Supporting Material for Chapter 8

Introduction

Some material that was previously in the main textbook has been moved to this supplement to allow space for new material in the textbook. All the summary of equations for calculating the performance analysis of the gas turbine engines with losses in Chapter 8 can be found here. In addition, detailed calculations of Example 8.8 are given here. This section concludes with the methodology for performance analysis of afterburning turbojet engine with variable gas properties.

The sections that follow are:

- 8.3.SM Summary of Performance Equations: Single-Spool Turbojet Engine Without Afterburner
- 8.4.SM Summary of Performance Equations: Dual-Spool Turbojet with and Without Afterburner
- 8.5a.SM Summary of Performance Equations: Turbofan Engine with Separate Exhausts and Convergent Nozzles
- 8.5b.SM Example 8.8 Calculations for Separate-Exhaust-Stream Turbofan
- 8.5c.SM Summary of Performance Equations: Turbofan Engine with Compressor Stages on Low-Pressure Spool
- 8.6.SM Summary of Performance Equations: Turbofan with Afterburning Mixed-Flow Exhaust
- 8.7.SM Summary of Performance Equations: Turboprop
- 8.8.SM Variable Gas Properties
- Performance Analysis: Dual-Spool Afterburning Turbojet Engine

8.3.SM Summary of Performance Equations: Single-Spool Turbojet Engine Without Afterburner

Inputs:

Choices

Flight parameters:	M_0, T_0 (K, °R), P_0 (kPa, psia)
Throttle setting:	T_{t4} (K, °R)
Exhaust nozzle setting:	P_0/P_9

Design constants

π:	$\pi_{d\,\max}$, π_b, π_t, π_n
τ:	τ_t
η:	η_c, η_b, η_m
Gas properties:	γ_c, γ_t, c_{pc}, c_{pt} [kJ/(Kg · K), Btu/(lbm · °R)]
Fuel:	h_{PR}, (kJ/kg, Btu/lbm)

Reference conditions

Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia), τ_{rR}, π_{rR}
Throttle setting:	T_{t4R} (K, °R)
Component behavior:	T_{dR}, π_{cR}, τ_{cR}

Outputs:

Overall performance:	F(N, lbf), $\dot{m}_0$ (kg/s, lbm/s), f, $S\left(\frac{\text{g/s}}{\text{kN}}, \frac{\text{lbm/h}}{\text{lbf}}\right)$, η_P, η_{Th}, η_O
Component behavior:	π_d, π_c, τ_c, f, M_9, N/N_R

Equations:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{SM8.1a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{SM8.1b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{SM8.1c}$$

$$V_0 = a_0 M_0 \tag{SM8.1d}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{SM8.1e}$$

$$\pi_r = \tau^{\gamma_c/(\gamma_c - 1)} \tag{SM8.1f}$$

$$\eta_r = 1 \quad \text{for } M_0 \le 1 \tag{SM8.1g}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \quad \text{for } M_0 > 1 \tag{SM8.1h}$$

$$\pi_d = \pi_{d\,\max} \eta_r \tag{SM8.1i}$$

$$T_{t2} = T_0 \tau_r \tag{SM8.1j}$$

$$\tau_c = 1 + (\tau_{cR} - 1) \frac{T_{t4}/T_{t2}}{(T_{t4}/T_{t2})_R} \tag{SM8.1k}$$

$$\pi_c = [1 + \eta_c(\tau_c - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{SM8.1l}$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} \tag{SM8.1m}$$

$$f = \frac{h_{t4} - h_{t3}}{h_{PR}\eta_b - h_{t4}} \tag{SM8.1n}$$

$$\dot{m}_0 = \dot{m}_{0R} \frac{P_0 \pi_r \pi_d \pi_c}{(P_0 \pi_r \pi_d \pi_c)_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{SM8.1o}$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9} \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n \tag{SM8.1p}$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t - 1)/\gamma_t} - 1\right]} \tag{SM8.1q}$$

$$\frac{T_9}{T_0} = \frac{T_{t4}\tau_t / T_0}{(P_{t9}/P_0)^{(\gamma_t - 1)/\gamma_t}} \tag{SM8.1r}$$

$$\frac{V_9}{a_0} = M_9 \sqrt{\frac{\gamma_t R_t}{\gamma_c R_c} \frac{T_9}{T_0}} \tag{SM8.1s}$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1 - P_0/P_9}{\gamma_c}\right] \tag{SM8.1t}$$

$$F = \dot{m}_0 \left(\frac{F}{\dot{m}_0}\right) \tag{SM8.1u}$$

$$S = \frac{f}{F/\dot{m}_0} \tag{SM8.1v}$$

$$\eta_{Th} = \frac{a_0^2[(1+f)(V_9/a_0)^2 - M_0^2]}{2 g_c f h_{PR}} \tag{SM8.1w}$$

$$\eta_P = \frac{2 g_c V_0 (F/\dot{m}_0)}{a_0^2[(1+f)(V_0/a_0)^2 - M_0^2]} \tag{SM8.1x}$$

$$\eta_o = \eta_P \eta_{Th} \tag{SM8.1y}$$

$$\frac{N}{N_R} = \sqrt{\frac{T_0 \tau_r}{T_{0R} \tau_{rR}} \frac{\pi_c^{(\gamma_c - 1)/\gamma_c} - 1}{\pi_{cR}^{(\gamma_c - 1)/\gamma_c} - 1}} \tag{SM8.1z}$$

$$\frac{A_9}{A_{9R}} = \left[\frac{P_{t9}/P_9}{(P_{t9}/P_9)_R}\right]^{(\gamma_t + 1)/(2\gamma_t)} \sqrt{\frac{(P_{t9}/P_9)_R^{(\gamma_t - 1)/\gamma_t} - 1}{(P_{t9}/P_9)^{(\gamma_t - 1)/\gamma_t} - 1}} \tag{SM8.1aa}$$

8.4.SM Summary of Performance Equations: Dual-Spool Turbojet with and Without Afterburner

Inputs:

Choices

Flight parameters:	M_0, T_0(K, °R), P_0(kPa, psia)
Throttle setting:	T_{t4} (K, °R), T_{t7} (K, °R)
Exhaust nozzle setting:	P_0/P_9

Design constants

π:	$\pi_{d\,\max}$, π_b, π_{tH}, π_{tL}, π_{AB}, π_n
τ:	τ_{tH}, τ_{tL}
η:	η_{cL}, η_{cH}, η_b, η_{AB}, η_{mH}, η_{mL}
Gas properties:	γ_c, γ_t, γ_{AB}, c_{pc}, c_{pt}, c_{pAB} [kJ/(kg · K), Btu/(lbm · °R)]
Fuel:	h_{PR} (kJ/kg, Btu/lbm)

Reference conditions

Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia) τ_{rR}, π_{rR}
Throttle setting:	T_{t4R} (K, °R)
Component behavior:	π_{dR}, π_{cLR}, π_{cHR}, τ_{cLR}, τ_{cHR}

Outputs:

Overall performance:	F(N, lbf), $\dot{m}_0$ (kg/s, lbm/s), f_O, $S\left(\frac{\text{g/s}}{\text{kN}}, \frac{\text{lbm/h}}{\text{lbf}}\right)$, η_P, η_{Th}, η_O
Component behavior:	π_{cL}, π_{cH}, τ_{cL}, τ_{cH}, f, f_{AB}, M_9, $(N/N_R)_{\text{LPspool}}$, $(N/N_R)_{\text{HPspool}}$

Equations:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{SM8.2a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{SM8.2b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{SM8.2c}$$

$$V_0 = a_0 M_0 \tag{SM8.2d}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{SM8.2e}$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c - 1)} \tag{SM8.2f}$$

$$\eta_r = 1 \quad \text{for } M_0 \leq 1 \tag{SM8.2g}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \quad \text{for } M_0 > 1 \tag{SM8.2h}$$

$$\pi_d = \pi_{d\,\max} \eta_r \tag{SM8.2i}$$

$$\tau_{cL} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R} \frac{(\tau_r)_R}{\tau_r} (\tau_{cL} - 1)_R \tag{SM8.2j}$$

$$\pi_{cL} = [1 + \eta_{cL}(\tau_{cL} - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{SM8.2k}$$

$$\tau_{cH} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/t_0)_R} \frac{(\tau_r \tau_{cL})_R}{\tau_r \tau_{cL}} (\tau_{cH} - 1)_R \tag{SM8.2l}$$

$$\pi_{cH} = [1 + \eta_{cH}(\tau_{cH} - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{SM8.2m}$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} \tag{SM8.2n}$$

$$f = \frac{h_{t4} - h_{t3}}{h_{PR} \eta_b - h_{t4}} \tag{SM8.1o}$$

$$\dot{m}_0 = \dot{m}_{0R} \frac{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH}}{(P_0 \pi_r \pi_d \pi_{cL} \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{SM8.2p}$$

