

## Project: Kinematics Pick & Place

**Writeup Template:** You can use this file as a template for your writeup if you want to submit it as a markdown file, but feel free to use some other method and submit a pdf if you prefer.

---

### Steps to complete the project:

1. Set up your ROS Workspace.
2. Download or clone the project repository into the *src* directory of your ROS Workspace.
3. Experiment with the forward\_kinematics environment and get familiar with the robot.
4. Launch in demo mode.
5. Perform Kinematic Analysis for the robot following the project rubric.
6. Fill in the IK\_server.py with your Inverse Kinematics code.

### Rubric Points

Here I will consider the rubric points individually and describe how I addressed each point in my implementation.

---

### Writeup / README

1. Provide a Writeup / README that includes all the rubric points and how you addressed each one. You can submit your writeup as markdown or pdf.

You're reading it!

### Kinematic Analysis

1. Run the forward\_kinematics demo and evaluate the kr210.urdf.xacro file to perform kinematic analysis of Kuka KR210 robot and derive its DH parameters.

We follow the rules for generating the DH parameters. The urdf indicates joint1 has a z offset of 0.33 and joint2 has a z offset of 0.42 and an x offset of 0.35. We move the joint1 origin to the accumulated z to provide a d1 of 0.75. We then twist the joint -90deg and translate 0.35 along the x axis resulting in

$\alpha_1 = -\pi/2$  and  $a_1 = 0.35$ ,  $d_2 = 0$ . We translate along the z axis 1.25 to reach joint3 with no rotation. Joints 4,5,6 are will have a coincident origin as they form the spherical wrist. We rotate  $-\pi/2$  to orient in the joint4 configuration and apply a translation of  $0.96 + 0.54$  for  $d_4$  and  $a_3$  has an offset of  $-0.054$ . A twist of  $+\pi/2$  moves from joint4 to joint5. A final twist of  $-\pi/2$  moves from joint5 to joint6. Finally, the end effector has  $d_7$  of  $0.193 + 0.11$  or  $0.303$  from the urdf.

**2. Using the DH parameter table you derived earlier, create individual transformation matrices about each joint. In addition, also generate a generalized homogeneous transform between base\_link and gripper\_link using only end-effector(gripper) pose.**

Links	$\alpha(i-1)$	$a(i-1)$	$d(i-1)$	$\theta(i)$
0->1	0	0	0.75	0
1->2	$-\pi/2$	0.35	0	$-\pi/2 + q_2$
2->3	0	1.25	0	0
3->4	$-\pi/2$	-0.054	1.50	0
4->5	$\pi/2$	0	0	0
5->6	$-\pi/2$	0	0	0
6->EE	0	0	0.303	0

The generic transform looks like

$$T_{i-1,i} = \begin{pmatrix} \cos(q_i) & -\sin(q_i) & 0 & a_{i-1} \\ \sin(q_i)\cos(\alpha_{i-1}) & \cos(q_i)\cos(\alpha_{i-1}) & -\sin(\alpha_{i-1}) & -\sin(\alpha_{i-1})d_i \\ \sin(q_i)\sin(\alpha_{i-1}) & \cos(q_i)\sin(\alpha_{i-1}) & \cos(\alpha_{i-1}) & \cos(\alpha_{i-1})d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For the  $T_{0,1}$  matrix we substitute  $\alpha_0 = 0$ ,  $a_0 = 0$ ,  $d_1 = 0.75$  noting that  $\cos(0) = 1$  and  $\sin(0) = 0$  yields

$$T_{0,1} = \begin{pmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0.75 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For the  $T_{1,2}$  matrix we substitute  $\alpha_1 = -\pi/2$ ,  $a_1 = 0.35$ ,  $d_2 = 0$  noting that  $\cos(-\pi/2) = 0$  and  $\sin(-\pi/2) = -1$  yields

$$T_{1,2} = \begin{pmatrix} \cos(q_2) & -\sin(q_2) & 0 & 0.35 \\ 0 & 0 & 1 & 0 \\ -\sin(q_2) & -\cos(q_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For the T2,3 matrix we substitute  $\alpha_2=0$ ,  $a_2=1.25$ ,  $d_3=0$  noting that  $\cos(0) = 1$  and  $\sin(0) = 0$  yields

