z dz$, unde L este segmentul de dreaptă, ce uneşte punctele $z_1 = \frac{\pi}{4} + i$, $z_2 = \frac{\pi}{4} - i$;

26. $\int\limits_L \sin \overline{z} \cdot dz$, unde $\overline{z} = x - iy$ şi L este arcul de parabolă $y = x^2$, ce uneşte punctele $z_1 = 1+i$, $z_2 = -1+i$;

27. $\int\limits_{1-i}^{1+i} ze^z dz$; 28. $\int\limits_L \sin iz \cdot dz$; 29. $\int\limits_0^{\frac{\pi}{2}+i} \cos iz \cdot dz$;

30. $\int\limits_L z \operatorname{Im} z^2 dz$, unde conturul L este mulţimea punctelor $\operatorname{Re} z = 1$, $|\operatorname{Im} z| \le 1$

**Problema 4.** Să se dezvolte funcţia $f(z)$ în seria Laurent în vecinătatea punctului $z_0$ şi să se afle domeniul de convergenţă al acestei serii.

1. $f(z) = (z+i)^2 \cdot e^{\frac{1}{z}}$, $z_0 = 0$; 2. $f(z) = \dfrac{1}{(z-2)(z+3)}$, $z_0 = 0$;

3. $f(z) = \dfrac{1}{(z+2)}$, $z_0 = i$; 4. $f(z) = \dfrac{1}{z(1-z)}$, $z_0 = 1$;

5. $f(z) = \dfrac{1}{(z^2+1)}$, $z_0 = 0$; 6. $f(z) = \dfrac{1}{z^2+z}$, $z_0 = i$;

7. $f(z) = z^2 \cos\dfrac{1}{z-2}$, $z_0 = 2$; 8. $f(z) = \dfrac{1}{3z-4}$, $z_0 = 2i$;

9. $f(z) = ze^{\frac{1}{z-1}}$, $z_0 = 1$; 10. $f(z) = \dfrac{z^2-1}{z^2+1}$, $z_0 = i$;

11. $f(z) = \sin\dfrac{z}{1+z}$, $z_0 = -1$; 12. $f(z) = \dfrac{1}{z(z-5)}$, $z_0 = 2$;

13. $f(z) = \dfrac{z+i}{z^2}$, $z_0 = -1$; 14. $f(z) = \dfrac{1}{(z+1)^2(z+2)}$, $z_0 = -i$;

15. $f(z) = \dfrac{1}{(z^2+1)(z^2+2)}$, $z_0 = 0$;

16. $f(z) = z\sin\dfrac{z+1}{z}$, $z_0 = 0$; 17. $f(z) = \dfrac{1}{z(z+1)}$, $z_0 = i$;

18. $f(z) = \dfrac{z}{(z-1)(z+2)}$, $z_0 = 2$;

19. $f(z) = \dfrac{1}{z^2-4}$, $z_0 = 1$;

20. $f(z) = \dfrac{1}{z^2+4}$, $z_0 = 2i$;

21. $f(z) = \dfrac{\cos z}{z^2}$, $z_0 = 0$;

22. $f(z) = \dfrac{1}{3z+5}$, $z_0 = 1$;

23. $f(z) = \sin\dfrac{z+1}{z-1}$, $z_0 = 1$;

24. $f(z) = \dfrac{z}{1+z^2}$, $z_0 = -i$;

25. $f(z) = ze^{\frac{1}{1+z}}$, $z_0 = -1$;

26. $f(z) = \dfrac{1}{z^2+9}$, $z_0 = 0$;

27. $f(z) = \cos\dfrac{z-1}{z+i}$, $z_0 = -i$;

28. $f(z) = \dfrac{1}{z^2-z}$, $z_0 = 1$;

29. $f(z) = \dfrac{1}{(z-2)^2(z-1)}$, $z_0 = 2$;

30. $f(z) = \dfrac{\sin z}{(z-1)^2}$, $z_0 = 1$;

**Problema 5.** Să se aplice teoremele Cauchy la calcularea următoarelor integrale (orientarea conturului L se consideră pozitivă):