Without afterburner:

$$R_{AB} = R_t \quad c_{pAB} = c_{pt} \quad \gamma_{AB} = \gamma_t \quad T_{t7} = T_{t4} \tau_{tH} \tau_{tL} \quad \pi_{AB} = 1 \quad f_{AB} = 0 \tag{SM8.2q}$$

With afterburner:

$$R_{AB} = \frac{\gamma_{AB} - 1}{\gamma_{AB}} c_{pAB} \tag{SM8.2r}$$

$$\tau_{\lambda AB} = \frac{c_{pAB} T_{t7}}{c_{pc} T_0} \tag{SM8.2s}$$

$$f_{\mathrm{AB}} = \frac{h_{t7} - h_{t4}\tau_{tH}\tau_{tL}}{h_{\mathrm{PR}}\eta_{\mathrm{AB}} - h_{t7}} \tag{SM8.2t}$$

Remainder of equations:

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r\pi_d\pi_{cL}\pi_{cH}\pi_b\pi_{tH}\pi_{tL}\pi_{\mathrm{AB}}\pi_n \tag{SM8.2u}$$

$$M_9 = \sqrt{\frac{2}{\gamma_{\mathrm{AB}-1}}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_{\mathrm{AB}}-1)/\gamma_{\mathrm{AB}}} - 1\right]} \tag{SM8.2v}$$

$$\frac{T_9}{T_0} = \frac{T_{t7}/T_0}{(P_{t9}/P_9)^{(\gamma_{\mathrm{AB}}-1)/\gamma_{\mathrm{AB}}}} \tag{SM8.2w}$$

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_{\mathrm{AB}}R_{\mathrm{AB}}}{\gamma_c R_c}\frac{T_9}{T_0}} \tag{SM8.2x}$$

$$f_O = f + f_{\mathrm{AB}} \tag{SM8.2y}$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f_0)\frac{V_9}{a_0} - M_0 + (1+f_O)\frac{R_{\mathrm{AB}}}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] \tag{SM8.2z}$$

$$F = \dot{m}_0\left(\frac{F}{\dot{m}_0}\right) \tag{SM8.2aa}$$

$$S = \frac{f_O}{F/\dot{m}_0} \tag{SM8.2ab}$$

$$\eta_{Th} = \frac{a_0^2[(1+f_O)(V_9/a_0)^2 - M_0^2]}{2g_c f_O h_{PR}} \tag{SM8.2ac}$$

$$\eta_P = \frac{2g_c V_0(F/\dot{m}_0)}{a_0^2[(1+f_O)(V_9/a_0)^2 - M_0^2]} \tag{SM8.2ad}$$

$$\eta_O = \eta_P\eta_{Th} \tag{SM8.2ae}$$

$$\left(\frac{N}{N_R}\right)_{\mathrm{LPspool}} = \sqrt{\frac{T_0\tau_r}{T_{0R}\tau_{rR}}\frac{\pi_{cL}^{(\gamma-1)/\gamma} - 1}{\pi_{cLR}^{(\gamma-1)/\gamma} - 1}} \tag{SM8.2af}$$

$$\left(\frac{N}{N_R}\right)_{\text{HPspool}} = \sqrt{\frac{T_0 \tau_r \tau_{cL}}{T_{0R} \tau_{rR} \tau_{cLR}} \frac{\pi_{cH}^{(\gamma-1)/\gamma} - 1}{\pi_{cHR}^{(\gamma-1)/\gamma} - 1}} \tag{SM8.2ag}$$

$$\frac{A_9}{A_{9R}} = \left[\frac{P_{t9}/P_9}{(P_{t9}/P_9)_R}\right]^{(\gamma_t+1)/(2\gamma_t)} \sqrt{\frac{(P_{t9}/P_9)_R^{(\gamma_t-1)/\gamma_t} - 1}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t} - 1}} \tag{SM8.2ah}$$

8.5a.SM Summary of Performance Equations: Turbofan Engine with Separate Exhausts and Convergent Nozzles

Inputs:

Choices	
Flight parameters:	M_0, T_0 (K, °R), P_0 (kPa, psia)
Throttle setting:	T_{t4} (K, °R)
Design constants	
π:	$\pi_{d\,\max}$, π_b, π_{tH}, π_n, π_{fn}
τ:	τ_{tH}
η:	η_f, η_{cH}, η_b, η_{mH}, η_{mL}
Gas properties:	γ_c, γ_t, c_{pc}, c_{pt} [kJ/(kg · K), Btu/(lbm · °R)]
Fuel:	h_{PR} (kJ/kg, Btu/lbm)
Reference conditions	
Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia), τ_{rR}, π_{rR}
Throttle setting:	T_{t4R} (K, °R)
Component behavior:	π_{dR}, π_{fR}, π_{cHR}, π_{tL}, τ_{fR}, τ_{cHR}, τ_{rLR}, α_R, M_{9R}, M_{19R}

Outputs:

Overall performance:	F(N, lbf), $\dot{m}_0$ (kg/s, lbm/s), f, $S\left(\frac{\text{g/s}}{\text{kN}}, \frac{\text{lbm/h}}{\text{lbf}}\right)$, η_P, η_{Th}, η_O
Component behavior:	α, π_f, π_{cH}, π_{tL}, τ_f, τ_{cH}, τ_{tL}, f, M_9, M_{19}, N_{fan}, N_{HPspool}
Exhaust nozzle pressure:	P_0/P_9, P_0/P_{19}

Equations:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{SM8.3a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{SM8.3b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{SM8.3c}$$

$$V_0 = a_0 M_0 \tag{SM8.3d}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{SM8.3e}$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c-1)} \tag{SM8.3f}$$

$$\eta_r = 1 \quad \text{for } M_0 \leq 1 \tag{SM8.3g}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \quad \text{for } M_0 > 1 \tag{SM8.3h}$$

$$\pi_d = \pi_{d\,\max} \eta_r \tag{SM8.3i}$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} \tag{SM8.3j}$$

Initial values:

$$\tau_{tL} = \tau_{tLR} \qquad \tau_f = \tau_{fR} \qquad \pi_{tL} = \pi_{tLR}$$

$$\tau_{cH} = 1 + \frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R} \frac{\tau_{fR}}{\tau_f} (\tau_{cHR} - 1) \tag{SM8.3k}$$

$$\pi_{cH} = [1 + (\tau_{cH} - 1)\eta_{cH}]^{\gamma_c/(\gamma_c-1)} \tag{SM8.3l}$$

$$\pi_f = [1 + (\tau_f - 1)\eta_f]^{\gamma_c/(\gamma_c-1)} \tag{SM8.3m}$$

Exhaust nozzles:

$$\frac{P_{t19}}{P_0} = \pi_r \pi_d \pi_f \pi_{\text{fn}} \tag{SM8.3n}$$

$$\text{If} \quad \frac{P_{t19}}{P_0} < \left(\frac{\gamma_c + 1}{2}\right)^{\gamma_c/(\gamma_c-1)} \quad \text{then} \quad \frac{P_{t19}}{P_{19}} = \frac{P_{t19}}{P_0}$$

$$\text{else} \quad \frac{P_{t19}}{P_{19}} = \left(\frac{\gamma_c + 1}{2}\right)^{\gamma_c/(\gamma_c-1)} \tag{SM8.3o}$$

$$M_{19} = \sqrt{\frac{2}{\gamma_c - 1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c-1)/\gamma_c} - 1\right]} \tag{SM8.3p}$$

$$\frac{P_{t9}}{P_0} = \pi_r \pi_d \pi_f \pi_{cH} \pi_b \pi_{tH} \pi_{tL} \pi_n \tag{SM8.3q}$$

$$\text{If} \quad \frac{P_{t9}}{P_0} < \left(\frac{\gamma_t + 1}{2}\right)^{\gamma_t/(\gamma_t - 1)} \quad \text{then} \quad \frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_0}$$

$$\text{else} \quad \frac{P_{t9}}{P_9} = \left(\frac{\gamma_t + 1}{2}\right)^{\gamma_t/(\gamma_t - 1)} \tag{SM8.3r}$$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t - 1)/\gamma_t} - 1\right]} \tag{SM8.3s}$$

$$\alpha = \alpha_R \frac{\pi_{cHR}}{\pi_{cH}} \sqrt{\frac{\tau_\lambda/(\tau_r \tau_f)}{[\tau_\lambda/(\tau_r \tau_f)]_R}} \frac{\text{MFP}(M_{19})}{\text{MFP}(M_{19R})} \tag{SM8.3t}$$

$$\tau_f = 1 + \frac{1 - \tau_{tL}}{(1 - \tau_{tL})_R} \frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R} \frac{1 + \alpha_R}{1 + \alpha} (\tau_{fR} - 1) \tag{SM8.3u}$$

$$\tau_{tL} = 1 - \eta_{tL}(1 - \pi_{tL}^{(\gamma_t - 1)/\gamma_t}) \tag{SM8.3v}$$

$$\pi_{tL} = \pi_{tLR} \sqrt{\frac{\tau_{tL}}{\tau_{tLR}}} \frac{\text{MFP}(M_{9R})}{\text{MFP}(M_9)} \tag{SM8.3w}$$

If τ_{tL} is not within 0.0001 of its previous value, return to Eq. (SM8.3k) and perform another iteration.