$$T_{2,3} = \begin{pmatrix} \cos(q_3) & -\sin(q_3) & 0 & 1.25 \\ \sin(q_3) & \cos(q_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For the T3,4 matrix we substitute  $\alpha_3=-\pi/2$ ,  $a_3=-0.054$ ,  $d_4=1.50$  noting that  $\cos(-\pi/2) = 0$  and  $\sin(-\pi/2) = -1$  yields

$$T_{3,4} = \begin{pmatrix} \cos(q_4) & -\sin(q_4) & 0 & -0.054 \\ 0 & 0 & 1 & 1.50 \\ -\sin(q_4) & -\cos(q_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For the T4,5 matrix we substitute  $\alpha_4=\pi/2$ ,  $a_4=0$ ,  $d_5=0$  noting that  $\cos(\pi/2) = 0$  and  $\sin(\pi/2) = 1$  yields

$$T_{4,5} = \begin{pmatrix} \cos(q_5) & -\sin(q_5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(q_5) & \cos(q_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For the T5,6 matrix we substitute  $\alpha_5=-\pi/2$ ,  $a_5=0$ ,  $d_6=0$  noting that  $\cos(-\pi/2) = 0$  and  $\sin(-\pi/2) = -1$  yields

$$T_{5,6} = \begin{pmatrix} \cos(q_6) & -\sin(q_6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(q_6) & -\cos(q_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

For the T6,G matrix we substitute  $\alpha_6 = 0$ ,  $a_6=0$ ,  $d_7=0.303$  noting that  $\cos(0) = 1$  and  $\sin(0) = 0$  yields

$$T_{6,G} = \begin{pmatrix} \cos(q_7) & -\sin(q_7) & 0 & 0 \\ \sin(q_7) & \cos(q_7) & 0 & 0 \\ 0 & 0 & 1 & 0.303 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transform from the base link to the end effector is then

$$T_{0,G} = T_{0,1}T_{1,2}T_{2,3}T_{3,4}T_{4,5}T_{5,6}T_{6,G}$$

The input into the system is the roll, pitch, and yaw of the end effector. We will transform the rotations about the end effector into the total rotation required from the origin of our robot. Here is the coordinate frame of our system:

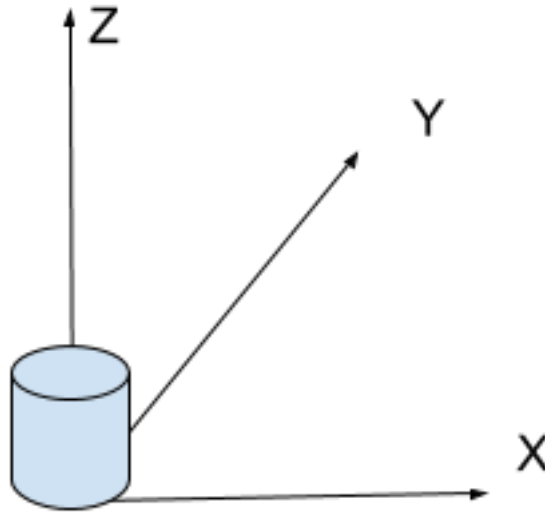


Figure 1: alt text

and here is the coordinate frame of the end effector:

To transform the orientation of our world frame into that of the input end effector rotations, we need to rotate 180 degrees about the z axis to get the coordinate frames equivalent. Additionally, a rotation of -90 degrees about the y axis is required from the base revolute joint such that the normal is in the direction of the end effector frame. We call this matrix

$$R_{corr}$$

.

