# Applied model specification

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> ⚠ Under construction
>
> Still working on this!

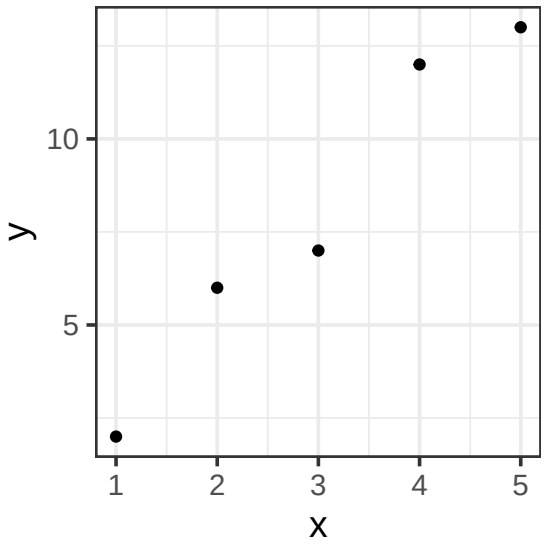
Now that we've covered the terminology and concepts, let's apply model specification to some real models.

```
library(tidyverse)
library(mosaic)
```

## 1 "Toy" data

Let's start with the simplest possible example, a dataset with two data points. Suppose you record how many days you study over the next two days. On day 1, you study for 2 hours. On day 2, you study for 3 hours. Your dataset might look something like this.

## 2 Plot




## 3 Data

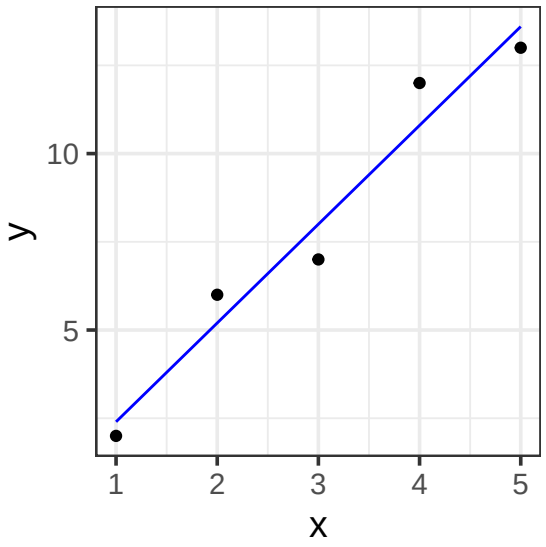
```
function (object, ...)
{
    UseMethod("model")
}
<bytecode: 0x149be8be0>
<environment: namespace:mosaic>
```

## 4 Code

```
toy_data <- tibble(
  x = c(1, 2, 3, 4, 5),
  y = c(2, 6, 7, 12, 13)
)

toy_data %>%
  ggplot(aes(x = x, y = y)) +
  geom_point() +
  theme_bw(base_size = 14)
```

1. **Specify our response variable**, $y$: the response variable ( data, output, prediction) is the variable you are trying to predict or explain with your model.
    - `y`
2. **Specify explantory variables**, $x_i$: the explanatory variables ( regressors, inputs, predictors) are the predictors in your data that could help explain the response variable. Our data has only one possible:
    - `x`
3. **Specify the functional form**: the functional form describes the relationship between the response and explanatory variables with a mathematical expresson. In a linear model, we express this relationship as a weighted sum of inputs:
    - $y = \sum_{i=1}^{n} w_i x_i$
4. **Specify model terms**: here we need to specify exactly *how* to express our explanatory variables in our functional form. The actual variables and constants that will be included in the model. There are four kinds of terms: (1) intercept, (2) main, (3) interaction, and (4) transformation. Here we have the simplest case of an intercept and one main term (no interactions or transformations necessary)
    - $y = w_1\mathbf{1} + w_2 x_2$
    - in R: `y ~ 1 + x`

## 5 Plot




Model specification: $y = w_1\mathbf{1} + w_2\mathbf{x}$

```
Call:
lm(formula = y ~ 1 + x, data = toy_data)

Coefficients:
(Intercept)            x
       -0.4          2.8
```