Remainder of calculations:

$$\dot{m}_0 = \dot{m}_{0R} \frac{1 + \alpha}{1 + \alpha_R} \frac{P_0 \pi_r \pi_d \pi_f \pi_{cH}}{(P_0 \pi_r \pi_d \pi_f \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{SM8.3x}$$

$$f = \frac{h_{t4} - h_{t3}}{h_{PR}\eta_b - h_{t4}} \tag{SM8.3y}$$

$$\frac{T_9}{T_0} = \frac{\tau_\lambda \tau_{tH} \tau_{tL}}{(P_{t9}/P_9)^{(\gamma_t - 1)/\gamma_t}} \frac{c_{pc}}{c_{pt}} \tag{SM8.3z}$$

$$\frac{V_9}{a_0} = M_9 \sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}} \tag{SM8.3aa}$$

$$\frac{T_{19}}{T_0} = \frac{\tau_r \tau_f}{(P_{t19}/P_{19})^{(\gamma_c - 1)/\gamma_c}} \tag{SM8.3ab}$$

$$\frac{V_{19}}{a_0} = M_{19}\sqrt{\frac{T_{19}}{T_0}} \tag{SM8.3ac}$$

$$\frac{F}{\dot{m}_0} = \frac{1}{1+\alpha}\frac{a_0}{g_c}\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] + \frac{\alpha}{1+\alpha}\frac{a_0}{g_c}\left[\frac{V_{19}}{a_0} - M_0 + \frac{T_{19}/T_0}{V_{19}/a_0}\frac{1-P_0/P_{19}}{\gamma_c}\right] \tag{SM8.3ad}$$

$$S = \frac{f}{(1+\alpha)(F/\dot{m}_0)} \tag{SM8.3ae}$$

$$F = \dot{m}_0\left(\frac{F}{\dot{m}_0}\right) \tag{SM8.3af}$$

$$\left(\frac{N}{N_R}\right)_{\text{fan}} = \sqrt{\frac{T_0\tau_r}{T_{0R}\tau_{rR}}\frac{\pi_f^{(\gamma_c-1)/\gamma_c}-1}{\pi_{fR}^{(\gamma_c-1)/\gamma_c}-1}} \tag{SM8.3ag}$$

$$\left(\frac{N}{N_R}\right)_{\text{HPspool}} = \sqrt{\frac{T_0\tau_r\tau_f}{(T_0\tau_r\tau_f)_R}\frac{\pi_{cH}^{(\gamma_c-1)/\gamma_c}-1}{\pi_{cHR}^{(\gamma_c-1)/\gamma_c}-1}} \tag{SM8.3ah}$$

$$\eta_{Th} = \frac{a_0^2[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2]}{2g_c f h_{PR}} \tag{SM8.3ai}$$

$$\eta_P = \frac{2g_c V_0(1+\alpha)(F/\dot{m}_0)}{a_0^2[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 - (1+\alpha)M_0^2]} \tag{SM8.3aj}$$

$$\eta_O = \eta_P\eta_{Th} \tag{SM8.3ak}$$

8.5b.SM Example 8.8 Calculations for Separate-Exhaust-Stream Turbofan

Given the reference engine (see data) sized for a mass flow rate of 600 lbm/s at 40 kft and Mach 0.8, determine the performance at sea-level static conditions (standard day) with $T_{t4} = 3200°\text{R}$.

$T_0 = 390°\text{R}$, $\gamma_c = 1.4$, $c_{pc} = 0.24\ \text{Btu}/(\text{lbm}\cdot°\text{R})$, $\gamma_t = 1.33$, $c_{pt} = 0.276\ \text{Btu}/(\text{lbm}\cdot°\text{R})$, $T_{t4} = 2750°\text{R}$, $M_0 = 0.8$, $\pi_c = 36$, $\pi_f = 1.7$, $\alpha = 8$, $\eta f = 0.8815$, $f = 0.02315$, $\eta_{cH} = 0.8512$, $\tau_{tH} = 0.7341$, $\pi_{tH} = 0.2505$, $\tau_{tL} = 0.6895$,

$$\pi_{tL} = 0.1892, \qquad \eta_{tL} = 0.9175, \qquad \eta_b = 0.99, \qquad \pi_{d\,\max} = 0.99,$$
$$\pi_b = 0.96, \qquad \pi_n = 0.99, \qquad \pi_{fn} = 0.99, \qquad \eta_{mH} = 0.9915,$$
$$\eta_{mL} = 0.997, \qquad P_0 = 2.730\,\text{psia}(40\,\text{kft}), \qquad \dot{m}_0 = 600\,\text{lbm/s},$$
$$F/\dot{m}_0 = 16.20\,\text{lbf/(lbm/s)}, \qquad F = 9720\,\text{lbf}$$

Performance Conditions:

$$T_0 = 518.7^\circ\text{R}, \qquad P_0 = 14.696 \text{ psia (sea level)}, \qquad T_{t4} = 3200^\circ\text{R}, \qquad M_0 = 0$$

Equations:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} = \frac{0.4}{1.4} 0.24 \times 778.16 = 53.36 \text{ ft} \cdot \text{lbf/(lbm} \cdot {}^\circ\text{R)}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} = \frac{0.33}{1.33} 0.276 \times 778.16 = 53.29 \text{ ft} \cdot \text{lbf/(lbm} \cdot {}^\circ\text{R)}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} = \sqrt{1.4 \times 53.36 \times 32.174 \times 518.7} = 1116.6 \text{ ft/s}$$

$$V_0 = a_0 M_0 = 1116.6 \times 0 = 0$$

$$\tau_r = 1 \qquad \text{and} \qquad \pi_r = 1$$

$$\pi_d = \pi_{d\,\max} \eta_r = 0.99 \times 1 = 0.99$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} = \frac{0.276}{0.240} \frac{3200}{518.7} = 7.095$$

Initial values:

$$\tau_{tL} = \tau_{tLR} = 0.6895 \qquad \tau_f = \tau_{fR} = 1.1857 \qquad \pi_{tL} = \pi_{tLR} = 0.1892$$

$$\tau_{cH} = 1 + \frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R} \frac{\tau_{fR}}{\tau_f} (\tau_{cHR} - 1)$$
$$= 1 + \frac{7.095/1.0}{8.846/1.128} \frac{1.1857}{1.1857} (2.636 - 1) = 2.6143 \tag{i}$$

$$\pi_{cH} = [1 + \eta_{cH}(\tau_{cH} - 1)]^{\gamma_c/(\gamma_c - 1)} = [1 + 0.8512(2.6143 - 1)]^{3.5} = 20.615$$

$$\pi_f = [1 + \eta_f(\tau_f - 1)]^{\gamma_c/(\gamma_c - 1)} = [1 + 0.8815(1.1857 - 1)]^{3.5} = 1.70$$

Fan exhaust nozzles:

$$\frac{P_{t19}}{P_0} = \pi_r \pi_d \pi_f \pi_{fn} = 1 \times 0.99 \times 1.70 \times 0.99 = 1.6663$$

Because $P_{t19}/P_0 < 1.893$, then $P_{19} = P_0$

$$M_{19} = \sqrt{\frac{2}{\gamma_c - 1}\left[\left(\frac{P_{t19}}{P_{19}}\right)^{(\gamma_c-1)/\gamma_c} - 1\right]} = \sqrt{\frac{2}{0.4}(1.6663^{1/3.5} - 1)} = 0.8862$$

Core exhaust nozzle:

$$\frac{P_{t9}}{P_0} = \pi_r \pi_d \pi_f \pi_{cH} \pi_b \pi_{tH} \pi_{tL} \pi_n$$

$$\frac{P_{t9}}{P_0} = 1 \times 0.99 \times 1.70 \times 20.615 \times 0.96 \times 0.2505 \times 0.1892 \times 0.99 = 1.5628$$

