Exam 2 Solutions

Data Science for Studying Language & the Mind

Instructions

The exam is worth **113 points**. You have **1 hour and 30 minutes** to complete the exam.

- The exam is closed book/note/computer/phone except for the provided reference sheets
- If you need to use the restroom, leave your exam and phone with the TAs
- If you finish early, you may turn in your exam and leave early

(5 points) Preliminary questions

Please complete these questions *before* the exam begins.

(a) **(1 point)** What is your full name?

(b) **(1 point)** What is your penn ID number?

(c) **(1 point)** What is your lab section TA's name?

(d) **(1 point)** Who is sitting to your left?

(e) **(1 point)** Who is sitting to your right?

1. (24 points) True or false

(a) **(2 points)** The goal of a regression model is to classify observations into distinct categories.

☐ True
☒ False

(b) **(2 points)** Model specification involves defining the functional form of the model.

☒ True
☐ False

(c) **(2 points)** The equation $y = ax + b$ expresses y as a weighted sum of inputs.

☒ True
☐ False

(d) **(2 points)** Regression is a type of supervised learning, while classification is unsupervised.

☐ True
☒ False

(e) **(2 points)** In gradient descent, we search through all possible parameters in the parameter space.

☐ True
☒ False

(f) **(2 points)** The ordinary least squares solution is an example of an iterative optimization algorithm.

☐ True
☒ False

(g) **(2 points)** Adding more predictors to a regression model will always increase the R^2 value.

☒ True
☐ False

(h) **(2 points)** An overfit model performs poorly on both the sample and predicting new values.

☐ True
☒ False

(i) **(2 points)** A reliable model will always be a highly accurate model.

☐ True
☒ False

(j) **(2 points)** The error bars on our parameter estimates will become smaller as we increase our sample size.

☒ True
☐ False

(k) **(2 points)** Support vector machines can be used for classification problems.

☒ True
☐ False

(l) **(2 points)** The logistic function always produces outputs between 0 and 1.