Fitted model: $y = (-0.4)1 + (2.8)x$

## 6 Data


```
Call:
lm(formula = y ~ 1 + x, data = toy_data)

Coefficients:
(Intercept)            x
       -0.4          2.8
```

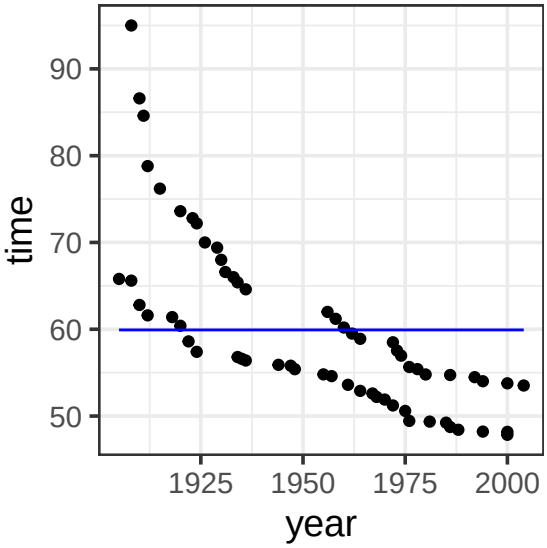

## 7 Code

```
model <- lm(y ~ 1 + x, data = toy_data)

toy_data <- toy_data %>%
  mutate(with_formula = -0.4*1 + 2.8*x) %>%
  mutate(with_predict= predict(model, toy_data))

toy_data %>%
  ggplot(aes(x = x, y = y)) +
  geom_point() +
  geom_line(aes(y = with_predict), color = "blue") +
  theme_bw(base_size = 14)
```

# 8 Swim records

## 8.1 One input

# 9 Plot




Model specification: $y = w_1\mathbf{1}$

```
Call:
lm(formula = time ~ 1, data = SwimRecords)

Coefficients:
(Intercept)
      59.92
```

Fitted model: $y = (59.92)1$

# 10 Data

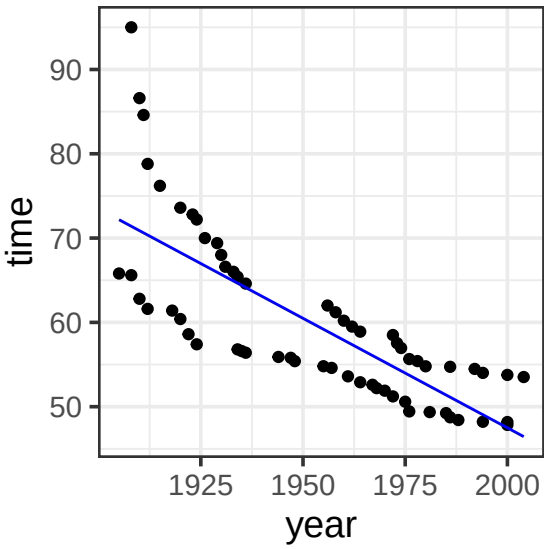
| year | time | sex | with_formula | with_predict |
|---|---|---|---|---|
| 1905 | 65.80 | M | 59.92 | 59.92419 |
| 1908 | 65.60 | M | 59.92 | 59.92419 |
| 1910 | 62.80 | M | 59.92 | 59.92419 |
| 1912 | 61.60 | M | 59.92 | 59.92419 |
| 1918 | 61.40 | M | 59.92 | 59.92419 |
| 1920 | 60.40 | M | 59.92 | 59.92419 |
| 1922 | 58.60 | M | 59.92 | 59.92419 |
| 1924 | 57.40 | M | 59.92 | 59.92419 |
| 1934 | 56.80 | M | 59.92 | 59.92419 |
| 1935 | 56.60 | M | 59.92 | 59.92419 |
| 1936 | 56.40 | M | 59.92 | 59.92419 |
| 1944 | 55.90 | M | 59.92 | 59.92419 |
| 1947 | 55.80 | M | 59.92 | 59.92419 |
| 1948 | 55.40 | M | 59.92 | 59.92419 |
| 1955 | 54.80 | M | 59.92 | 59.92419 |
| 1957 | 54.60 | M | 59.92 | 59.92419 |
| 1961 | 53.60 | M | 59.92 | 59.92419 |
| 1964 | 52.90 | M | 59.92 | 59.92419 |
| 1967 | 52.60 | M | 59.92 | 59.92419 |
| 1968 | 52.20 | M | 59.92 | 59.92419 |
| 1970 | 51.90 | M | 59.92 | 59.92419 |
| 1972 | 51.22 | M | 59.92 | 59.92419 |
| 1975 | 50.59 | M | 59.92 | 59.92419 |
| 1976 | 49.44 | M | 59.92 | 59.92419 |
| 1981 | 49.36 | M | 59.92 | 59.92419 |
| 1985 | 49.24 | M | 59.92 | 59.92419 |
| 1986 | 48.74 | M | 59.92 | 59.92419 |
| 1988 | 48.42 | M | 59.92 | 59.92419 |
| 1994 | 48.21 | M | 59.92 | 59.92419 |
| 2000 | 48.18 | M | 59.92 | 59.92419 |
| 2000 | 47.84 | M | 59.92 | 59.92419 |
| 1908 | 95.00 | F | 59.92 | 59.92419 |
| 1910 | 86.60 | F | 59.92 | 59.92419 |
| 1911 | 84.60 | F | 59.92 | 59.92419 |
| 1912 | 78.80 | F | 59.92 | 59.92419 |
| 1915 | 76.20 | F | 59.92 | 59.92419 |
| 1920 | 73.60 | F | 59.92 | 59.92419 |
| 1923 | 72.80 | F | 59.92 | 59.92419 |
| 1924 | 72.20 | F | 59.92 | 59.92419 |
| 1926 | 70.00 | F | 59.92 | 59.92419 |
| 1929 | 69.40 | F | 59.92 | 59.92419 |
| 1930 | 68.00 | F | 59.92 | 59.92419 |
| 1931 | 66.60 | F | 59.92 | 59.92419 |
| 1933 | 66.00 | F | 59.92 | 59.92419 |
| 1934 | 65.40 | F | 59.92 | 59.92419 |
| 1936 | 64.60 | F | 59.92 | 59.92419 |
| 1956 | 62.00 | F | 59.92 | 59.92419 |
| 1958 | 61.20 | F | 59.92 | 59.92419 |
| 1960 | 60.20 | F | 59.92 | 59.92419 |
| 1962 | 59.50 | F | 59.92 | 59.92419 |
| 1964 | 58.90 | F | 59.92 | 59.92419 |

