An EMPIRICAL FORMULA for the measure of OPTICAL ILLUSION

Objectives-

1) To find out the parameters related in creating eye illusion
2) To derive a mathematical formula for deriving the measure of eye illusion
3) To trace out relation between individual parameters
4) To develop strategies for future constructions in creation of eye illusion

Introduction-

Consider two congruent sectors of two concentric circles. Name them as alpha (α) and beta (β). Let the centre most point of the large arc on α represent α, and its corresponding point on β represent β. Let α be stationary. Draw three mutually perpendicular lines arising from α, making an imaginary co-ordinate system (refer figure)

Variables-

$\bar{d}$ is the displacement vector whose initial point is at α and terminal point is at β (vector from the stationary object to the mobile object).

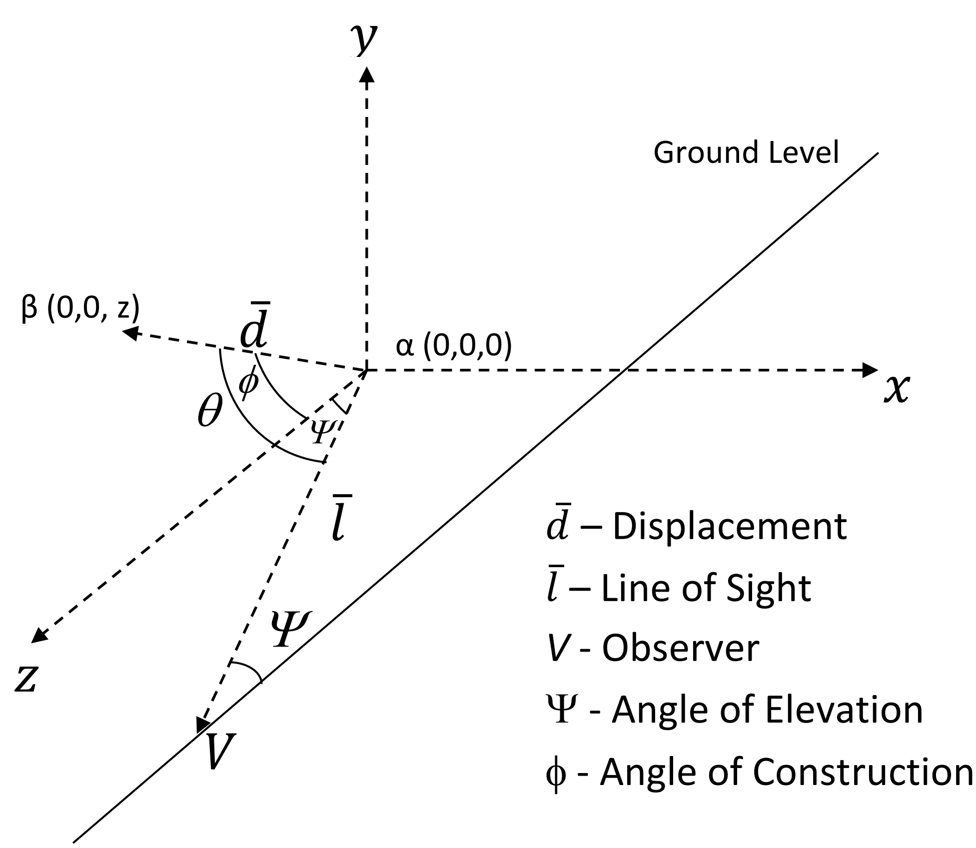
$\bar{l}$ is the line of sight vector whose initial point is at α and terminal point is at the eye of the observer (vector from the stationary object to the viewer /observer).

θ is the angle between $\bar{d}$ and $\bar{l}$, i.e., angle between displacement vector and line of sight vector.

Ψ is the angle made by $\bar{l}$ with positive z-axis, i.e., angle of elevation.

Φ is the angle made by β with positive z-axis, i.e., angle of construction.

p is the range of the region in which the desired illusion is possible. It is applicable only for constructions.