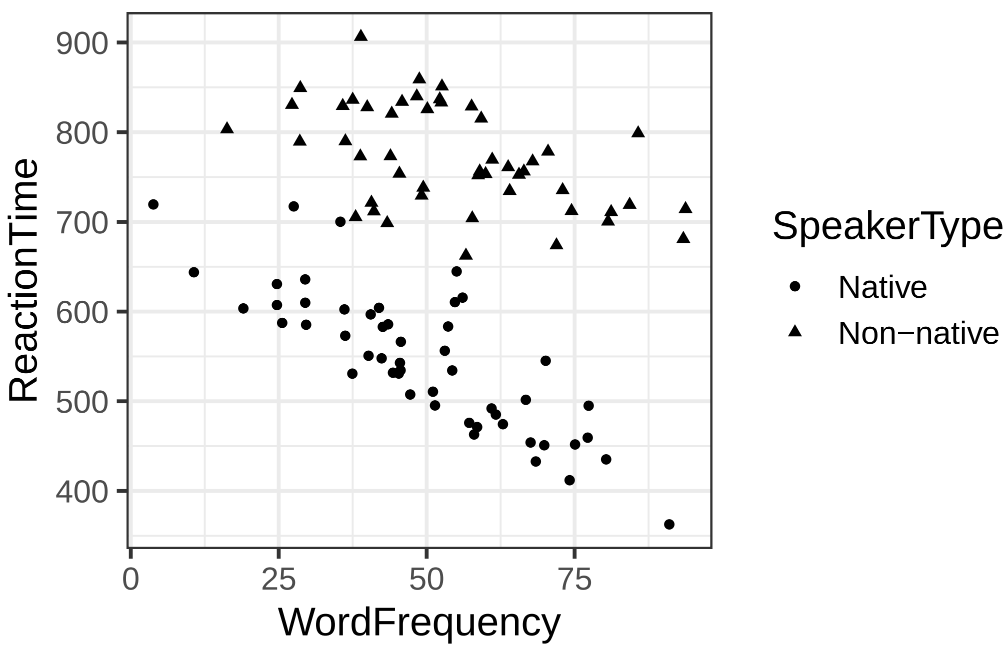
☒ True
☐ False

2. (12 points) Model specification

Suppose we measure the reaction times (in milliseconds) of both native and non-native speakers as they process words of varying frequency in English (measured as occurrences per million words). We store these data in a tibble called `rt_by_speaker`. The first 6 rows of this tibble are printed below for your reference.

```
# A tibble: 6 x 3
  WordFrequency ReactionTime SpeakerType
          <dbl>        <dbl> <chr>
1          38.8         773. Non-native
2          45.4         754. Non-native
3          81.2         711. Non-native
4          51.4         495. Native
5          52.6         851. Non-native
6          84.3         719. Non-native
```

We've also included an exploratory plot of these data.



Suppose we specify the following model with `lm`:

```
model <- lm(ReactionTime ~ 1 + WordFrequency + SpeakerType, data = rt_by_speaker)
```

(a) **(3 points)** Write the model's specification as a mathematical expression:

$$y = x$$

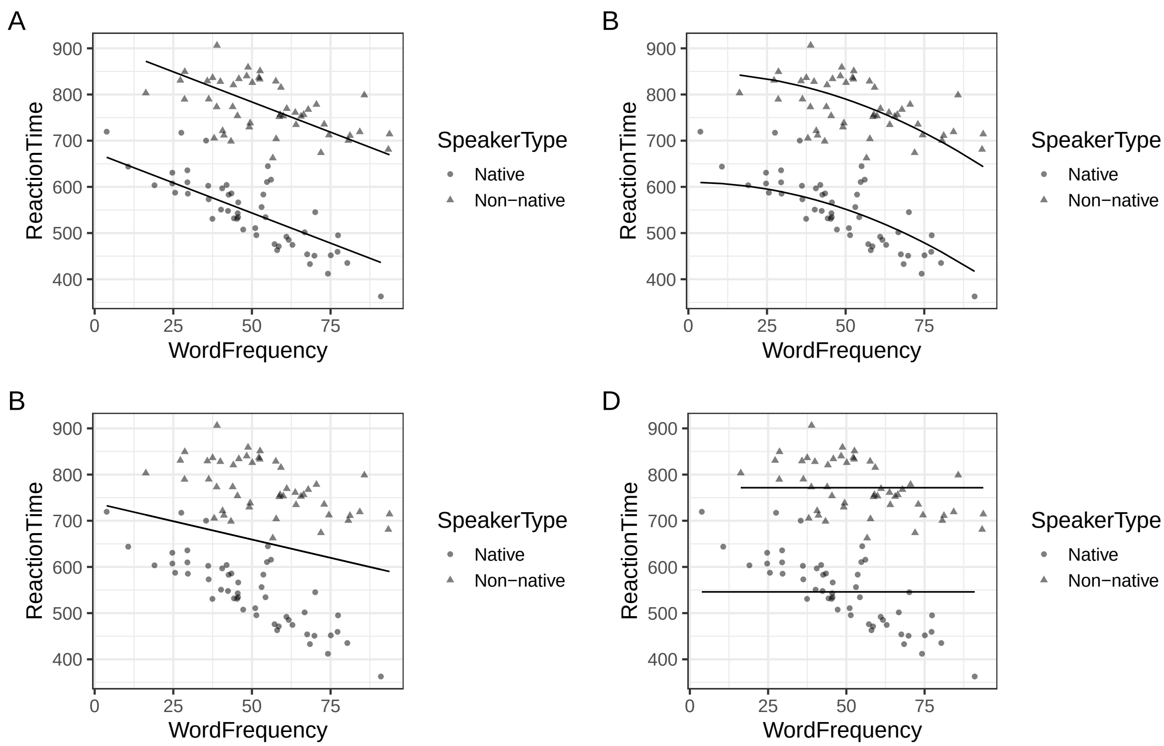
(b) **(3 points)** For each of the following, circle the option that best describes the type of model we fit.