# 11 Code

```
model <- lm(time ~ 1, data = SwimRecords)

SwimRecords_predict <- SwimRecords %>%
  mutate(with_formula = 59.92*1) %>%
  mutate(with_predict= predict(model, SwimRecords))

SwimRecords_predict %>%
  ggplot(aes(x = year, y = time)) +
  geom_point() +
  geom_line(aes(y = with_predict), color = "blue") +
  theme_bw(base_size = 14)
```

## 11.1 Two inputs

# 12 Plot




Model specification: $y = w_1\mathbf{1} + w_2\mathbf{year}$

```
Call:
lm(formula = time ~ 1 + year, data = SwimRecords)
```

```
Coefficients:
(Intercept)         year
   567.2420      -0.2599
```

Fitted model: $y = (567.2420)\mathbf{1} + (-0.2599)\mathbf{year}$

## 13 Data

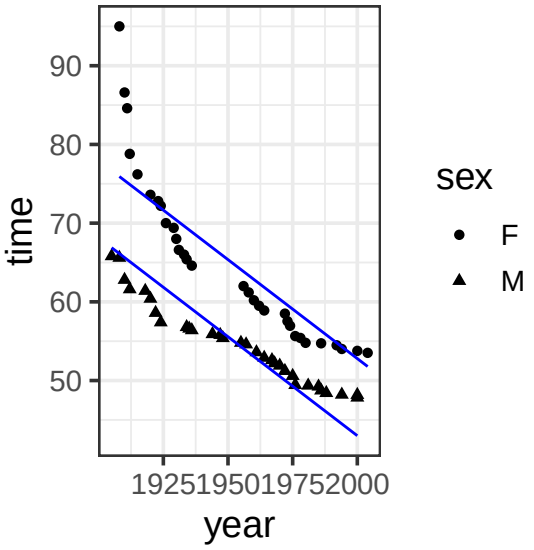
## 14 Code

```
model <- lm(time ~ 1 + year, data = SwimRecords)

SwimRecords_predict <- SwimRecords %>%
  mutate(with_formula = 567.2420*1 + -0.2599*year) %>%
  mutate(with_predict= predict(model, SwimRecords))

SwimRecords_predict %>%
  ggplot(aes(x = year, y = time)) +
  geom_point() +
  geom_line(aes(y = with_predict), color = "blue") +
  theme_bw(base_size = 14)
```