We define a sequence of extrinsic rotations in order to calculate the end effector rotation.

$$R_{EE} = R_z R_y R_x R_{corr}$$

The rotations about

$$R_z, R_y, \text{ and } R_x$$

are by the book and are:

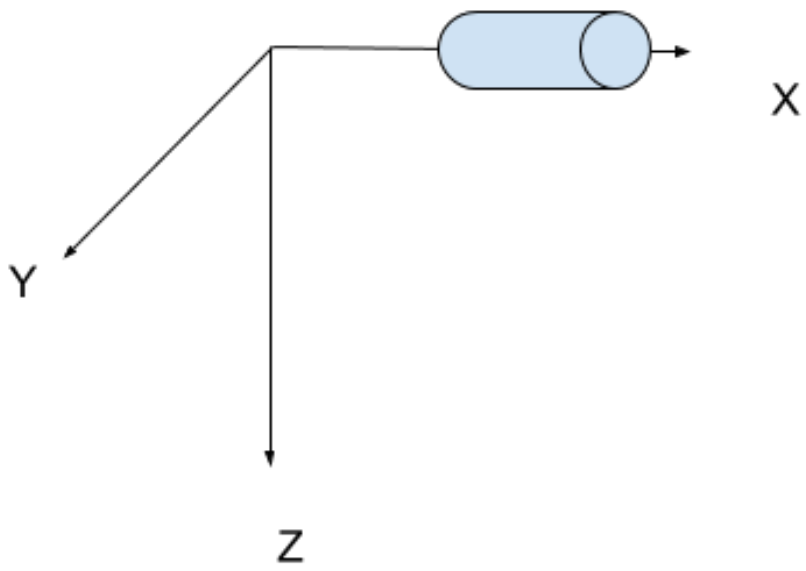


Figure 2: alt text

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(r) & -\sin(r) \\ 0 & \sin(r) & \cos(r) \end{pmatrix}$$

where  $r$  is the input roll.

$$R_y = \begin{pmatrix} \cos(p) & 0 & \sin(p) \\ 0 & 1 & 0 \\ -\sin(p) & 0 & \cos(p) \end{pmatrix}$$

where  $p$  is the input pitch.

$$R_z = \begin{pmatrix} \cos(y) & -\sin(y) & 0 \\ \sin(y) & \cos(y) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where  $y$  is the input yaw.

$$R_{corr} = R_{z,y=180deg} R_{y,p=-90deg}$$

$$R_{z,y=180deg} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{y,p=-90deg} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_{corr} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

at this point we have

$$R_{EE}$$

in the world coordinates and the end effector position is also part of the input. Given the rotation of the end effector and the length of the end effector, we can now define the coordinates of the wrist center (WC). Lesson 15 defines the wrist center equations as

$$wc_x = p_x - (d_6 + l)n_x$$

$$wc_y = p_y - (d_6 + l)n_y$$

$$wc_z = p_z - (d_6 + l)n_z$$

where the  $\mathbf{n}$  vector may be extracted from the just calculated

$$R_{EE}$$

to give the resulting **WC** position vector.

**3. Decouple Inverse Kinematics problem into Inverse Position Kinematics and inverse Orientation Kinematics; doing so derive the equations to calculate all individual joint angles.**

We solve the inverse kinematics problem by considering the first three joints as those that resolve the place the wrist center in the correct position and the last three joints are those that specify the orientation of the end effector. We will start by solving for  $\theta_1$ , which happens to be the only joint to provide any rotation about the z axis. We start by taking a top-down view of the robot in order to calculate the required rotation about z in order to calculate  $\theta_1$ .