(i) **(1 point)** Supervised or unsupervised

(ii) **(1 point)** Regression or classification

(iii) **(1 point)** Linear or linearlizable nonlinear

(c) **(3 points)** Each of the figures below show a model's predictions for these data plotted with black lines. Circle the figure that is most likely to be the plot of the model spcified to `lm`? Choose one.



(d) **(3 points)** Suppose we also fit the model with `infer`, which returns the parameter estimates below. Which of the following could be the predicted reaction time for a Native speaker with a word frequency of 10?

```
# A tibble: 3 x 2
  term                  estimate
  <chr>                    <dbl>
1 intercept               674.
2 WordFrequency            -2.61
3 SpeakerTypeNon-native   240.
```

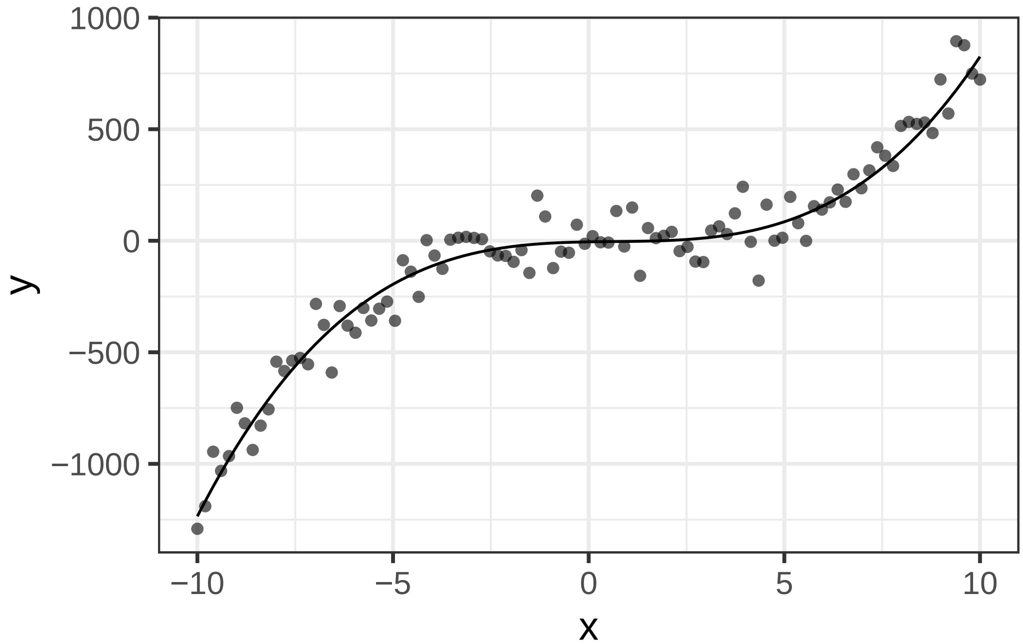
- ☐ 647.9
- ☐ 695.1
- ☐ 887.9
- ☐ Not enough information to determine this

You may show your work here, if you wish:

3. (12 points) Applied model specification

Suppose we encounter the following dataset, glimpsed and plotted here.

```
Rows: 100
Columns: 2
$ x <dbl> -10.000000, -9.797980, -9.595960, -9.393939, -9.191919, -8.989899, -~
$ y <dbl> -1291.0476, -1190.0226, -945.7013, -1031.6017, -965.2677, -748.6480,~
```



We specify and fit these data with `lm` as below:

```
lm(y ~ poly(x, 3) , data = data)
```

```
Call:
lm(formula = y ~ poly(x, 3), data = data)

Coefficients:
(Intercept)  poly(x, 3)1  poly(x, 3)2  poly(x, 3)3
     -63.97      3816.56      -514.32      1568.49
```

(a) **(2 points)** What type of polynomial is included in the model specification?