| year | time | sex | with_formula | with_predict |
|---|---|---|---|---|
| 1905 | 65.80 | M | 72.1325 | 72.17614 |
| 1908 | 65.60 | M | 71.3528 | 71.39651 |
| 1910 | 62.80 | M | 70.8330 | 70.87676 |
| 1912 | 61.60 | M | 70.3132 | 70.35700 |
| 1918 | 61.40 | M | 68.7538 | 68.79774 |
| 1920 | 60.40 | M | 68.2340 | 68.27798 |
| 1922 | 58.60 | M | 67.7142 | 67.75823 |
| 1924 | 57.40 | M | 67.1944 | 67.23848 |
| 1934 | 56.80 | M | 64.5954 | 64.63971 |
| 1935 | 56.60 | M | 64.3355 | 64.37983 |
| 1936 | 56.40 | M | 64.0756 | 64.11995 |
| 1944 | 55.90 | M | 61.9964 | 62.04093 |
| 1947 | 55.80 | M | 61.2167 | 61.26130 |
| 1948 | 55.40 | M | 60.9568 | 61.00143 |
| 1955 | 54.80 | M | 59.1375 | 59.18229 |
| 1957 | 54.60 | M | 58.6177 | 58.66253 |
| 1961 | 53.60 | M | 57.5781 | 57.62302 |
| 1964 | 52.90 | M | 56.7984 | 56.84339 |
| 1967 | 52.60 | M | 56.0187 | 56.06376 |
| 1968 | 52.20 | M | 55.7588 | 55.80388 |
| 1970 | 51.90 | M | 55.2390 | 55.28413 |
| 1972 | 51.22 | M | 54.7192 | 54.76438 |
| 1975 | 50.59 | M | 53.9395 | 53.98474 |
| 1976 | 49.44 | M | 53.6796 | 53.72487 |
| 1981 | 49.36 | M | 52.3801 | 52.42548 |
| 1985 | 49.24 | M | 51.3405 | 51.38597 |
| 1986 | 48.74 | M | 51.0806 | 51.12610 |
| 1988 | 48.42 | M | 50.5608 | 50.60634 |
| 1994 | 48.21 | M | 49.0014 | 49.04708 |
| 2000 | 48.18 | M | 47.4420 | 47.48782 |
| 2000 | 47.84 | M | 47.4420 | 47.48782 |
| 1908 | 95.00 | F | 71.3528 | 71.39651 |
| 1910 | 86.60 | F | 70.8330 | 70.87676 |
| 1911 | 84.60 | F | 70.5731 | 70.61688 |
| 1912 | 78.80 | F | 70.3132 | 70.35700 |
| 1915 | 76.20 | F | 69.5335 | 69.57737 |
| 1920 | 73.60 | F | 68.2340 | 68.27798 |
| 1923 | 72.80 | F | 67.4543 | 67.49835 |
| 1924 | 72.20 | F | 67.1944 | 67.23848 |
| 1926 | 70.00 | F | 66.6746 | 66.71872 |
| 1929 | 69.40 | F | 65.8949 | 65.93909 |
| 1930 | 68.00 | F | 65.6350 | 65.67921 |
| 1931 | 66.60 | F | 65.3751 | 65.41934 |
| 1933 | 66.00 | F | 64.8553 | 64.89958 |
| 1934 | 65.40 | F | 64.5954 | 64.63971 |
| 1936 | 64.60 | F | 64.0756 | 64.11995 |
| 1956 | 62.00 | F | 58.8776 | 58.92241 |
| 1958 | 61.20 | F | 58.3578 | 58.40266 |
| 1960 | 60.20 | F | 57.8380 | 57.88290 |
| 1962 | 59.50 | F | 57.3182 | 57.36315 |
| 1964 | 58.90 | F | 56.7984 | 56.84339 |

