Computer Vision: Homework 3

Katherine A. Scott

October 17, 2011

kas2221@columbia.edu

1 Problem 1

1.1 Part A

Let's call our 3x3 image that we want to multiple our kernel and define a couple variables:

$$img(i,j) = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}; A = \frac{1}{16}; B = 2A = \frac{2}{16}; E = 4A = \frac{4}{16}$$

This gives us the convolution kernel of the form:

$$img(i,j) = \begin{bmatrix} A & B & A \\ B & E & B \\ A & B & A \end{bmatrix}$$

From this we can see that the value of the edge image at (i,j) can be computed as:

$$img(i,j) = aA + bB + cA + dB + eE + fB + gA + hB + kA$$

This is $8MN$ additions and $9MN$ multiplications, but we can do a lot better really quickly if we gather terms:

$$img(i,j) = A(a+c+g+k) + B(b+d+f+h) + Ee$$

So now we still have $8MN$ additions but only $3MN$ multiplications.

1.2 Part B

We can do a lot better than $8MN$ additions but only $3MN$ multiplications. If we assume we are raster scanning the image and moving the kernel from left to right then from the top to the bottom we can save terms between computations. Using the same notation we define:

$$\alpha = A(c+k) + Bf$$

Now we let α' be the previous alpha value for the calculation at the pixel to the left of the current pixel (i.e. $j-1$), and α'' be the value of alpha two steps prior (i.e. $j-2$) This means our kernel becomes

$$img(i,j) = \begin{bmatrix} | & | & | \\ \alpha'' & 2\alpha' & \alpha \\ | & | & | \end{bmatrix}$$

and our equation at each step becomes:

$$img(i, j) = \alpha'' + 2\alpha' + A(c + k) + Bf$$

This means the total number of calculations (ignoring the calculations need to start a new row) is $4MN$ additions and $3MN$ additions. For large images the setup cost is negligible and these terms dominate.

2 Problem 2

Given the Hough equation

$$0 = x \sin\theta - y \cos\theta + \rho$$

we rearrange to get:

$$\rho = y \cos\theta - x \sin\theta$$

Now we want to get this in the form of $y = A \cos(\theta + \phi)$ so we can rationalize about the underlying sinusoid. We convert x and y to polar coordinates, namely $x = r \cos\phi$ and $y = r \sin\phi$ to get:

$$\rho = r \cos\theta \sin\phi - r \sin\theta \cos\phi$$

We now use the following trig identity:

$$\sin(\theta - \beta) = \sin\theta \cos\beta - \cos\theta \sin\beta$$

Which yields (replacing r with its true value):

$$\rho = \sqrt{x^2 + y^2} \sin(\phi - \theta)$$

Where

$$\phi = cos^{-1}\left(\frac{x}{\sqrt{x^2 + y^2}}\right)$$

It is clear from this equation that in Hough space x and y directly relate to the phase and amplitude of the resulting sinusoid, with the length of the point from the origin relating to the amplitude and the angle from y-axis relating to the phase shift. The period/frequency of the sinusoid is completely independent of the x and y values.