- ☐ Constant
- ☐ Linear
- ☐ Quadratic
- ☐ Cubic
- ☐ Quartic

(b) **(3 points)** Write the *fitted model* as a mathematical expression:

(c) **(2 points)** In class we learned about two ways to linearlize a nonlinear model. Which option best describes what we have done here?

- ☐ Expanding the input space by adding new terms
- ☐ Transforming an existing term

(d) **(2 points)** Given the predicted model (shown with the black line on the figure), what does the model predict for the value of y when $x = 1$?

(e) **(3 points)** Suppose we fit the model specification `y ~ poly(x, 100)`. Explain why this would be an overfit model.

4. (13 points) Model fitting

Section 4 refers to the `rt_by_speaker` tibble from section 2. We have returned the first 6 rows of the tibble here for your reference.

```
# A tibble: 6 x 3
  WordFrequency ReactionTime SpeakerType
          <dbl>        <dbl> <chr>
1          38.8         773. Non-native
2          45.4         754. Non-native
3          81.2         711. Non-native
4          51.4         495. Native
5          52.6         851. Non-native
6          84.3         719. Non-native
```

Suppose we estimate the free parameters with `optimg` and `lm`, which return the following results:

```
optimg(data = rt_by_speaker, par = c(0,0, 0), fn=SSE, method = "STGD")


$par
[1] 674.046758  -2.612294 240.353670

$value
[1] 244250.2

$counts
[1] 24

$convergence
[1] 0

lm(ReactionTime ~ 1 + WordFrequency + SpeakerType, data = rt_by_speaker)



Call:
lm(formula = ReactionTime ~ 1 + WordFrequency + SpeakerType,
    data = rt_by_speaker)

Coefficients:
          (Intercept)          WordFrequency  SpeakerTypeNon-native
              674.052                 -2.613                240.361
```

(a) **(2 points)** Explain why `optimg` and `lm` return slightly different parameter estimates?

1

(2 points) What is the cost function used by `optimg`? Choose one.

- ☐ SSE
- ☐ STGD
- ☐ Gradient descent
- ☐ R^2
- ☐ Not enough information to determine this

(b) **(2 points)** How many steps did our iterative optimization algorithm take?

(c) **(2 points)** What was the sum of squared error of the optimal parameters according to `optimg`? Choose one.

- ☐ 24
- ☐ 0
- ☐ 244250.2
- ☐ 244250.2^2
- ☐ Not enough information to determine this

(d) **(2 points)** Which approach does `lm` use to estimate the free parameters? Choose one.

- ☐ Ordinary least-squares solution
- ☐ Gradient descent
- ☐ Another iterative optimzation algorith
- ☐ All of the above

(e) **(3 points)** Given the model specified in the code to `lm`, fill in the missing values for the first 6 rows of the input matrix **X**.

$$\begin{bmatrix} 773 \\ 754 \\ 711 \\ 495 \\ 851 \\ 719 \end{bmatrix} = \begin{bmatrix} 1 & 38.8 & \underline{\hspace{2cm}} \\ 1 & 45.4 & \underline{\hspace{2cm}} \\ 1 & 81.2 & \underline{\hspace{2cm}} \\ 1 & 51.4 & \underline{\hspace{2cm}} \\ 1 & 52.6 & \underline{\hspace{2cm}} \\ 1 & 84.3 & \underline{\hspace{2cm}} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \end{bmatrix}$$

5. (12 points) Model accuracy

Suppose we want to determine how accurate our model is for the `rt_by_speaker` dataset. Section 5 refers to the following code and output.