### 14.1 Three inputs

## 15 Plot




Model specification: $y = w_1\mathbf{1} + w_2\mathbf{year} + w_3\mathbf{sex}$

```
Call:
lm(formula = time ~ 1 + year + sex, data = SwimRecords)

Coefficients:
(Intercept)         year         sexM
   555.7168      -0.2515      -9.7980
```

Fitted model: $y = (555.7168)\mathbf{1} + (-0.2515)\mathbf{year} + (-9.7980)\mathbf{sex}$

## 16 Data

## 17 Code

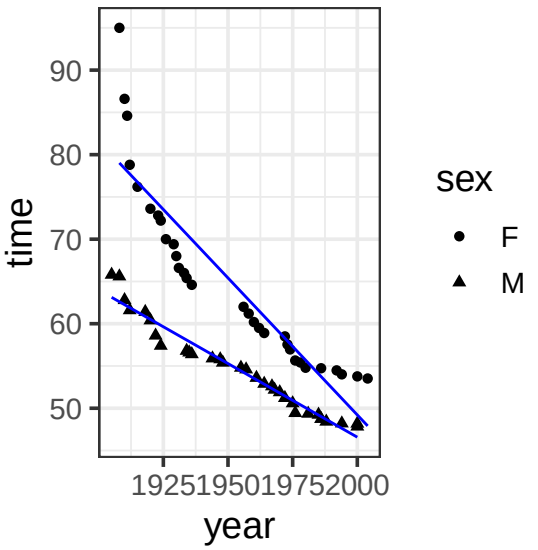
```
model <- lm(time ~ 1 + year, data = SwimRecords)
```

| year | time | sex | with_formula | with_predict |
|---|---|---|---|---|
| 1905 | 65.80 | M | 72.1325 | 66.88051 |
| 1908 | 65.60 | M | 71.3528 | 66.12612 |
| 1910 | 62.80 | M | 70.8330 | 65.62319 |
| 1912 | 61.60 | M | 70.3132 | 65.12026 |
| 1918 | 61.40 | M | 68.7538 | 63.61148 |
| 1920 | 60.40 | M | 68.2340 | 63.10855 |
| 1922 | 58.60 | M | 67.7142 | 62.60563 |
| 1924 | 57.40 | M | 67.1944 | 62.10270 |
| 1934 | 56.80 | M | 64.5954 | 59.58806 |
| 1935 | 56.60 | M | 64.3355 | 59.33660 |
| 1936 | 56.40 | M | 64.0756 | 59.08513 |
| 1944 | 55.90 | M | 61.9964 | 57.07343 |
| 1947 | 55.80 | M | 61.2167 | 56.31903 |
| 1948 | 55.40 | M | 60.9568 | 56.06757 |
| 1955 | 54.80 | M | 59.1375 | 54.30732 |
| 1957 | 54.60 | M | 58.6177 | 53.80440 |
| 1961 | 53.60 | M | 57.5781 | 52.79854 |
| 1964 | 52.90 | M | 56.7984 | 52.04415 |
| 1967 | 52.60 | M | 56.0187 | 51.28976 |
| 1968 | 52.20 | M | 55.7588 | 51.03830 |
| 1970 | 51.90 | M | 55.2390 | 50.53537 |
| 1972 | 51.22 | M | 54.7192 | 50.03244 |
| 1975 | 50.59 | M | 53.9395 | 49.27805 |
| 1976 | 49.44 | M | 53.6796 | 49.02659 |
| 1981 | 49.36 | M | 52.3801 | 47.76927 |
| 1985 | 49.24 | M | 51.3405 | 46.76341 |
| 1986 | 48.74 | M | 51.0806 | 46.51195 |
| 1988 | 48.42 | M | 50.5608 | 46.00902 |
| 1994 | 48.21 | M | 49.0014 | 44.50024 |
| 2000 | 48.18 | M | 47.4420 | 42.99146 |
| 2000 | 47.84 | M | 47.4420 | 42.99146 |
| 1908 | 95.00 | F | 71.3528 | 75.92408 |
| 1910 | 86.60 | F | 70.8330 | 75.42115 |
| 1911 | 84.60 | F | 70.5731 | 75.16969 |
| 1912 | 78.80 | F | 70.3132 | 74.91822 |
| 1915 | 76.20 | F | 69.5335 | 74.16383 |
| 1920 | 73.60 | F | 68.2340 | 72.90651 |
| 1923 | 72.80 | F | 67.4543 | 72.15212 |
| 1924 | 72.20 | F | 67.1944 | 71.90066 |
| 1926 | 70.00 | F | 66.6746 | 71.39773 |
| 1929 | 69.40 | F | 65.8949 | 70.64334 |
| 1930 | 68.00 | F | 65.6350 | 70.39188 |
| 1931 | 66.60 | F | 65.3751 | 70.14041 |
| 1933 | 66.00 | F | 64.8553 | 69.63749 |
| 1934 | 65.40 | F | 64.5954 | 69.38602 |
| 1936 | 64.60 | F | 64.0756 | 68.88310 |
| 1956 | 62.00 | F | 58.8776 | 63.85382 |
| 1958 | 61.20 | F | 58.3578 | 63.35090 |
| 1960 | 60.20 | F | 57.8380 | 62.84797 |
| 1962 | 59.50 | F | 57.3182 | 62.34504 |
| 1964 | 58.90 | F | 56.7984 | 61.84211 |

```r
SwimRecords_predict <- SwimRecords %>%
  mutate(with_formula = 567.2420*1 + -0.2599*year) %>%
  mutate(with_predict= predict(model, SwimRecords))

SwimRecords_predict %>%
  ggplot(aes(x = year, y = time)) +
  geom_point() +
  geom_line(aes(y = with_predict), color = "blue") +
  theme_bw(base_size = 14)
```