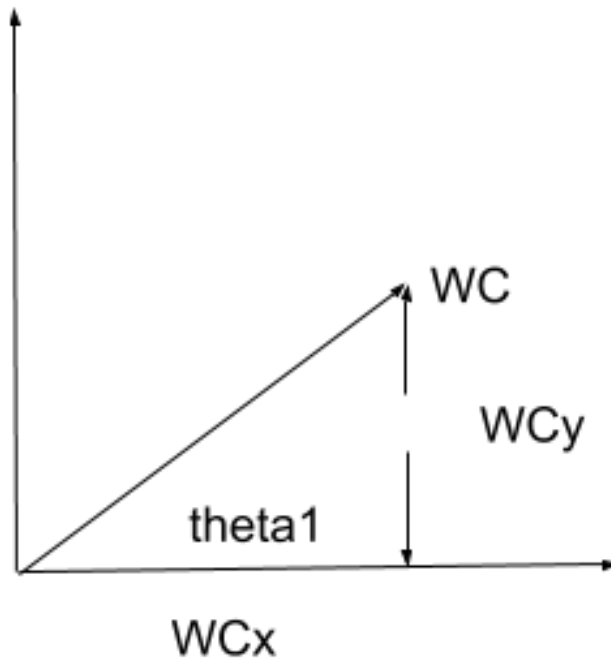


Figure 3: alt text

From here it is easy to solve for  $\theta_1$ .

$$\theta_1 = \tan^{-1}\left(\frac{WC_y}{WC_x}\right)$$

Next up, we move to  $\theta_2$  and we will start with a side-view.

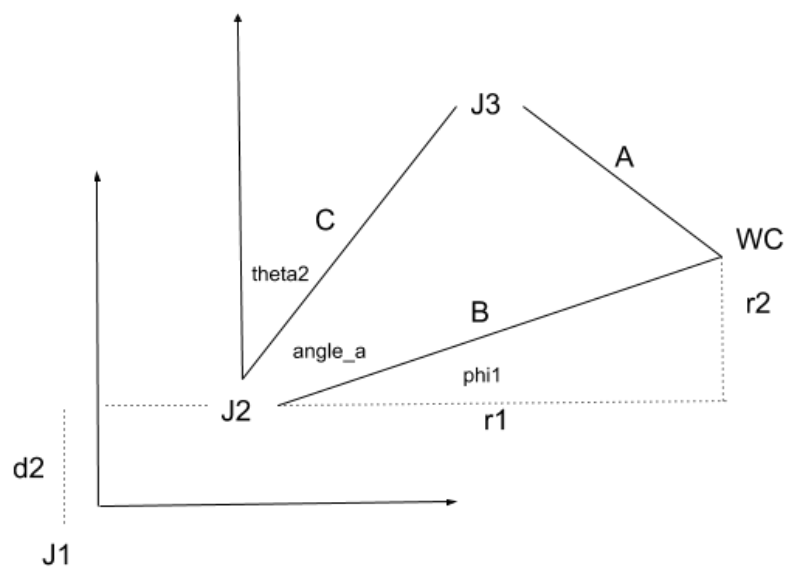


Figure 4: alt text



From this drawing we can see that

$$\theta_2 = \pi/2 - \phi_1 - \alpha$$

Furthermore we can see that

$$\phi_1 = \tan^{-1}\left(\frac{r_2}{r_1}\right)$$

and from the law of cosines

$$\alpha = \cos^{-1}\left(\frac{B^2 + C^2 - A^2}{2BC}\right)$$

Some of these components are immediately solvable:

$$C = \text{length}_{J2toJ3}$$

and extracted from the DH table is

$$C = 1.25$$

$$\begin{aligned} A &= \text{length}_{J3toWC} \\ A &= \sqrt{(-0.054)^2 + 1.50^2} \\ A &= 1.501 \end{aligned}$$

$$\begin{aligned} r_2 &= WC_z - d_1 \\ r_2 &= WC_z - 0.75 \end{aligned}$$

to solve for B and r1 requires we return to the top view.

In this coordinate frame we see that r1 is now the remaining distance to the wrist center from joint1 once the length of a1 has been subtracted.

$$\begin{aligned} r_1 &= \sqrt{WC_y^2 + WC_x^2} - a_1 \\ B &= \sqrt{r_1^2 + r_2^2} \end{aligned}$$

and now all required elements to solve for  $\theta_2$  have been resolved.

We advance to joint3 to solve for  $\theta_3$  in a similar manner.