First we specify and fit our model with `lm` and return the model summary.

```
model <- lm(ReactionTime ~ 1 + WordFrequency + SpeakerType, data = rt_by_speaker)
summary(model)
```

```
Call:
lm(formula = ReactionTime ~ 1 + WordFrequency + SpeakerType,
    data = rt_by_speaker)

Residuals:
     Min       1Q   Median       3Q      Max
-109.805  -31.329   -2.827   26.158  118.645

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)           674.0520    15.4094  43.743  < 2e-16 ***
WordFrequency          -2.6125     0.2796  -9.342 3.53e-15 ***
SpeakerTypeNon-native 240.3609    10.1616  23.654  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 50.18 on 97 degrees of freedom
Multiple R-squared:  0.8593,    Adjusted R-squared:  0.8564
F-statistic: 296.2 on 2 and 97 DF,  p-value: < 2.2e-16
```

Then we perform cross-validation and return the validation metrics with `collect_metrics()`

```
set.seed(2)
splits <- vfold_cv(rt_by_speaker)

model_spec <-
  linear_reg() %>%
  set_engine(engine = "lm")

our_workflow <-
  workflow() %>%
  add_model(model_spec) %>%
  add_formula(ReactionTime ~ 1 + WordFrequency + SpeakerType)

fitted_models <-
  fit_resamples(
    object = our_workflow,
    resamples = splits
    )

fitted_models %>%
    collect_metrics()
```

```
# A tibble: 2 x 6
  .metric .estimator   mean     n std_err .config
  <chr>   <chr>       <dbl> <int>   <dbl> <chr>
1 rmse    standard   50.7      10  2.19   Preprocessor1_Model1
2 rsq     standard    0.865    10  0.0300 Preprocessor1_Model1
```

(a) **(2 points)** What is the R^2 value for our original sample?

(b) **(2 points)** What is the R^2 estimate for the population?

(c) **(2 points)** What kind of cross-validation did we perform? Choose one.

- ☐ k-fold
- ☐ boostrapping
- ☐ leave-one out
- ☐ Not enough information to determine this

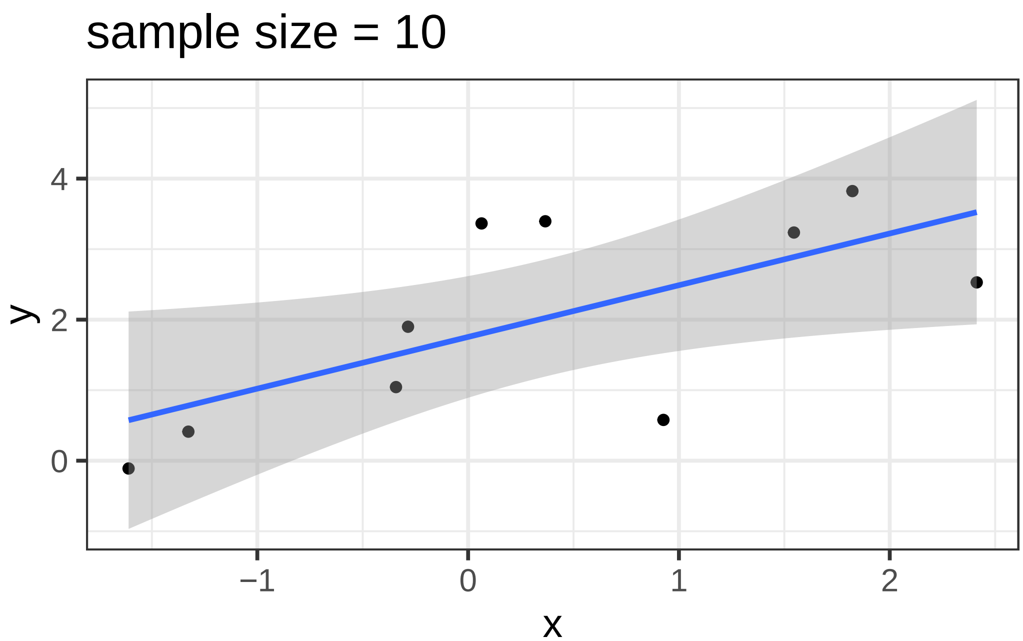
(d) **(2 points)** How many splits of our data does our code generate?

