

Problem set 3

due Thursday, October 26, 2023 at 11:59pm

⚠ Similar exercises will be worked through in the R tutorial on Oct 19

💡 Estimated time: 6 hours

Allocate about **1 hour per problem**, though some will take longer than others. You may need more time if programming is completely new to you, or less if you have some experience already.

Instructions Upload your .ipynb notebook to gradescope by 11:59pm on the due date.

- Note that each problem will be graded according to this [rubric](#). Solutions that include packages or functions not covered in this course will receive a score no higher than 2.
- You may collaborate with any of your classmates, but you must write your own code/solutions, understand all parts of the problem, and name your collaborators.
- You should also cite any outside sources you consulted, like Stack Overflow or ChatGPT, with a comment near the relevant lines of code (see example below). Recycled code that has not been cited will be considered plagiarism and receive a zero.

```
# code here was inspired by user2554330 on stack overflow:  
# https://stackoverflow.com/questions/69091812/is-everything-a-vector-in-r
```

Problem 0

not graded

Create a new colab R notebook. Please include the title “Problem set 3”, your name, the date, and any collaborators somewhere at the top.

Problem 1

The dataset below includes data simulated from work done by Carolyn Rovee-Collier. Dr. Rovee-Collier developed a new way to study very young babies' ability to remember things over time: the "mobile conjugate reinforcement paradigm". See a video of this paradigm [here](#) and a nice description from Merz et al (2017) [here](#):

"In this task, one end of a ribbon is tied around an infant's ankle and the other end is connected to a mobile hanging over his/her crib. Through experience with this set-up, the infant learns the contingency between kicking and movement of the mobile. After a delay, the task is repeated, and retention is measured by examining whether the infant kicks more during the retention phase than at baseline (i.e., spontaneous kicking prior to the learning trials; Rovee-Collier, 1997). Developmental research using the mobile conjugate reinforcement paradigm has demonstrated that both the speed of learning and length of retention increase with age"

```
"https://kathrynschuler.com/datasets/rovee_collier_1989.csv"
```

The simulated dataset includes 4 variables:

1. **ratio** - the measure of retention
2. **day** - the delay in days (1 through 14)
3. **age** - the age group: 2 month olds or 3 month olds
4. **age_recoded** - the age group recoded as 0 (2 month olds) and 1 (3 month olds)

Explore these data with (at least) `glimpse` and a scatterplot. Include `ratio` on the y-axis, `day` on the x-axis, and color the dots by age. You may include any other explorations you wish to perform.

Problem 2

Suppose you have specified that you will use a linear regression model to predict the simulated Rovee-Collier babies' retention ratio by day and age. Your model can be represented by the following equation:

$y = w_0 + w_1x_1 + w_2x_2$, where:

- y = ratio
- x_1 = day
- x_2 = age

Fit the specified model using ordinary least squares approach with each of the three different functions we learned in the tutorial: (1) with `lm`, (2) with `infer`, and (3) with `parsnip`. Did all three ways return the same parameter estimates? Explain why or why not.

Problem 3

Given the specified model and the parameters estimated in problem 2, compute the sum of squared error for the fitted model.

Note: if you are stuck on Problem 2, you may proceed with this problem by using all zeros as your parameter estimates.

Problem 4

Expanding on problem 3, write a more general function that would allow you to compute the sum of squared errors for the model specified in problem 2. Your function should take two inputs: (1) the data and (2) the parameter estimates. Your function should return a single value as output. Test your function with each of the following parameter values:

1. 0, 0, 0
2. 2, -3, 5
3. 1, 2, 3

Which of these three options fit the data best? How do you know?

Problem 5

Use the `optim` package to find the optimal parameter estimates for the model specified in problem 2 via gradient descent. Initialize your search with $b_0 = 0$, $b_1 = 0$, and $b_2 = 0$. How many iterations were necessary to estimate the parameters? Are the parameters estimated by your gradient descent the same as those returned by `lm()`? Explain why or why not.

Problem 6

The function given below finds the ordinary least squares estimate via matrix operations given two inputs: X , a matrix containing the input/explanatory variables, and Y , a matrix containing the output/response variable.

```
ols_matrix_way <- function(X, Y){  
  solve(t(X) %*% X) %*% t(X) %*% Y  
}
```

Use this function to estimate the free parameters of the model specified in problem 2. Are the parameters estimated by the matrix operation the same as those returned by `lm()`? Explain why or why not.