Connecting the discrete Fourier transform to the Fourier series

The discrete Fourier transform represents a sequence x_n as the sum of complex exponentials

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{i2\pi kn/N} \quad ①$$

The complex coeficients can be computed:

$$X_k = \sum_{k=0}^{N-1} x_n e^{-2\pi kn/N} \quad ②$$

Equation ② is called the forward transform and ① is the inverse transform.
Euler's formula is a fundamental relationship between trigonometric functions and the complex exponential function:

$$e^{ix} = \cos x + i \sin x. \quad ③$$

Use equation ③ to remove the complex exponential in equation ① and replace X_n with $(A_n + iB_n)$:

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} (A_k + iB_k)\left(\cos(2\pi kn/N) + i\sin(2\pi kn/N)\right)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} \left[A_k \cos(2\pi kn/N) - B_k \sin(2\pi kn/N) + i A_k \sin(2\pi kn/N) + iB_k \cos(2\pi kn/N)\right] \quad ④$$

Khan's video on the Fouries series was about a real sequence. Focus attention on the real part of x_n:

$$Re(x_n) = \frac{1}{N} \sum_{k=0}^{N-1} A_k \cos(k\, 2\pi n/N) - B_k \sin(k\, 2\pi n/N)$$

Substitute $2\pi n/N = t$

$$Re(x_n) = \frac{1}{N} \sum_{k=0}^{N-1} A_k \cos(kt) - B_k \sin(kt)$$

This now looks like the Fourier series equation:

$$f(t) = \sum_{k=0}^{\infty} a_k \cos kt + b_k \sin(kt)$$ provided a_k and b_k are zero for $k > N-1$.