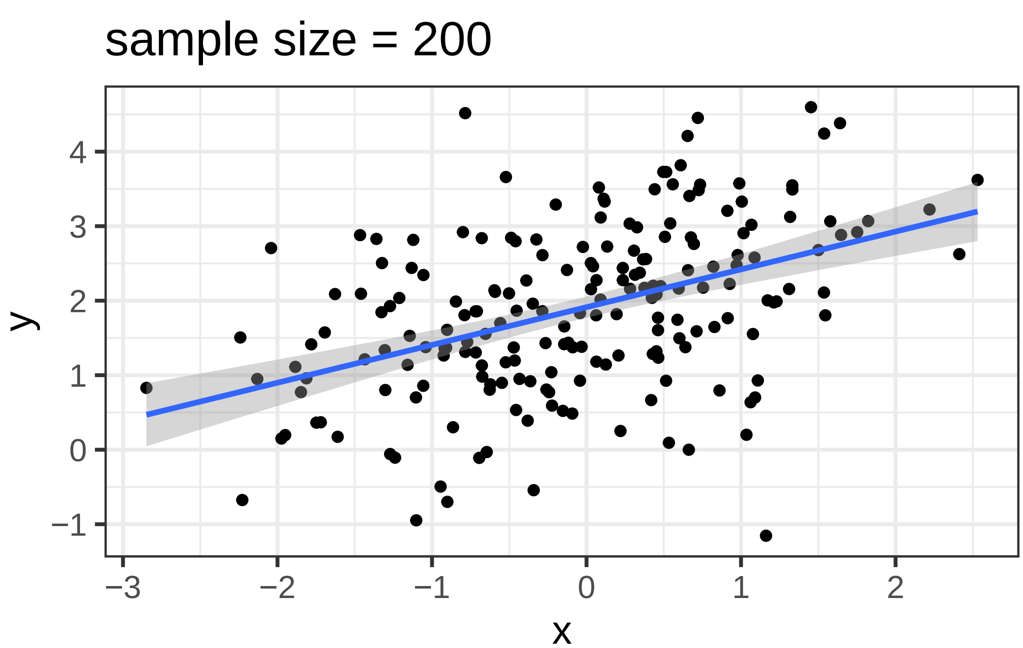
- ☐ 1000
- ☐ 100
- ☐ 10
- ☐ Not enough information to determine this

(e) **(3 points)** Explain the 3-step process that applies to all types of cross-validation.

6. (12 points) Model reliability

Section 6 refers to two datasets: `data_n10` and `data_n200` which have 10 and 200 observations respectively. Here we plot the data and the fitted model `y ~ 1 + x` for each dataset.





Here we return the model summary for each.

```
Call:
lm(formula = y ~ x, data = data_n10)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.8557 -0.6285 -0.0113  0.6370  1.5624 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)   
(Intercept)   1.7548     0.3740   4.692  0.00156 **
x             0.7333     0.2862   2.562  0.03352 * 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.138 on 8 degrees of freedom
Multiple R-squared:  0.4508,	Adjusted R-squared:  0.3821 
F-statistic: 6.566 on 1 and 8 DF,  p-value: 0.03352


Call:
lm(formula = y ~ x, data = data_n200)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.6565 -0.6757  0.0689  0.6032  3.0019 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.91308    0.07233  26.448  < 2e-16 ***
x            0.50704    0.07236   7.007 3.72e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.021 on 198 degrees of freedom
Multiple R-squared:  0.1987,	Adjusted R-squared:  0.1947 
F-statistic:  49.1 on 1 and 198 DF,  p-value: 3.724e-11
```

(a) **(2 points)** Which model is more accurate? Choose one.