### 17.1 Interaction

## 18 Plot




Model specification: $y = w_1\mathbf{1} + w_2\mathbf{year} + w_3\mathbf{sex} + w_4\mathbf{year} * \mathbf{sex}$

```
Call:
lm(formula = time ~ 1 + year * sex, data = SwimRecords)

Coefficients:
(Intercept)         year         sexM    year:sexM
   697.3012      -0.3240    -302.4638       0.1499
```

Fitted model: $y = (697.3012)\mathbf{1} + (-0.3240)\mathbf{year} + (-302.4638)\mathbf{sex} + (0.1499)\mathbf{year} \times \mathbf{sex}$

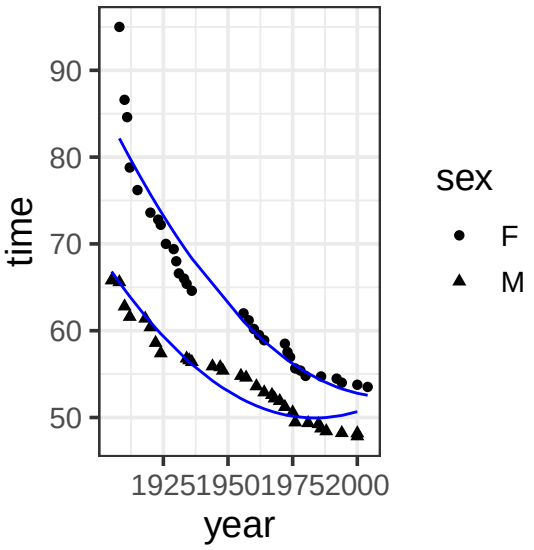
# 19 Data

# 20 Code

```
model <- lm(time ~ 1 + year, data = SwimRecords)

SwimRecords_predict <- SwimRecords %>%
  mutate(with_formula = 567.2420*1 + -0.2599*year) %>%
  mutate(with_predict= predict(model, SwimRecords))

SwimRecords_predict %>%
  ggplot(aes(x = year, y = time)) +
  geom_point() +
  geom_line(aes(y = with_predict), color = "blue") +
  theme_bw(base_size = 14)
```