Because $P_{t9}/P_0 < 1.851$, then $P_9 = P_0$

$$M_9 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma_t} - 1\right]} = \sqrt{\frac{2}{0.33}(1.5628^{0.33/1.33} - 1)}$$
$$= 0.8426$$

Bypass ratio, fan temperature ratio, and LP turbine

$$\alpha = \alpha_R \frac{\pi_{cHR}}{\pi_{cH}} \sqrt{\frac{\tau_\lambda/(\tau_r \tau_f)}{[\tau_\lambda/(\tau_r \tau_f)]_R}} \frac{\text{MFP}(M_{19})}{\text{MFP}(M_{19R})}$$

$$\alpha = 8.0 \frac{21.177}{20.615} \sqrt{\frac{7.095/1.1857}{8.846/(1.128 \times 1.1857)}} \left(\frac{0.5197}{0.5318}\right) = 7.639$$

$$\tau_f = 1 + \frac{1 - \tau_{tL}}{1 - \tau_{tLR}} \frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R} \frac{1 + \alpha_R}{1 + \alpha} (\tau_{fR} - 1)$$

$$\tau_f = 1 + \frac{1 - 0.6895}{1 - 0.6895} \frac{7.095/1}{8.846/1.128} \frac{1 + 8}{1 + 7.639} (1.1857 - 1) = 1.1750$$

$$\tau_{tL} = 1 - \eta_{tL}\left[1 - \pi_{tL}^{(\gamma_t-1)/\gamma_t}\right] = 1 - 0.9175\left[1 - 0.1892^{0.33/1.33}\right] = 0.6895$$

$$\pi_{tL} = \pi_{tLR} \sqrt{\frac{\tau_{tL}}{\tau_{tLR}}} \frac{\text{MFP}(M_{9R})}{\text{MFP}(M_9)} = 0.1892 \sqrt{\frac{0.6895}{0.6895}} \frac{0.5316}{0.5197} = 0.1935$$

Because τ_{tL} is not within 0.0001 of its previous value, we return to Eq. (i) and do another iteration. After numerous iterations, we get the following values:

$$\tau_{cH} = 2.6203, \quad \pi_{cH} = 20.771, \quad \pi_f = 1.6803, \quad M_{19} = 0.8753, \quad M_9 = 0.8591,$$
$$\alpha = 8.005, \quad \tau_f = 1.1813, \quad \tau_{tL} = 0.6926, \quad \pi_{tL} = 0.1932$$

Remainder of calculations:

$$\dot{m}_0 = \dot{m}_{0R} \frac{1+\alpha}{1+\alpha_R} \frac{P_0 \pi_r \pi_d \pi_f \pi_{cH}}{(P_0 \pi_r \pi_d \pi_f \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}}$$

$$\dot{m}_0 = 600 \frac{1+8.005}{1+8} \frac{14.696 \times 1.0 \times 0.99 \times 1.6804 \times 20.770}{2.730 \times 1.524 \times 0.99 \times 1.7 \times 21.176} \sqrt{\frac{2750}{3200}}$$
$$= 1905.6\,\text{lbm/s}$$

$$T_{t3} = T_0 \tau_r \tau_f \tau_{cH} = 518.7 \times 1 \times 1.1813 \times 2.6203 = 1605.5^\circ\text{R}$$

$$f = \frac{h_{t4} - h_{t3}}{h_{PR}\eta_b - h_{t4}} = \frac{886.90 - 397.08}{18{,}400 \times 0.99 - 886.90} = 0.028266$$

$$\frac{T_9}{T_0} = \frac{T_{t4}\tau_{tH}\tau_{tL}/T_0}{(P_{t9}/P_9)^{(\gamma_t - 1)/\gamma_t}} = \frac{3200 \times 0.7341 \times 0.6926/518.7}{(1.5890)^{0.33/1.33}} = 2.7962$$

$$\frac{V_9}{a_0} = M_9 \sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}} = 0.8426 \sqrt{\frac{1.33 \times 53.29}{1.4 \times 53.36} 2.7962} = 1.3724$$

$$\frac{T_{19}}{T_0} = \frac{\tau_r \tau_f}{(P_{t19}/P_{19})^{(\gamma_c - 1)/\gamma_c}} = \frac{1.0 \times 1.1813}{(1.6470)^{0.4/1.4}} = 1.0243$$

$$\frac{V_{19}}{a_0} = M_{t9} \sqrt{\frac{T_{19}}{T_0}} = 0.8591\sqrt{1.0243} = 0.8694$$

$$\frac{F}{\dot{m}_0} = \frac{a_0}{(1+\alpha)g_c} \left[(1+f)\frac{V_9}{a_0} - M_0 + \alpha\left(\frac{V_{19}}{a_0} - M_0\right) \right]$$

$$\frac{F}{\dot{m}_0} = \frac{1116.6/32.174}{1+8.005}[1.02823 \times 1.3724 + 8.005 \times 0.8694]$$
$$= 32.26\,\text{lbf/(lbm/s)}$$

$$S = \frac{f}{(1+\alpha)F/\dot{m}_0} = \frac{3600 \times 0.02823}{9.005 \times 32.26} = 0.3498\ \text{(lbm/h)/lbf}$$

$$\left(\frac{N}{N_R}\right)_{\text{fan}} = \sqrt{\frac{T_0\tau_r}{(T_0\tau_r)_R}\frac{\pi_f^{(\gamma_c-1)/\gamma_c}-1}{\pi_{fR}^{(\gamma_c-1)/\gamma_c}-1}} = \sqrt{\frac{518.7\times 1.0}{390\times 1.128}\frac{1.6804^{0.4/1.4}-1}{1.7^{0.4/1.4}-1}}$$
$$= 1.073$$

$$\left(\frac{N}{N_R}\right)_{\text{HP spool}} = \sqrt{\frac{T_0\tau_r\tau_f}{(T_0\tau_r\tau_f)_R}\frac{\pi_{cH}^{(\gamma_c-1)/\gamma_c}-1}{\pi_{cHR}^{(\gamma_c-1)/\gamma_c}-1}}$$
$$= \sqrt{\frac{518.7\times 1.0\times 1.1813}{390\times 1.128\times 1.1857}\frac{20.771^{0.4/1.4}-1}{21.176^{0.4/1.4}-1}} = 1.079$$

8.5c.SM Summary of Performance Equations: Turbofan Engine with Compressor Stages on Low-Pressure Spool

Inputs:

Choices

Flight parameters:	M_0, T_0 (K, °R), P_0 (kPa, psia)
Throttle setting:	T_{t4} (K, °R)

Design constants

π:	$\pi_{d\,\max}$, π_b, π_{tH}, π_{AB}, π_n, π_{fn}
τ:	τ_{tH}
η:	η_f, η_{cL}, η_{cH}, η_b, η_{AB}, η_{mH}, η_{mL}
Gas properties:	γ_c, γ_t, c_{pc}, c_{pt} [kJ/(Kg · K), Btu/(lbm · °R)]
Fuel:	h_{PR} (kJ/kg, Btu/lbm)

Reference conditions

Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia), τ_{rR}, π_{rR}
Throttle setting:	T_{t4R} (K, °R)
Component behavior:	π_{dR}, π_{fR}, π_{cLR}, π_{cHR}, π_{tL}, τ_{fR}, τ_{cHR}, τ_{tLR}, α_R, M_{9R}, M_{19R}

Outputs:

Overall performance:	F(N, lbf), $\dot{m}_0$ (kg/s, lbm/s), f, $S\left(\frac{\text{g/s}}{\text{kN}}, \frac{\text{lbm/h}}{\text{lbf}}\right)$ η_P, η_{Th}, η_O
Component behavior:	α, π_f, π_{cL}, π_{cH}, π_{tL}, τ_f, τ_{cH}, τ_{tL}, f, M_9, M_{19}, N_{fan}, N_{HPspool}
Exhaust nozzle pressure:	P_0/P_9, P_0/P_{19}

Equations (in order of calculation):

Equations (SM8.3a–SM8.3j)

Set initial values: $\tau_{tL} = \tau_{tLR}$ $\quad \tau_f = \tau_{fR}$ $\quad \tau_{cL} = \tau_{cLR}$ $\quad \pi_{tL} = \pi_{tLR}$

Equations (8.54a – 8.54o)

$$\dot{m}_0 = \dot{m}_{0R} \frac{1+\alpha}{1+\alpha_R} \frac{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH}}{(P_0 \pi_r \pi_d \pi_{cL} \pi_{cH})_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{8.52b}$$

$$f = \frac{h_{t4} - h_{t3}}{h_{PR}\eta_b - h_{t4}} \tag{SM8.3y}$$

Equations (SM8.3z – SM8.3ag)

$$\left(\frac{N}{N_R}\right)_{\text{HP spool}} = \sqrt{\frac{T_0 \tau_r \tau_{cL}}{(T_0 \tau_r \tau_{cL})_R} \frac{\pi_{cH}^{(\gamma-1)/\gamma} - 1}{\pi_{cHR}^{(\gamma-1)/\gamma} - 1}} \tag{SM8.4}$$