1. $\oint\limits_L \dfrac{\sin\pi(z+1)}{z^2-2z+2}dz$, unde L: $|z-1-i| = 2$;

2. $\oint\limits_L \dfrac{e^z}{z^2(z+1)}dz$, unde L: $|z| = 2$;

3. $\oint\limits_L \dfrac{e^z}{z^4-z^2-2}dz$, unde L: $|z+i| = 1$;

4. $\oint\limits_L \dfrac{\cos z}{z^2-4}dz$, unde L: $\dfrac{x^2}{9}+\dfrac{y^2}{4} = 1$;

5. $\oint\limits_L \dfrac{dz}{z^3+1}$, unde L: $x^2+y^2+2x = 0$;

6. $\oint\limits_L \dfrac{\sin\pi z}{(z^2-1)^2}dz$, unde L: $x^2+y^2-2x = 0$;

7. $\oint\limits_L \frac{dz}{(z-1)^n(z-2)}$, unde L: $|z| = \frac{3}{2}$, $n \in N$;

8. $\oint\limits_L z^3 \sin\frac{1}{z}dz$, unde L: $|z-i| = 2$;

9. $\oint\limits_L \frac{z-1}{z^2+z-2}dz$, unde L: $|z+2| = 4$;

10. $\oint\limits_L \frac{zdz}{z^3+8}$, unde L: $|z-1-i\sqrt{3}| = 2{,}5$;

11. $\oint\limits_L \frac{dz}{(z-1)^2(z^2+1)}$, unde L: $|z-1-i| = 2$;

12. $\oint\limits_L \frac{\sin z \cdot \sin(z+1)}{z^2-z}dz$, unde L: $|z| = 3$;

13. $\oint\limits_L \frac{dz}{(z^2+4)(z+4)}$, unde L: $|z-i| = 4$;

14. $\oint\limits_L \frac{\sin(z+\pi i)}{z(e^z+1)}dz$, unde L: $|z| = 4$;

15. $\oint\limits_L \frac{shz}{z^4-1}dz$, unde L: $|z-1| = 1{,}5$;

16. $\oint\limits_L \frac{ch(z+1)}{z^2+1}dz$, unde L: $x^2 + \frac{y^2}{4} = 1$;

17. $\oint\limits_L \frac{e^z \cos \pi z}{z^2-2z}dz$, unde L: $|z| = 3$;

18. $\oint\limits_L \frac{chzdz}{(z+1)^2(z-1)}$, unde L: $|z| = 2$;

19. $\oint_L \frac{zshz}{(z^2+1)^2}dz$, unde L: $|z+i| = 1{,}5$;

20. $\oint_L \frac{e^{iz}}{(z^2-1)^2}dz$, unde L: $|z-1| = 1$;

21. $\oint_L \frac{dz}{(z^2+2)^3(z^2-1)}$, unde L: $|z| = 1{,}5$;

22. $\oint_L ctg(\pi z)dz$, unde L: $|z+i| = \sqrt{2}$;

23. $\oint_L \sin\frac{1}{z}dz$, unde L: $|z-i| = 2$;

24. $\oint_L \left(\sin\frac{1}{z^2}+\frac{1}{z^3}e^{z^2}\right)dz$, unde L: $|z-1-i| = 2$;

25. $\oint_L (z-1)e^{\frac{1}{z}}dz$, unde L: $|z+1| = 2$;

26. $\oint_L \frac{e^{iz}}{(z-\pi)^3}dz$, unde L: $|z| = 4$;

27. $\oint_L \frac{ctg\pi z}{z^2-\frac{\pi^2}{4}}dz$, unde L: $|z-1| = 0{,}8$;

28. $\oint_L \frac{e^z ch z\, dz}{(z^2-9)^3(e^z+1)^2}$, unde L: $|z+i\cdot\frac{\pi}{2}| = 2$;

29. $\oint_L \frac{e^z}{z(z-1)^2}dz$, unde L: $\frac{x^2}{9}+\frac{y^2}{4} = 1$;

30. $\oint_L \frac{z\,dz}{\sin^2 z\cos z}$, unde L: $|z-1| = 2$.