## 20.1 Transformation

# 21 Plot




Model specification:

$$y = w_1\mathbf{1} + w_2\mathbf{year} + w_3\mathbf{sex} + w_4\mathbf{year} \times \mathbf{sex} + w_5\mathbf{year}^2$$

| year | time | sex | with_formula | with_predict |
|---|---|---|---|---|
| 1905 | 65.80 | M | 72.1325 | 63.12106 |
| 1908 | 65.60 | M | 71.3528 | 62.59867 |
| 1910 | 62.80 | M | 70.8330 | 62.25041 |
| 1912 | 61.60 | M | 70.3132 | 61.90215 |
| 1918 | 61.40 | M | 68.7538 | 60.85738 |
| 1920 | 60.40 | M | 68.2340 | 60.50912 |
| 1922 | 58.60 | M | 67.7142 | 60.16086 |
| 1924 | 57.40 | M | 67.1944 | 59.81260 |
| 1934 | 56.80 | M | 64.5954 | 58.07131 |
| 1935 | 56.60 | M | 64.3355 | 57.89718 |
| 1936 | 56.40 | M | 64.0756 | 57.72305 |
| 1944 | 55.90 | M | 61.9964 | 56.33002 |
| 1947 | 55.80 | M | 61.2167 | 55.80763 |
| 1948 | 55.40 | M | 60.9568 | 55.63350 |
| 1955 | 54.80 | M | 59.1375 | 54.41459 |
| 1957 | 54.60 | M | 58.6177 | 54.06634 |
| 1961 | 53.60 | M | 57.5781 | 53.36982 |
| 1964 | 52.90 | M | 56.7984 | 52.84743 |
| 1967 | 52.60 | M | 56.0187 | 52.32504 |
| 1968 | 52.20 | M | 55.7588 | 52.15091 |
| 1970 | 51.90 | M | 55.2390 | 51.80266 |
| 1972 | 51.22 | M | 54.7192 | 51.45440 |
| 1975 | 50.59 | M | 53.9395 | 50.93201 |
| 1976 | 49.44 | M | 53.6796 | 50.75788 |
| 1981 | 49.36 | M | 52.3801 | 49.88723 |
| 1985 | 49.24 | M | 51.3405 | 49.19072 |
| 1986 | 48.74 | M | 51.0806 | 49.01659 |
| 1988 | 48.42 | M | 50.5608 | 48.66833 |
| 1994 | 48.21 | M | 49.0014 | 47.62355 |
| 2000 | 48.18 | M | 47.4420 | 46.57878 |
| 2000 | 47.84 | M | 47.4420 | 46.57878 |
| 1908 | 95.00 | F | 71.3528 | 79.02170 |
| 1910 | 86.60 | F | 70.8330 | 78.37361 |
| 1911 | 84.60 | F | 70.5731 | 78.04956 |
| 1912 | 78.80 | F | 70.3132 | 77.72552 |
| 1915 | 76.20 | F | 69.5335 | 76.75338 |
| 1920 | 73.60 | F | 68.2340 | 75.13315 |
| 1923 | 72.80 | F | 67.4543 | 74.16101 |
| 1924 | 72.20 | F | 67.1944 | 73.83697 |
| 1926 | 70.00 | F | 66.6746 | 73.18887 |
| 1929 | 69.40 | F | 65.8949 | 72.21674 |
| 1930 | 68.00 | F | 65.6350 | 71.89269 |
| 1931 | 66.60 | F | 65.3751 | 71.56864 |
| 1933 | 66.00 | F | 64.8553 | 70.92055 |
| 1934 | 65.40 | F | 64.5954 | 70.59651 |
| 1936 | 64.60 | F | 64.0756 | 69.94842 |
| 1956 | 62.00 | F | 58.8776 | 63.46750 |
| 1958 | 61.20 | F | 58.3578 | 62.81941 |
| 1960 | 60.20 | F | 57.8380 | 62.17131 |
| 1962 | 59.50 | F | 57.3182 | 61.52322 |
| 1964 | 58.90 | F | 56.7984 | 60.87513 |

```
Call:
lm(formula = time ~ 1 + year * sex + I(year^2), data = SwimRecords)

Coefficients:
(Intercept)         year         sexM    I(year^2)    year:sexM
  1.110e+04   -1.098e+01   -3.171e+02    2.729e-03    1.575e-01
```

Fitted model: $y = (697.3012)\mathbf{1} + (-0.3240)\mathbf{year} + (-302.4638)\mathbf{sex} + (0.1499)\mathbf{year} \times \mathbf{sex}$