Equations (SM8.3ai – SM8.3ak)

8.6.SM Summary of Performance Equations: Turbofan with Afterburning Mixed-Flow Exhaust

Inputs:

Choices	
Flight parameters:	M_0, T_0 (K, °R), P_0 (kPa, psia)
Throttle setting:	T_{t4} (K, °R), T_{t7}(K, °R)
Exhaust nozzle:	P_0/P_9 or A_9/A_8
Engine control	TR($\theta_{0\ \text{break}}$)
Design constants	P_0/P_9 or A_9/A_8
π:	$\pi_{d\ \max}$, π_b, π_{tH}, π_{AB}, π_n, $\pi_{M\ \max}$
τ:	τ_{tH}
η:	η_f, η_{cH}, η_b, η_{aB}, η_{mH}, η_{mL}
Gas properties:	γ_c, γ_t, γ_{AB}, c_{pc}, c_{pt}, c_{PAB} [kJ/(Kg · K), Btu/(lbm · °R)]
Reference conditions	
Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia), τ_{rR}, π_{rR}, θ_{0R}
Throttle setting:	T_{t4R} (K, °R), T_{t7R} (K, °R)
Component behavior:	π_{dR}, π_{fR}, π_{cHR}, π_{tLR}, τ_{fR}, τ_{cHR}, τ_{tLR}, α_R, M_{6R}, A_6/A_{16}

Outputs:

Overall performance:	F(N, lbf), $\dot{m}_0$ (kg/s, lbm/s), f_O, $S\left(\frac{\text{g/s}}{\text{kN}}, \frac{\text{lbm/h}}{\text{lbf}}\right)$, η_P, η_{Th}, η_O
Component behavior:	α, π_d, π_f, π_{cH}, π_{tL}, τ_f, τ_{cH}, τ_{tL}, f, f_{AB}, M_9, N_{LPspool}, N_{HPspool}
Exhaust nozzle:	A_9/A_8 or P_0/P_9

Equations (in order of calculation):

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{SM8.5a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{SM8.5b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{SM8.5c}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{SM8.5d}$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c - 1)} \tag{SM8.5e}$$

$$T_{t2} = T_0 \tau_r \tag{SM8.5f}$$

$$\eta_r = 1 \quad \text{for } M_0 \leq 1 \tag{SM8.5g}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \quad \text{for } M_0 > 1 \tag{SM8.5h}$$

$$\pi_d = \pi_{d\,\max} \eta_r \tag{SM8.5i}$$

$$\theta_{0R} = \tau_{rR} \frac{T_{0R}}{T_{\text{ref}}} \tag{SM8.5j}$$

$$\alpha'_R = \frac{\alpha_R}{1 + f_R} \tag{SM8.5k}$$

$$\text{If} \quad \theta_{0R} \geq \text{TR} \quad \text{then} \quad T_{t4\ \max} = T_{t4R} \tag{SM8.5l}$$

$$\text{else} \quad T_{t4\ \max} = T_{t4R} \left(\frac{TR}{\theta_{0R}} \right) \tag{SM8.5m}$$

$$\theta_0 = \tau_r \frac{T_0}{T_{\text{ref}}} \tag{SM8.5n}$$

$$\text{If} \quad \theta_0 \geq \text{TR} \quad \text{then} \quad T_{t4\,\text{lim}} = T_{t4\ \max} \tag{SM8.5o}$$

$$\text{else} \quad T_{t4\,\text{lim}} = T_{t4\ \max} \frac{\theta_0}{\text{TR}} \tag{SM8.5p}$$

$$\text{If} \quad \text{TR} \geq 1 \quad \text{and} \quad T_{t4} > T_{t4\text{lim}} \quad \text{then} \quad T_{t4} = T_{t4\text{lim}} \quad \text{and} \quad \alpha' = \alpha'_R \tag{SM8.5q}$$

$$\text{else} \quad \alpha' = \alpha'_R \frac{\theta_0}{\theta_{0R}} \tag{SM8.5r}$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} \tag{SM8.5s}$$

Set initial values:

$$\tau_{tL} = \tau_{tLR} \quad \tau_f = \tau_{fR} \quad \pi_{tL} = \pi_{tLR} \tag{SM8.5t}$$

$$\pi_{tL} = \left(1 - \frac{1-\tau_{tL}}{\eta_{tL}}\right)^{\gamma_t/(\gamma_t-1)} \tag{SM8.5u}$$

$$\alpha = \alpha_R \frac{\alpha'}{\alpha'_R} \tag{SM8.5v}$$

$$\tau_f = 1 + \frac{1-\tau_{tL}}{(1-\tau_{tL})_R} \frac{\tau_\lambda/\tau_r}{(\tau_\lambda/\tau_r)_R} \frac{1+\alpha_R}{1+\alpha}(\tau_{fR} - 1) \tag{SM8.5w}$$

$$\pi_f = [1 + (\tau_f - 1)\eta_f]^{\gamma_c/(\gamma_c-1)} \tag{SM8.5x}$$

$$\tau_{cH} = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R} \frac{(\tau_r\tau_f)_R}{\tau_r\tau_f}(\tau_{cH} - 1)_R \tag{SM8.5y}$$

$$\pi_{cH} = [1 + \eta_{cH}(\tau_{cH} - 1)]^{\gamma_c/(\gamma_c-1)} \tag{SM8.5z}$$

Set mixer entrance gas properties:

$$R_6 = R_t, \quad R_{16} = R_c, \quad C_{p6} = C_{pt}, \quad C_{p16} = C_{pc}, \quad \gamma_6 = \gamma_t, \quad \gamma_{16} = \gamma_c$$

$$\text{MFP}(M_6) = \text{MFP}(M_{6R})\sqrt{\frac{\tau_{tL}}{\tau_{tLR}} \frac{\pi_{tLR}}{\pi_{tL}}} \tag{SM8.5aa}$$

$$\frac{P_{t16}}{P_{16}} = \frac{(\pi_{cH}\pi_{tL})_R}{\pi_{cH}\pi_{tL}} \left(\frac{P_{t16}}{P_{t9}}\right)_R \left(1 + \frac{\gamma_6 - 1}{2} M_6^2\right)^{\gamma_6/(\gamma_6-1)} \tag{SM8.5ab}$$

$$M_{16} = \left\{ \frac{2}{\gamma_{16} - 1} \left[\left(\frac{P_{t16}}{P_{16}}\right)^{(\gamma_{16}-1)/\gamma_{t6}} - 1 \right] \right\}^{1/2} \tag{SM8.5ac}$$

$$\tau_{tLN} = \frac{T_{t2}}{T_{t4}} \frac{\tau_f}{\tau_{tH}} \left(\frac{A_6}{A_{16}} \frac{M_6}{M_{16}} \alpha'\right)^2 \frac{\gamma_6}{\gamma_{16}} \frac{R_{16}}{R_6} \frac{1 + \frac{\gamma_6 - 1}{2} M_6^2}{1 + \frac{\gamma_{16} - 1}{2} M_{16}^2} \tag{SM8.5ad}$$

If $|\tau_{tLN} - \tau_{tL}| > 0.0001$, then

if $\tau_{tLN} - \tau_{tL} > 0.0001$, increase τ_{tL} and go to Eq. (SM8.5u) (SM8.5ae)

if $\tau_{tLN} - \tau_{tL} < -0.0001$, decrease τ_{tL} and go to Eq. (SM8.5u)