Hypothesis-

Null Hypothesis

H_0- Architectural Illusion is undefined through measure

Alternate Hypothesis

H_1- Illusion of having a decrease in the size of the main building as the observer nears it, can be created by following certain construction strategies.

H_2- The architectural illusion created, can be measured through a specific relation between the variables.

H_3- Measuring the degree of illusion of having one object relatively larger, among two congruent objects (Note: Such an illusion is possible only with certain shaped objects).

Experimental Study-

Congruent sectors of two concentric circles, alpha and beta, are placed in several orientations taking care that in all cases the smaller arcs of both the pieces face the same direction. The observations as to which of the two congruent pieces is appearing larger in different orientations are as follows.

Observation-

Whenever β is placed on the imaginary z-axis with x and y co-ordinates as 0 each, no illusion is created and both are appearing as of the same size and congruent pieces. Whenever β is placed over α, an illusion is created and β is appearing smaller than the other. Whenever β is placed under α, once again an illusion is created and β is appearing larger than the other.

When seen from different angles at the same orientation, though the illusion remains same, difference in degree of illusion is observed. This helps us infer that a person's line of sight has a relation with the degree of illusion created.

Of the easily conceivable contributors towards the degree of illusion, is the observer's line of sight, which is larger when seen from a greater distance and smaller when close by. This plays a major role in determining the measure of illusion as this is the only factor which keeps changing from situation to situation, while $\bar{d}$, Φ, θ and Ψ remain constant in all cases for a specific orientation / construction.

Analysis-

One of the prime objectives of this experiment is to formulate an expression ***Q*** for the degree and nature of illusion, whose magnitude gives the measure of illusion and sign determines which of the two appears larger.

$$Q = [|\bar{d}| + |(\bar{d} . \bar{l})|] \sin\Phi$$

$$Q = [|\bar{d}| + |(|\bar{d}| . |\bar{l}| . \cos\theta)|] \sin\Phi$$

Cases:

i) If Q > 0, then the stationary object/main construction will appear larger.

ii) If Q = 0, then both the objects/constructions appear to be of the same size.

iii) If Q <0, then the mobile object will appear larger. It is not practically possible in constructions as we get a negative value only when the arch is at a lower height than the main construction.

The expression for Q ($\bar{d}$, $\bar{l}$, ϕ, θ) satisfies the following bi-linear properties:

i) Q ($\bar{d}_1$+$\bar{d}_2$, $\bar{l}$, ϕ, θ) = Q($\bar{d}_1$, $\bar{l}$, ϕ, θ) + Q($\bar{d}_2$, l, ϕ, θ)

ii) Q (λ$\bar{d}$., $\bar{l}$, ϕ, θ) = λ Q ($\bar{d}$, $\bar{l}$, ϕ, θ), where λ > 0

iii) Q (λ$\bar{d}$, λ$\bar{l}$, ϕ, θ) = λ Q ($\bar{d}$, λ$\bar{l}$, ϕ, θ), where λ > 0

Relation between the individual parameters-

i) Distance between objects ($\bar{d}$) is inversely proportional to range of region in which the illusion is possible (p), when Φ is constant.

ii) Angle between two objects/ constructions (Φ) is directly proportional to range of region in which the illusion is possible (p), when $\bar{d}$ is constant.

iii) Angle between two objects/ constructions (Φ) is directly proportional to distance between objects ($\bar{d}$) when p is constant.

Diagrammatically the relation between the individual parameters has been proved (refer figures).

Construction Strategies-

1) The tip of the arch should not be at the same level or lower than the tip of the main construction.
2) The displacement vector between the arch and the main construction should be maintained as low as possible in order to attain a wider range of illusion.
3) The angle between the main construction and the arch should neither be very low nor very high.

Conclusion-

By the above derived empirical formula, creating optical illusion which is an attention captivating factor, can be achieved successfully in future constructions.

Future Enhancements-

The following concept can be further carried forward onto calculating the degree of anaglyph (3d) created. Depending on person’s maximum anaglyph degree, the effect can be modulated, so as to avoid head-aches and strain of eyes.