## „Calculul operațional”

**Problema 1.** Să se restabilească originalul după imaginea lui:

1. $F(p)=\dfrac{1}{p^3+8}$;

2. $F(p)=\dfrac{p+2}{p^2-4p+7}$;

3. $F(p)=\dfrac{e^{-3p}}{(p+1)^2}$;

4. $F(p)=\dfrac{1}{(p^2-1)^2(p^2+2p+2)}$;

5. $F(p)=\dfrac{p}{(p^2+1)(p^2+2)}$;

6. $F(p)=\dfrac{1}{(p^2+2)^2}$;

7. $F(p)=\dfrac{p}{(p-1)(p^2+4)}$;

8. $F(p)=\dfrac{p^2}{p^4+4p^2+3}$;

9. $F(p)=\dfrac{1}{p^2(p^2+9)}$;

10. $F(p)=\dfrac{p}{p^3-1}$;

11. $F(p)=\dfrac{2p}{p^2-1}e^{-p}$;

12. $F(p)=\dfrac{p^3}{p^4-1}$;

13. $F(p)=\dfrac{p}{(p+1)(p+2)^2}$;

14. $F(p)=\dfrac{p+1}{(p-3)^3(p-1)^3}$;

15. $F(p)=\dfrac{p^2}{p^4+4p^2+3}$;

16. $F(p)=\dfrac{1}{(p-1)^2(p^2-9)}$;

17. $F(p)=\dfrac{p^2+2}{(p+3)(p^2-1)}$;

18. $F(p)=\dfrac{13p+1}{p(p-1)^2(p+2)}$;

19. $F(p)=\dfrac{p+4}{(p^3-1)}$;

20. $F(p)=\dfrac{1}{p^2}\sin\dfrac{1}{p}$;

21. $F(p) = \frac{e^p}{p^2} + \frac{5e^{-3p}}{p^2+2}$;  22. $F(p) = \frac{1}{p}\cos\frac{1}{p}$;

23. $F(p) = \frac{p^3+2p^2+3}{p^5+2p^4+p^3}$;  24. $F(p) = \frac{p}{(p^2-1)^2}$;

25. $F(p) = \frac{p^2+p+1}{p^3-3p^2+3p-1}$;  26. $F(p) = \frac{1}{p^2(p^2+1)}$;

27. $F(p) = \frac{p-3}{(p+1)^2(p^2-2p)}$;  28. $F(p) = \frac{p^2}{(p^2+1)^2}$;

29. $F(p) = \frac{1}{p^4-5p^2+6}$;  30. $F(p) = \frac{e^{-\frac{p}{3}}}{p(p^2+2)}$.

**Problema 2.** Să se afle soluția particulară a ecuației diferențiale cu condițiile inițiale indicate:

1. $x'''+x'' = \sin t$, $x(0) = 1, x'(0) = 1, x''(0) = 0$.
2. $x''+2x' = \cos t$, $x(0) = x'(0) = 0$.
3. $x'''-x''+x' = 4$, $x(0) = 1, x'(0) = 2, x''(0) = -2$.
4. $x''+2x' - 3x = e^{-x}$, $x(0) = 0, x'(0) = 1$.
5. $x'''+x'' = 1$, $x(0) = x'(0) = 0, x''(0) = 1$.
6. $x''-x' = t\, e^{t}$, $x(0) = 0, x'(0) = -1$.
7. $x'''-x' = \cos t$, $x(0) = 1, x'(0) = x''(0) = 0$.
8. $x''-2x'+3x = t - \sin t$, $x(0) = 1, x'(0) =0$.
9. $x''+9x = \cos 3t$, $x(0) = x'(0) = 0$.
10. $x''+3x'+x = 1+t+t^2$, $x(0) = 0, x'(0) = 2$.
11. $x''+2x'+x = \sin t$, $x(0) = x'(0) = 1$.
12. $x''-3x'-4x = 10\sin t$, $x(0) = 0, x'(0) =1$.
13. $x''+2x'+x = 9\, e^{2t}$, $x(0) = x'(0) = 0$.
14. $x''+16x = e^{-t}$, $x(0) = 0, x'(0) = 1$.
15. $x'''-3x''+3x'-x = 5(1+5t)$, $x(0) = x'(0) = x''(0) = 0$.
16. $x''-3x'+2x = 2(3+2t)$, $x(0) = -1, x'(0) = 1$.
17. $x'''+x'' = 2t^2$, $x(0) = -3,\ x'(0) = 1,\ x''(0) = 0$.