End if

$$c_{p6A} = \frac{c_{p6} + \alpha' c_{p16}}{1 + \alpha'} \qquad R_{6A} = \frac{R_6 + \alpha' R_{16}}{1 + \alpha'}$$

$$\gamma_{6A} = \frac{c_{p6A}}{c_{p6A} - R_{6A}} \tag{SM8.5af}$$

$$\frac{T_{t16}}{T_{t6}} = \frac{T_{t2}}{T_{t4}} \frac{\tau_f}{\tau_{tH}\tau_{tL}} \tag{SM8.5ag}$$

$$\tau_M = \frac{c_{p6}}{c_{p6A}} \frac{1 + \alpha'(c_{p16}/c_{p6})(T_{t16}/T_{t6})}{1 + \alpha'} \tag{SM8.5ah}$$

$$\phi(M_6, \gamma_6) = \frac{M_6^2\left(1 + \dfrac{\gamma_6 - 1}{2} M_6^2\right)}{(1 + \gamma_6 M_6^2)^2}$$

$$\phi(M_{16}, \gamma_{16}) = \frac{M_{16}^2\left(1 + \dfrac{\gamma_{16} - 1}{2} M_{16}^2\right)}{(1 + \gamma_{16} M_{16}^2)^2} \tag{SM8.5ai}$$

$$\Phi = \left[\frac{1 + \alpha'}{\dfrac{1}{\sqrt{\phi(M_6, \gamma_6)}} + \alpha' \sqrt{\dfrac{R_{16}\gamma_6}{R_6 \gamma_{16}} \dfrac{T_{t6}/T_{t16}}{\phi(M_{16}, \gamma_{16})}}}\right]^2 \frac{R_{6A}\gamma_6}{R_6 \gamma_{6A}} \tau_M \tag{SM8.5aj}$$

$$M_{6A} = \sqrt{\frac{2\Phi}{1 - 2\gamma_{6A}\Phi + \sqrt{1 - 2(\gamma_{6A} + 1)\Phi}}} \tag{SM8.5ak}$$

$$\pi_{M\,\text{ideal}} = \frac{(1 + \alpha')\sqrt{\tau_M}}{1 + A_{16}/A_6} \frac{\text{MFP}(M_6, \gamma_6, R_6)}{\text{MFP}(M_{6A}, \gamma_{6A}, R_{6A})} \tag{SM8.5al}$$

$$\pi_M = \tau_{M\,\max}\pi_{M\,\text{ideal}} \tag{SM8.5am}$$

$$\pi_{\text{AB dry}} = 1 - \frac{1}{2}(1 - \pi_{\text{AB}}) \tag{SM8.5an}$$

$$\left(\frac{P_{t9}}{P_9}\right)_{\text{dry}} \geq \frac{P_0}{P_9} \pi_r \pi_d \pi_f \pi_{cH} \pi_b \pi_{tH} \pi_{tL} \pi_M \pi_{\text{AB dry}} \pi_n \tag{SM8.5ao}$$

Is

$$\left(\frac{P_{t9}}{P_9}\right)_{\text{dry}} \geq \left(\frac{\gamma_{6A} + 1}{2}\right)^{\gamma_{6A}/(\gamma_{6A} - 1)} \tag{SM8.5ap}$$

true? If so, then the exhaust nozzle is choked and $M_{8\,\text{dry}} = 1$. If not, then the exhaust nozzle is unchoked and

$$M_{8\,\text{dry}} = \sqrt{\frac{2}{\gamma_{6A} - 1}\left[\left(\frac{P_{t9}}{P_9}\right)_{\text{dry}}^{(\gamma_{6A}-1)/\gamma_{6A}} - 1\right]} \tag{SM8.5aq}$$

$$\alpha'_N = (1 + \alpha'_R)\frac{\pi_{tL}\pi_M}{(\pi_{tL}\pi_M)_R}\sqrt{\frac{(\tau_{tL}\tau_M)_R}{\tau_{tL}\tau_M}}\frac{\text{MFP}(M_{8\,\text{dry}}, \gamma_{6A}, R_{6A})}{\text{MFP}(M_{8\,\text{dry}}, \gamma_{6A}, R_{6A})_R} - 1 \tag{SM8.5ar}$$

If $|\alpha'_N - \alpha'| > 0.0001$, then set $\alpha' = \alpha'_N$ and return to Eq. (SM8.5u).

$$\alpha = \alpha_R\frac{\alpha'}{\alpha'_R} \tag{SM8.5as}$$

$$\dot{m}_0 = \dot{m}_{0R}\frac{1 + \alpha}{1 + \alpha_R}\frac{P_0\pi_r\pi_d\pi_f\pi_{cH}}{(P_0\pi_r\pi_d\pi_f\pi_{cH})_R}\sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{SM8.5at}$$

$$f = \frac{h_{t4} - h_{t3}}{h_{PR}\eta_b - h_{t4}} \tag{SM8.5au}$$

Is $T_{t7} \geq T_{t7R}$? If so, then $c_{p\text{AB}}$, R_{AB}, and γ_{AB} are equal to their reference values, $x = 1$, and go to Eq. (SM.8.5az). Else, calculate the ratio

$$x = \frac{T_{t7} - T_{t6A}}{T_{t7R} - T_{t6A}} \tag{SM8.5av}$$

and then determine the revised values of $c_{p\text{AB}}$, R_{AB}, and γ_{AB} using

$$c_{pa\text{B}} = c_{p6A} + x(c_{p\text{AB}R} - c_{p6A}) \tag{SM8.5aw}$$

$$R_{\text{AB}} = R_{6A} + x(R_{\text{AB}R} - R_{6a}) \tag{SM8.5ax}$$

$$\gamma_{\text{AB}} = \frac{c_{p\text{AB}}}{c_{p\text{AB}} - R_{\text{AB}}} \tag{SM8.5ay}$$

$$\pi_{\text{AB}} = 1 - (1 + x)(1 - \pi_{\text{AB dry}}) \tag{SM8.5az}$$

$$\tau_{\lambda\text{AB}} = \frac{c_{p\text{AB}}T_{t7}}{c_{pc}T_0} \tag{SM8.5ba}$$

$$f_{\text{AB}} = \left(1 + \frac{f}{1 + \alpha}\right)\frac{h_{t7} - h_{t4}\tau_{tH}\tau_{tL}\tau_M}{\eta_{\text{AB}}h_{PR} - h_{t7}} \tag{SM8.5bb}$$

$$f_O = \frac{f}{1 + \alpha} + f_{\text{AB}} \tag{SM8.5bc}$$

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r\pi_d\pi_f\pi_{cH}\pi_b\pi_{tH}\pi_{tL}\pi_M\pi_{\text{AB}}\pi_n \tag{SM8.5bd}$$

$$M_9 = \sqrt{\frac{2}{\gamma_9 - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_9-1)/\gamma_9} - 1\right]} \tag{SM8.5be}$$

$$\frac{T_9}{T_0} = \frac{T_{t7}/T_0}{(P_{t9}/P_9)^{(\gamma_9-1)/\gamma_9}} \tag{SM8.5bf}$$

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_9 R_9}{\gamma_c r_c}\frac{T_9}{T_0}} \tag{SM8.5bg}$$

$$\frac{E}{\dot{m}_0} = \frac{a_0}{g_c}\left[(1+f_O)\frac{V_9}{a_0} - M_0 + (1+f_O)\frac{R_9}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] \tag{SM8.5bh}$$

$$S = \frac{f_O}{F/\dot{m}_0} \tag{SM8.5bi}$$

$$\eta_P = \frac{2g_c V_0(F/\dot{m}_0)}{a_0^2[(1+f_O)(V_9/a_0)^2 - M_0^2]} \tag{SM8.5bj}$$

$$\eta_T = \frac{a_0^2[(1+f_O)(V_9/a_0)^2 - M_0^2]}{2g_c f_o h_{PR}} \tag{SM8.5bk}$$

$$\eta_O = \eta_P \eta_T \tag{SM8.5bl}$$

$$\dot{m}_{c2} = \dot{m}_{c2R}\frac{1+\alpha}{1+\alpha_R}\frac{\pi_f \pi_{cH}}{(\pi_f \pi_{cH})_R}\sqrt{\frac{(T_{t4}/T_{t2})_R}{T_{t4}/T_{t2}}} \tag{SM8.5bm}$$

$$\left(\frac{N}{N_R}\right)_{\text{LPspool}} = \sqrt{\frac{T_0\tau_r}{(T_0\tau_r)_R}\frac{\pi_f^{(\gamma_c-1)/\gamma_c} - 1}{\pi_{fR}^{(\gamma_c-1)/\gamma_c} - 1}} \tag{SM8.5bn}$$

$$\left(\frac{N}{N_R}\right)_{\text{HPspool}} = \sqrt{\frac{T_0\tau_r\tau_f}{(T_0\tau_r\tau_f)_R}\frac{\pi_{cH}^{(\gamma_c-1)/\gamma_c} - 1}{\pi_{cHR}^{(\gamma_c-1)/\gamma_c} - 1}} \tag{SM8.5bo}$$

If the exhaust nozzle is choked, then

$$\frac{A_9}{A_8} = \frac{\Gamma_8}{\pi_n}\sqrt{\frac{\gamma_8 - 1}{2\gamma_8}}\frac{(P_{t9}/P_9)^{(\gamma_8+1)/(2\gamma_8)}}{\sqrt{(P_{t9}/P_9)^{(\gamma_8-1)/\gamma_8} - 1}} \tag{SM8.5bp}$$

Else

$$\frac{A_9}{A_8} = \frac{1}{\pi_n} \tag{SM8.5bq}$$

End if

8.7.SM Summary of Performance Equations: Turboprop

Inputs:

Choices

Flight parameters:	M_0, T_0 (K, °R), P_0 (kPa, psia)
Throttle setting:	T_{t4} (K, °R)

Design constants

π:	$\pi_{d\,\max}$, π_b, π_{tH}, π_n
τ:	τ_{tH}
η:	η_c, η_b, η_{tL}, η_{mL}, η_g, $\eta_{\text{prop max}}$
Gas properties:	γ_c, γ_t, c_{pc}, c_{pt} [KJ/(kg · K), Btu/(lbm · °R)]
Fuel:	h_{PR} (kJ/kg, Btu/lbm)

Reference conditions

Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia), τ_{rR}, π_{rR}
Throttle setting:	T_{t4R} (K, °R)
Component behavior:	π_{dR}, $\pi_{c\,R}$, $\pi_{tL\,R}$, $\tau_{tL\,R}$
Exhaust nozzle:	M_{9R}

Outputs:

Overall performance:	$F(\text{N, lbf})$, $\dot{W}(kW, hp)$, $\dot{m}_0\left(\frac{\text{kg}}{\text{s}}, \frac{\text{lbm}}{\text{s}}\right)$, $S\left(\frac{\text{g/s}}{\text{kN}}, \frac{\text{lbm/h}}{\text{lbf}}\right)$, $S_P\left(\frac{\text{g/s}}{\text{MW}}, \frac{\text{lbm/h}}{\text{hp}}\right)$, f, η_p, η_{Th}, η_O, C_C, C_{prop}, C_{tot}
Component behavior:	π_c, τ_c, π_{tL}, τ_{tL}, f, M_9, $N_{\text{core spool}}$, $N_{\text{power spool}}$

Equations:

$$R_c = \frac{\gamma_c - 1}{\gamma_c} c_{pc} \tag{SM8.6a}$$

$$R_t = \frac{\gamma_t - 1}{\gamma_t} c_{pt} \tag{SM8.6b}$$

$$a_0 = \sqrt{\gamma_c R_c g_c T_0} \tag{SM8.6c}$$

$$V_0 = a_0 M_0 \tag{SM8.6d}$$

$$\tau_r = 1 + \frac{\gamma_c - 1}{2} M_0^2 \tag{SM8.6e}$$

$$\pi_r = \tau_r^{\gamma_c/(\gamma_c - 1)} \tag{SM8.6f}$$

$$\eta_r = 1 \tag{SM8.6g}$$

$$\pi_d = \pi_{d\,\max} \eta_r \tag{SM8.6h}$$

$$\tau_c = 1 + \frac{T_{t4}/T_0}{(T_{t4}/T_0)_R} \frac{(\tau_r)_R}{\tau_r} (\tau_c - 1)_R \tag{SM8.6i}$$

$$\pi_c = [1 + \eta_c(\tau_c - 1)]^{\gamma_c/(\gamma_c - 1)} \tag{SM8.6j}$$

$$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_0} \tag{SM8.6k}$$

$$f = \frac{h_{t4} - h_{t3}}{h_{PR}\eta_b - h_{t4}} \tag{SM8.6l}$$

$$\dot{m}_0 = \dot{m}_{0R} \frac{P_0 \pi_r \pi_d \pi_c}{(P_0 \pi_r \pi_d \pi_c)_R} \sqrt{\frac{T_{t4R}}{T_{t4}}} \tag{SM8.6m}$$

Initial value of π_{tL}:

$$\pi_{tL} = \pi_{tLR} \tag{SM8.6n}$$

Low-pressure turbine and exhaust nozzle:

$$\tau_{tL} = 1 - \eta_{tL}(1 - \pi_{tL}^{(\gamma_t - 1)/\gamma_t}) \tag{SM8.6o}$$

$$\frac{P_{t9}}{P_0} = \pi_r \pi_d \pi_c \pi_b \pi_{tH} \pi_{tL} \pi_n \tag{SM8.6 p}$$

If

$$\frac{P_{t9}}{P_0} \geq \left(\frac{\gamma_t + 1}{2}\right)^{\gamma_t/(\gamma_t - 1)}$$

then

$$M_9 = 1 \qquad \frac{P_{t9}}{P_9} = \left(\frac{\gamma_t + 1}{2}\right)^{\gamma_t/(\gamma_t - 1)}$$

and

$$\frac{P_0}{P_9} = \frac{P_{t9}/P_9}{P_{t9}/P_0} \tag{SM8.6q}$$

else

$$\frac{P_0}{P_9} = 1 \qquad \frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_0}$$

and

$$M_9 = \sqrt{\frac{2}{\gamma_1 - 1}\left[\left(\frac{P_{t9}}{P_0}\right)^{(\gamma_t - 1)/\gamma_t} - 1\right]} \tag{SM8.6r}$$

$$\pi_{tLN} = \pi_{tLR}\sqrt{\frac{\tau_{tL}}{\tau_{tLR}}}\frac{\text{MFP}(M_{9R})}{\text{MFP}(M_9)} \tag{SM8.6s}$$

Is $|\tau_{tLN} - \pi_{tL}| \le 0.0001$? If so, then continue. If not, set $\pi_{tL} = \pi_{tLN}$ and return to Eq. (SM8.6o).

$$\frac{T_9}{T_0} = \frac{T_{t4}\tau_{tH}\tau_{tL}}{(P_{t9}/P_9)^{(\gamma_t-1)/\gamma_t}} \tag{SM8.6t}$$

$$\frac{V_9}{a_0} = M_9\sqrt{\frac{\gamma_t R_t T_9}{\gamma_c R_c T_0}} \tag{SM8.6u}$$

$$C_C = (\gamma_c - 1)M_0\left[(1+f)\frac{V_9}{a_0} - M_0 + (1+f)\frac{R_t}{R_c}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_c}\right] \tag{SM8.6v}$$

$$C_{\text{prop}} = \eta_{\text{prop}}\eta_g\eta_{mL}(1+f)\tau_\lambda\tau_{tH}(1-\tau_{tL}) \tag{SM8.6w}$$

$$C_{\text{tot}} = C_{\text{prop}} + C_C \tag{SM8.6x}$$

$$\frac{F}{\dot{m}_0} = \frac{C_{\text{tot}}c_{pc}T_0}{V_0} \tag{SM8.6y}$$

$$S = \frac{f}{F/\dot{m}_0} \tag{SM8.6z}$$

$$\frac{\dot{W}}{\dot{m}_0} = C_{\text{tot}}c_{pc}T_0 \tag{SM8.6aa}$$

$$S_P = \frac{f}{C_{\text{tot}}c_{pc}T_0} \tag{SM8.6ab}$$

$$F = \dot{m}_0\left(\frac{F}{\dot{m}_0}\right) \tag{SM8.6ac}$$

$$\dot{W} = \dot{m}_0\left(\frac{\dot{W}}{\dot{m}_0}\right) \tag{SM8.6ad}$$

$$\eta_P = \frac{C_{\text{tot}}}{C_{\text{prop}}/\eta_{\text{prop}} + ([\gamma_c - 1)/2][(1+f)(V_9/a_0)^2 - M_0^2]} \tag{SM8.6ae}$$

$$\eta_{Th} = \frac{C_{\text{tot}}c_{pc}T_0}{fh_{PR}} \tag{SM8.6af}$$

$$\eta_O = \eta_P\eta_{Th} \tag{SM8.6ag}$$

$$\left(\frac{N}{N_R}\right)_{\text{core spool}} = \sqrt{\frac{T_0\tau_r}{T_{0R}\tau_{rR}}\frac{\tau_c - 1}{\tau_{cR} - 1}} \tag{SM8.6ah}$$

$$\left(\frac{N}{N_R}\right)_{\text{power spool}} = \sqrt{\frac{T_{t4}}{T_{t4R}}\frac{1 - \tau_{tL}}{1 - \tau_{tLR}}} \tag{SM8.6ai}$$

8.8.SM Variable Gas Properties

The effect of variable gas properties can be included in the analysis of gas turbine engine performance. One first needs a method to calculate the thermodynamic state of the gas, given the fuel/air ratio f and two independent properties. Equations (2.71–2.73), (2.75a–2.75d), and (2.66), and the constants of Table 2.4 from the textbook permit direct calculation of h, c_p, ϕ, and P_r given the fuel/air ratio f and temperature T. Calculation of the temperature T given the fuel/air ratio f and one of h, P_r, or ϕ requires iteration. The methods developed in this section can be used to hand-calculate the performance of a gas turbine engine by using Appendix L. However, use of a computer is recommended due to the iterative nature of the calculations.