## 22 Data

## 23 Code

```
model <- lm(time ~ 1 + year, data = SwimRecords)

SwimRecords_predict <- SwimRecords %>%
  mutate(with_formula = 567.2420*1 + -0.2599*year) %>%
  mutate(with_predict= predict(model, SwimRecords))

SwimRecords_predict %>%
  ggplot(aes(x = year, y = time)) +
  geom_point() +
  geom_line(aes(y = with_predict), color = "blue") +
  theme_bw(base_size = 14)
```

## 24 Brain size (log)

## 25 Plant heights (polynomials)

## 26 Further reading

- Ch 6: Language of models in Statistical Modeling

| year | time | sex | with_formula | with_predict |
|---|---|---|---|---|
| 1905 | 65.80 | M | 72.1325 | 66.81874 |
| 1908 | 65.60 | M | 71.3528 | 65.55576 |
| 1910 | 62.80 | M | 70.8330 | 64.74106 |
| 1912 | 61.60 | M | 70.3132 | 63.94819 |
| 1918 | 61.40 | M | 68.7538 | 61.70057 |
| 1920 | 60.40 | M | 68.2340 | 60.99502 |
| 1922 | 58.60 | M | 67.7142 | 60.31130 |
| 1924 | 57.40 | M | 67.1944 | 59.64941 |
| 1934 | 56.80 | M | 64.5954 | 56.66741 |
| 1935 | 56.60 | M | 64.3355 | 56.39922 |
| 1936 | 56.40 | M | 64.0756 | 56.13650 |
| 1944 | 55.90 | M | 61.9964 | 54.23115 |
| 1947 | 55.80 | M | 61.2167 | 53.60669 |
| 1948 | 55.40 | M | 60.9568 | 53.40946 |
| 1955 | 54.80 | M | 59.1375 | 52.18160 |
| 1957 | 54.60 | M | 58.6177 | 51.87991 |
| 1961 | 53.60 | M | 57.5781 | 51.34200 |
| 1964 | 52.90 | M | 56.7984 | 50.99587 |
| 1967 | 52.60 | M | 56.0187 | 50.69886 |
| 1968 | 52.20 | M | 55.7588 | 50.61078 |
| 1970 | 51.90 | M | 55.2390 | 50.45097 |
| 1972 | 51.22 | M | 54.7192 | 50.31300 |
| 1975 | 50.59 | M | 53.9395 | 50.14697 |
| 1976 | 49.44 | M | 53.6796 | 50.10254 |
| 1981 | 49.36 | M | 52.3801 | 49.96226 |
| 1985 | 49.24 | M | 51.3405 | 49.94827 |
| 1986 | 48.74 | M | 51.0806 | 49.95841 |
| 1988 | 48.42 | M | 50.5608 | 49.99508 |
| 1994 | 48.21 | M | 49.0014 | 50.23605 |
| 2000 | 48.18 | M | 47.4420 | 50.67349 |
| 2000 | 47.84 | M | 47.4420 | 50.67349 |
| 1908 | 95.00 | F | 71.3528 | 82.16082 |
| 1910 | 86.60 | F | 70.8330 | 81.03116 |
| 1911 | 84.60 | F | 70.5731 | 80.47451 |
| 1912 | 78.80 | F | 70.3132 | 79.92332 |
| 1915 | 76.20 | F | 69.5335 | 78.30250 |
| 1920 | 73.60 | F | 68.2340 | 75.71028 |
| 1923 | 72.80 | F | 67.4543 | 74.22044 |
| 1924 | 72.20 | F | 67.1944 | 73.73474 |
| 1926 | 70.00 | F | 66.6746 | 72.77971 |
| 1929 | 69.40 | F | 65.8949 | 71.38810 |
| 1930 | 68.00 | F | 65.6350 | 70.93515 |
| 1931 | 66.60 | F | 65.3751 | 70.48765 |
| 1933 | 66.00 | F | 64.8553 | 69.60603 |
| 1934 | 65.40 | F | 64.5954 | 69.17790 |
| 1936 | 64.60 | F | 64.0756 | 68.33203 |
| 1956 | 62.00 | F | 58.8776 | 61.07389 |
| 1958 | 61.20 | F | 58.3578 | 60.46814 |
| 1960 | 60.20 | F | 57.8380 | 59.88422 |
| 1962 | 59.50 | F | 57.3182 | 59.32213 |
| 1964 | 58.90 | F | 56.7984 | 58.78187 |