18. $x''-2x' = t\sin t$, $x(0) = 0, x'(0) = -1$.
19. $x'''-x' = 1-t$, $x(0) = 0,\ x'(0) = 1, x''(0) = -1$.
20. $x'''-x'' = t\,e^{t}$, $x(0) = 1, x'(0) = -1 = x''(0)$.
21. $x''+4x = \sin\frac{3}{2}t\cos\frac{t}{2}$, $x(0) = 1, x'(0) = 0$.

22. $x'''+3x' - 4x = 0$, $x(0) = x'(0) = 0, x''(0) = 2$.
23. $x''+2\alpha x'+(\alpha^2+\beta^2) = 0$, $x(0) = 0, x'(0) = 1$.
24. $x'''+x' = e^{2t}$, $x(0) = x''(0) = 0, x'(0) = 1$.
25. $x''-x'+x = t\,e^{t}$, $x(0) = 0, x'(0) = 2$.
26. $x'''+x = t^2e^{t}$, $x(0) = x'(0) = x''(0) = 0$.
27. $x''-x = t\cos 2t$, $x(0) = 1, x'(0) = 0$.
28. $x'''-2x''+x' = 4$, $x(0) = x'(0) = 1, x''(0) = 2$.
29. $x''+4x' = 2\cos t\cos 3t$, $x(0) = x'(0) = 1$.
30. $x'''+x' = t\,e^{t} + \sin t$, $x(0) = x'(0) = 0, x''(0) = 1$.

**Problema 3.** Să se rezolve sistemul de ecuații diferențiale liniare cu condițiile inițiale indicate:

1. $\begin{cases} x_t' = y - z, \\ y_t' = x + y, \\ z_t' = x + y, \end{cases}$ $x(0) = 1, y(0) = 2, z(0) = 3$.

2. $\begin{cases} x_t' + y_t' - y = e^{t}, \\ 2x_t' + y_t' - 2y = 2\cos t, \end{cases}$ $x(0) = 1, y(0) = -1$.

3. $\begin{cases} x_t' = -x + y + z, \\ y_t' = x - y + z, \\ z_t' = x + y - z, \end{cases}$ $x(0) = y(0) = 1, z(0) = -1$.

4. $\begin{cases} x_t' + y_t' = 0, \\ x_t' + 2y_t' + x = 0, \end{cases}$ $x(0) = 1, y(0) = -1$.

5. $\begin{cases} x_t' = -2x - 2y - 4z, \\ y_t' = -2x + y - 2z, \\ z_t' = x + y + 2z, \end{cases}$ $x(0) = y(0) = 1,\ z(0) = -1.$

6. $\begin{cases} x_t' + x + 2y = 2t, \\ -2x_t' + y_t' - y = e^t, \end{cases}$ $x(0) = y(0) = 1.$

7. $\begin{cases} x_t' + y - z = 0, \\ y_t' - z = 0, \\ x + z - z_t' = 0, \end{cases}$ $x(0) = 1,\ y(0) = 2,\ z(0) = 3.$

8. $\begin{cases} x_t' + y = 0, \\ y_t' - 2x - 2y = 0, \end{cases}$ $x(0) = 1,\ y(0) = -1.$

9. $\begin{cases} x_t' - x + 2y = 3, \\ 3x_t' + y_t' - 4x + 2y = 0, \end{cases}$ $x(0) = y(0) = 1.$