- ☐ The model fitted to `data_n10`
- ☐ The model fitted to `data_n200`
- ☐ Both models are equally accurate
- ☐ Not enough information to determine this

(b) **(2 points)** Which model is more reliable? Choose one.

- ☐ The model fitted to `data_n10`
- ☐ The model fitted to `data_n200`
- ☐ Both models are equally reliable
- ☐ Not enough information to determine this

(c) **(2 points)** Which value in the model summary quantifies the model's reliability?

- ☐ Multiple R-squared
- ☐ Adjusted R-squared
- ☐ Estimate
- ☐ Std. Error
- ☐ Pr(>|t|)

(d) **(3 points)** Suppose we bootstrap a 95% confidence interval for our parameter estimates for the `data_n10` dataset. What would happen if we changed the level of the confidence interval to 68%? Choose one.

- ☐ It would get smaller (narrower)
- ☐ It would get bigger (wider)
- ☐ It would stay the same

(e) **(3 points)** Explain why there is uncertainy on our model parameter estimates.

7. (13 points) Classification

Suppose we want to predict the Fruit_Type (0 = apple, 1 = banana) based on its Weight, Color (1 = red, 2 = yellow, 3 = green), and Diameter. Our data is stored in the tibble `fruit_data`, glimpsed below.

```
Rows: 1,000
Columns: 4
$ Weight     <int> 113, 149, 217, 142, 113, 217, 189, 190, 190, 191, 236, 198,~
$ Color      <int> 3, 2, 1, 1, 1, 2, 1, 1, 3, 2, 3, 3, 3, 2, 1, 3, 2, 1, 3, 2,~
$ Diameter   <dbl> 21.8, 13.4, 19.3, 19.7, 24.7, 24.2, 7.8, 18.3, 5.7, 14.9, 2~
$ Fruit_Type <dbl> 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1,~
```

We fit this model with `glm` and return the following output:

```
glm(Fruit_Type ~ Weight + Color + Diameter, family = "binomial", data = fruit_data)
```

```
Call:  glm(formula = Fruit_Type ~ Weight + Color + Diameter, family = "binomial",
    data = fruit_data)

Coefficients:
(Intercept)       Weight        Color     Diameter
  -2.994585     0.001124    -0.005461     0.101965

Degrees of Freedom: 999 Total (i.e. Null);  996 Residual
Null Deviance:      1093
Residual Deviance: 1034     AIC: 1042
```

(a) **(3 points)** For each of the following, circle the option that best describes the type of model we fit.

(i) **(1 point)** Supervised or unsupervised
(ii) **(1 point)** Regression or classification
(iii) **(1 point)** Linear or linearlizable nonlinear

(b) **(2 points)** How many free parameters is this model estimating?

☐ 1
☐ 2
☐ 3
☐ 4
☐ Not enough information to determine this

(c) **(2 points)** Which of the following parsnip specifications could specify and fit a generalized linear model?

☐ `linear_reg() %>% set_engine("lm")`
☐ `logistic_reg() %>% set_enging("glm")`
☐ Both work

(d) **(2 points)** Which of the following expresses the link function for the `glm` we fit?

☐ $f(a) = \frac{1}{1+e^{-a}}$
☐ $\sum_{i=i}^{n}(d_i - m_i)^2$
☐ $y = \sum_{i=1}^{n} w_i x_i$
☐ $R^2 = 100 \times (1 - \frac{SSE_{model}}{SSE_{reference}})$

(e) **(2 points)** What do we call the type of classification we performed via our `glm`?

☐ linear regression
☐ logistic regression
☐ nearest-prototype regression
☐ support vector machine

(f) **(2 points)** What accuracy metric is best applied to classification models?

☐ R^2
☐ RMSE - root mean squared error
☐ Percent correct
☐ Adjusted R^2