From this drawing we can see that

$$\theta_2 = \frac{\pi}{2} - \phi_3$$

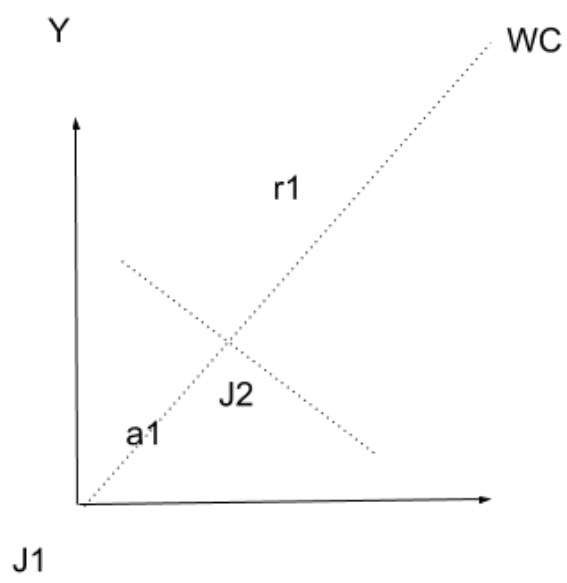


Figure 5: alt text

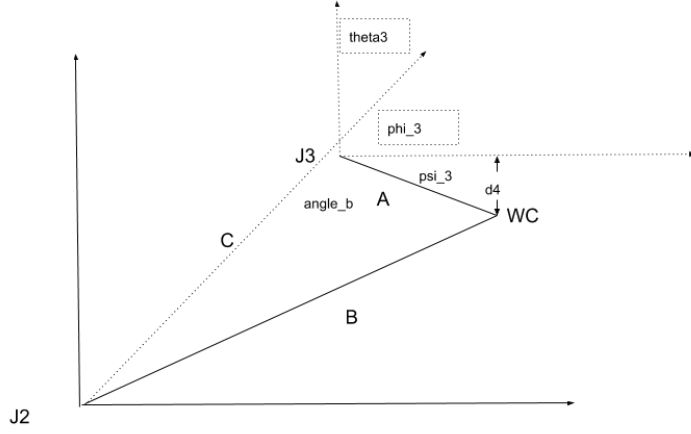


Figure 6: alt text

meanwhile we can see that

$$pi = phi_3 + psi_3 + angle_b$$

or

$$phi_3 = pi - psi_3 - angle_b$$

and

$$psi_3 = \sin^{-1}\left(\frac{d4}{A}\right)$$

where d4 is from the table and A was previously resolved.

$$psi_3 = \sin^{-1}\left(\frac{0.054}{1.501}\right)$$

resulting in

$$psi_3 = 0.36$$

We can solve angle\_b from the law of cosines

$$angle_b = \cos^{-1}\left(\frac{A^2 + C^2 - B^2}{2AC}\right)$$

and all unknowns are now resolved for theta3. However, the sign of theta3 needs to be inverted to reflect the negative rotation. This completes the inverse kinematics code for position determination.

We now move to the inverse kinematics of the orientation of the spherical wrist. First, we already have the total orientation required and we know the orientation applied to the end effector by the first three joints. So, we will first solve for the required orientation applied by the spherical wrist alone.

$$R_{3,6} = R_{0,3}^{-1} R_{EE}$$

where

$$R_{0,3} = T_{0,1;q1=theta1} T_{1,2;q2=theta2} T_{2,3;q3=theta3}$$

Here we just applied the homogenous transforms using the solved rotational angles for the first three joints to find the total orientation contribution of the first three joints. Therefore, we have also found the total orientation contribution required by the last three joints and the problem is to now solve for the angles on each of the last three joints.

The transform from joint3 to the wrist center is:

$$T_{3,6} = T_{3,4} T_{4,5} T_{5,6}$$

to make the matrix multiplications less daunting we will write “c4” to mean  $\cos(\theta_4)$  and “s4” to mean  $\sin(\theta_4)$ .

$$\begin{aligned} T_{3,6} &= T_{3,5} T_{3,6} \\ T_{3,5} &= T_{3,4} T_{4,5} \\ T_{3,5} &= \begin{pmatrix} c4c5 & -c4s5 & s4 \\ s5 & c5 & 0 \\ -s4c5 & s4s5 & c4 \end{pmatrix} \\ T_{3,6} &= \begin{pmatrix} c4c5c6 - s4s6 & -c4c5s6 - s4c6 & -c4s5 \\ s5c6 & -s5s6 & c5 \\ -s4c5s6 - c4s6 & s4c5s6 - c4c6 & s4s5 \end{pmatrix} \end{aligned}$$

