Report on "The TestIdeals Package for Macaulay2"

This paper describes the `TestIdeals.m2` package for *Macaulay2*, which can be used for computations in positive characteristic commutative algebra. In recent decades, there has been a large body work developing methods in this field, especially in the study of singularities, which has largely been developed through the theory of test ideals. Many papers in that literature have given methods for computing various of the invariants developed in this study, and this paper represents a step forward by collecting and implementing those theoretical results in *Macaulay2*, by building off and improving the package `PosChar.m2`. It further provides an excellent guide to using `TestIdeals.m2`, with both motivation and plenty of examples.

It has been long understood that the Frobenius endomorphism plays a central role in the study of singularities in positive characteristic, so it is no surprise that the first functions introduced are `frobeniusPower` and `frobeniusRoot` to compute Frobenius powers and roots of ideals, and that these serve as a basis for many of the other functions in the package. The paper goes on to introduce several functions in the `TestIdeals.m2` which measure various notions of singularity in prime characteristic; it can check if a ring is F-injective, F-pure, F-rational, or F-regular. Further, this package can also compute various types of test ideals and modules, namely parameter test modules, parameter test ideals, test ideals, and HSLG-modules, and the last of these can be used to compute the level of a polynomial. It can further compute ideals compatible with a given p^{-e}-linear map.

This paper presents each of these functions by defining and motivating the invariants they compute, and then giving a description of the algorithms used to compute them, noting the strengths and limitations of these methods. The authors also provide some examples of *Macaulay2* code implementing each method and demonstrating their applications. They comment on modifications which can be used when implementing their functions, including some which increase speed in certain scenarios. The paper is clearly written and contains many helpful examples for any reader wanting to run computations in positive characteristic commutative algebra; it serves as a fantastic manual to a much needed and powerful tool. I recommend this paper for publication.

Minor comments:
-page 4: `o4` and `o8` should read `Ideal of S`

-page 11: it might be helpful to comment that (2) is not unique (similarly for (2) on page 14, or make a general comment on canonical module computations in *Macaulay2*)
-page 15, last example: there seems to be a typo; I got:

```
i3 : frobeniusPower( 1/11, ideal f )
o3 = ideal 1
o3 : Ideal of R
```

-page 16: first line should be `i4: u = f^(121-1);`