Multiplying Frequency Spectra
Amplitude and Phase

Before explaining why we do these operations, I want to explain a processing sequence. I want to make sure you understand the complex number operations before going into the Fourier theory.

We often have two time sequences, a_n and b_n. These can be Fourier transformed to get A_f and B_f:

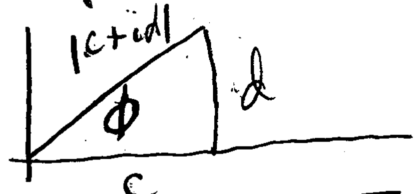
$$A_f = \sum_{n=0}^{N-1} a_n e^{-2\pi f n/N}$$

$$B_f = \sum_{n=0}^{N-1} b_n e^{-2\pi f n/N}$$

We multiply the two spectra to get a third P_f, the Product:

$$P_f = A_f B_f \quad \text{for each } f$$

Complex numbers are usually represented as the real and imaginary parts (eg $c+id$) but sometimes we want to represent them as amplitude and phase.



$|c+id| = \sqrt{c^2+d^2}$ is called the magnitude or amplitude or absolute value of $a+id$

$\phi(c+id) = \arctan d/c$ is called the phase of $c+id$

You can represent any complex number with its amplitude and phase

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You can compute the product P_f using the standard form

$$P_f = \{ Re(A_f) + i Im(A_f) \} \{ Re(B_f) + i\, Im(B_f) \}$$

$$= Re(A_f) Re(B_f) - Im(A_f) Im(A_f) + i \left(Re(A_f) Im(B_f) + Im(A_f) Im(B_f) \right)$$

Or you can work in the polar form

$$|P_f| e^{i\phi(P_f)} = |A_f| \, e^{i\phi(A_f)} \, |B_f| e^{i\phi(B_f)}$$

$$= |A_f||B_f| \, e^{i(\phi(A_f) + \phi(B_f))}$$

We say:

"The amplitude of the product is the product of the amplitudes.
The phase of the product is the sum of the phases"