$\theta_5$  could be easily plucked out of the  $T_{3,6}[1,2]$  entry but the guidance is to use the tangent allowing for the specification of quadrants. Observe:

$$(-c4s5)^2 = R_{3,6}[0, 2]^2$$

$$(-s4s5)^2 = R_{3,6}[2, 2]^2$$

then

$$c4^2 s5^2 + s4^2 s5^2 = R_{3,6}[0, 2]^2 + R_{3,6}[2, 2]^2$$

$$s5^2(c4^2 + s4^2) = R_{3,6}[0, 2]^2 + R_{3,6}[2, 2]^2$$

$$s5 = \sqrt{R_{3,6}[0, 2]^2 + R_{3,6}[2, 2]^2}$$

leading to

$$\theta_5 = \tan^{-1}\left(\frac{\sqrt{R_{3,6}[0, 2]^2 + R_{3,6}[2, 2]^2}}{R_{3,6}[1, 2]}\right)$$

theta4 and theta6 are easier to resolve as

$$\theta_4 = \tan^{-1}\left(\frac{R_{3,6}[2, 2]}{-R_{3,6}[0, 2]}\right)$$

$$\theta_6 = \tan^{-1}\left(\frac{-R_{3,6}[1, 1]}{R_{3,6}[1, 0]}\right)$$

and all angles have now been resolved. The inverse kinematics problem is complete.

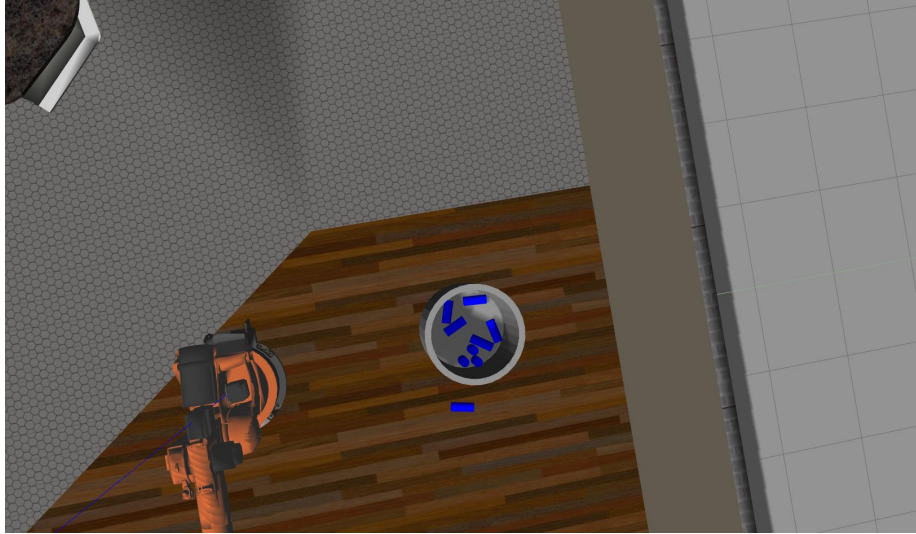
### Project Implementation

1. Fill in the `IK_server.py` file with properly commented python code for calculating Inverse Kinematics based on previously performed Kinematic Analysis. Your code must guide the robot to successfully complete 8/10 pick and place cycles. Briefly discuss the code you implemented and your results.

The result of this implementation was 8/10 successes. These results can be seen in



the following two images.



The first image shows a target object that has fallen on its side and the second image shows one target object not in the bin. The reason behind the second failure is that the robot did not account for the length of the target while orienting into the bin. The result was that the target hit the side of the bin during one of the maneuvers moving the bin and the target fell to the ground. The first failure was a similar result. The arm hit the target during orientation - before the grasping stage. Both of these could be accounted for but are difficult to consider in this scenario where we are given the required pose but not the contextual information such as “am I already holding the target”.