10. $\begin{cases} x_t' = y + z, \\ y_t' = x + y, \\ z_t' = x - z, \end{cases}$ $x(0) = y(0) = z(0) = 1.$

11. $\begin{cases} x_t' + 7x - 2y = 0, \\ y_t' + 2x - 5y = 0, \end{cases}$ $x(0) = 0,\ y(0) = 0.$

12. $\begin{cases} x_t' = x + z, \\ y_t' - x = 0, \\ z_t' = x + y - z, \end{cases}$ $x(0) = 1,\ y(0) = -1,\ z(0) = 2.$

13. $\begin{cases} x_t' = x - y, \\ y_t' = 2x + 2y, \end{cases}$ $x(0) = 1,\ y(0) = -1.$

14. $\begin{cases} x_t' = 2x - 2y + 4z, \\ y_t' = -2x + y + 2z, \\ z_t' = 3x + y + 2z, \end{cases}$ $x(0) = y(0) = z(0) = 0.$

15. $\begin{cases} x_t' - x - 2y = t, \\ 2x + 2y_t' - 3y = t, \end{cases}$ $x(0) = 3, y(0) = 2.$

16. $\begin{cases} x_t' + 2x + y = 1, \\ x_t' + 4y_t' + 3y = 0, \end{cases}$ $x(0) = y(0) = 0.$

17. $\begin{cases} x_t' + y = 2x + z, \\ y_t' = x + z, \\ z_t' + 2z = y - 3x, \end{cases}$ $x(0) = 0, y(0) = z(0) = 0.$

18. $\begin{cases} x_t' + y_t' + y = e^t, \\ 2x + y_t' + 2y = \sin t, \end{cases}$ $x(0) = y(0) = 0.$

19. $\begin{cases} x_t' + y + z = 0, \\ y_t' + x + z = 0, \\ z_t' + x - y = 0, \end{cases}$ $x(0) = -1, y(0) = 0, z(0) = 1.$

20. $\begin{cases} x + x_t' = y + e^t, \\ y + y_t' = x + e^t, \end{cases}$ $x(0) = 0, y(0) = 1.$

21. $\begin{cases} x_t' = y + z, \\ y_t' = 3x + z, \\ z_t' = 3y + x, \end{cases}$ $x(0) = y(0) = 0, z(0) = 1.$

22. $\begin{cases} x_t' - 2x + 2y = 1 + 2t, \\ y_t' + 2x + y = 0, \end{cases}$ $x(0) = y(0) = 0.$

23. $\begin{cases} x_t' = 2x + y + z, \\ y_t' = x - z, \\ z_t' = 3x - y + 2z, \end{cases}$ $x(0) = y(0) = 1, z(0) = 0.$

24. $\begin{cases} x_t' = 3y + x, \\ y_t' = x + y + e^t, \end{cases}$ $x(0) = 0, y(0) = 1.$

25. $\begin{cases} x_t' = x - y + z + t, \\ y_t' = x + y - z + t^2, \\ z_t' = x + y + z, \end{cases}$ $x(1) = y(1) = z(1) = 0.$

26. $\begin{cases} x_t' + y_t' + x = e^t, \\ y_t' + 2x + y = 1, \end{cases}$ $x(0) = 1, y(0) = 0.$

27. $\begin{cases} 3x_t' = 2x + y - z, \\ 2y_t' = x + 2y + z, \\ 6z_t' = x - y - z, \end{cases}$ $x(1) = y(1) = z(1) = 1.$

28. $\begin{cases} x_t' = 3y - x + 1, \\ y_t' = x + y + e^t, \end{cases}$ $x(0) = 1, y(0) = -1.$

29. $\begin{cases} x_t' = y - z + 1, \\ y_t' = 2x + y + 2z, \\ z_t' = x + y + z + 4, \end{cases}$ $x(0) = y(0) = z(0) = 0.$

30. $\begin{cases} 3x_t' + 2x + y_t' = 1, \\ x_t' + 4y_t' + 5y = 0, \end{cases}$ $x(0) = 1, y(0) = -1.$