The subroutine FAIR [for products of combustion from air and hydrocarbon fuels of the type $(CH_2)_n$] was developed for use in the AFPROP program, first introduced in Chapter 2 of the textbook and mentioned again in the Supporting Material for Chapters 5 and 7. It contains Eqs. (2.71–2.73), (2.75a–2.75d), and (2.66), and the constants of Table 2.4 from the textbook, and provides direct calculation of h, c_p, ϕ, γ, P_r, R, and the speed of sound α for a given value of the fuel/air ratio f and temperature T. Given the fuel/air ratio f and one of h, P_r, or ϕ, the temperature T can be found by the addition of simple iteration algorithms.

For convenience, we use the nomenclature for the subroutine FAIR first introduced in the Supporting Material for Chapter 7. The primary input of FAIR (one of T, h, P_r or ϕ) is indicated by first listing a corresponding number from 1 to 4, followed by a list of the variables. Table 7.1 in the textbook identifies the four sets of knowns for FAIR. The subroutine FAIR with the first three sets of unknowns is used extensively in this section.

An additional property such as pressure P, density ρ, or entropy s is needed to completely define the thermodynamic state of the gas. We will normally use pressure or entropy as the additional property and Eqs. (2.28) and (2.68) from the textbook to obtain the remaining unknowns first given in Chapter 2 of the textbook:

$$\rho = \frac{P}{RT} \tag{2.28}$$

$$s_2 - s_1 = \phi_2 - \phi_1 - R\ell n\frac{P_2}{P_1} \tag{2.68}$$

The 1-D flow of a perfect gas with variable specific heats can be studied in a manner similar to the material presented in Chapter 3 of the textbook for a calorically perfect gas. The mass flow parameter (MFP) can be written as

$$\mathrm{MFP} = \frac{\dot{m}\sqrt{T_t}}{P_t A} = \rho V \frac{\sqrt{T_t}}{P_t} = \frac{M\sqrt{\gamma g_c R T}}{RT}\frac{\sqrt{T_t}}{P_t/P} = M\sqrt{\frac{\gamma g_c}{R}}\frac{\sqrt{T_t/T}}{P_t/P}$$

where R is a function of the fuel/air ratio f and the terms γ, T_t/T, and P_t/P are functions of the Mach number M, the static or total temperature T or T_t, and the fuel/air ratio f. For convenience, we choose the total temperature T_t for expressing the mass flow parameter in its functional form, and we write

$$\mathrm{MFP} \equiv \frac{\dot{m}\sqrt{T_t}}{P_t A} = M\sqrt{\frac{\gamma g_c}{R}}\frac{\sqrt{T_t/T}}{P_t/P} = \mathrm{MFP}(M, T_t, f) \tag{SM8.7}$$

The mass flow parameter can be calculated for given values of M, T_t, and f by using functional iteration and the subroutine FAIR as outlined in Fig. SM8.1 for the subroutine MASSFP. It starts with an initial guess for the velocity V based on an approximate speed of sound a. Then the velocity is used in a functional iteration loop to obtain the static state.

For the performance analysis of gas turbine engines, two important characteristics of a perfect gas with variable specific heats are the following:

1. The variations of the mass flow parameter, T/T_t, and P/P_t with Mach number, total temperature T_t, and fuel/air ratio f
2. The variation of the mass flow parameter for choked flow with total temperature T_t and fuel/air ratio f

Figure SM8.2a shows the variation of MFP with Mach number for two cases: air alone ($f = 0$) with a total temperature of 1000°R, and products of combustion for a fuel/air ratio of 0.03 with a total temperature of 3000°R. The MFP peaks at Mach 1.0 (this corresponds to choked flow) for both cases, and these curves are very similar to those of Fig. 3.9 in the textbook. Figure SM8.2b gives the variations of T/T_t and P/P_t with Mach number for the same two cases as Fig. SM8.2a: air alone ($f = 0$) with a total temperature of 1000°R, and products of combustion for a fuel/air ratio of 0.03 with a total temperature of 3000°R. The P/P_t curves in Fig. SM8.2b are about the same and are similar to the P/P_t curve in Fig. 3.7 in the textbook; however, the T/T_t curves in Fig. SM8.2b are distinctly different, and the T/T_t curve for air alone at 1000°R is very similar to the T/T_t curve in Fig. 3.7.

Figure SM8.3 shows the variation with total temperature T_t of the maximum mass flow parameter (MFP at $M = 1$) for air alone ($f = 0$) and products of combustion for fuel/air ratios of 0.02, 0.04, and 0.06. From this figure, one can estimate the variation of MFP with engine throttle for choked flow at engine stations 4, 4.5, and 8. The conditions at engine station 4 may vary from a total temperature of 3500°R with a fuel/air ratio

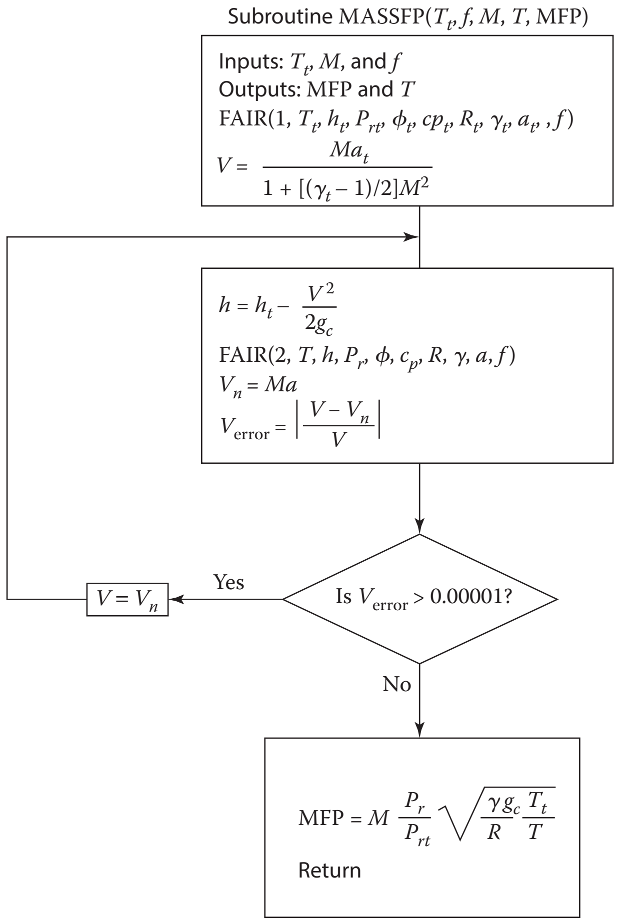

Fig. SM8.1 Flowchart of subroutine MASSFP.

of 0.05 to a total temperature of 1000°R with a fuel/air ratio of nearly zero. For this range, the maximum MFP will vary only about 3%. Likewise, the conditions at engine station 8 may vary over similar ranges. It is this small variation in the corrected mass flow rate per unit area that permitted us to consider it a constant when an engine station was choked ($M = 1$) in earlier sections of this chapter.

To predict engine performance, we need details about the behavior of the engine components. For our computer analysis, the most useful details are analytical expressions for the engine's dependent variables in terms of its independent variables. Table SM8.1 gives these variables for the performance analysis of a dual-spool afterburning turbojet engine with variable gas properties. Note that the fuel/air ratios (f and f_{AB}) show up as dependent variables for the first time.

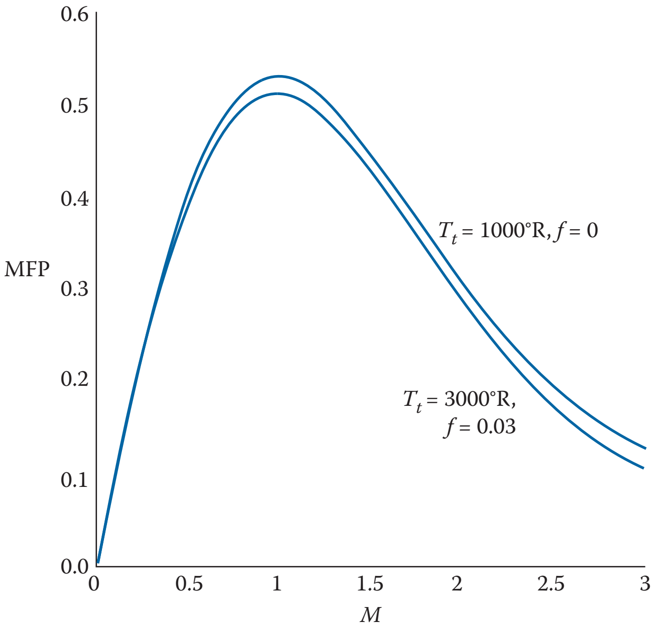

Fig. SM8.2a Variation of MFP with Mach number.

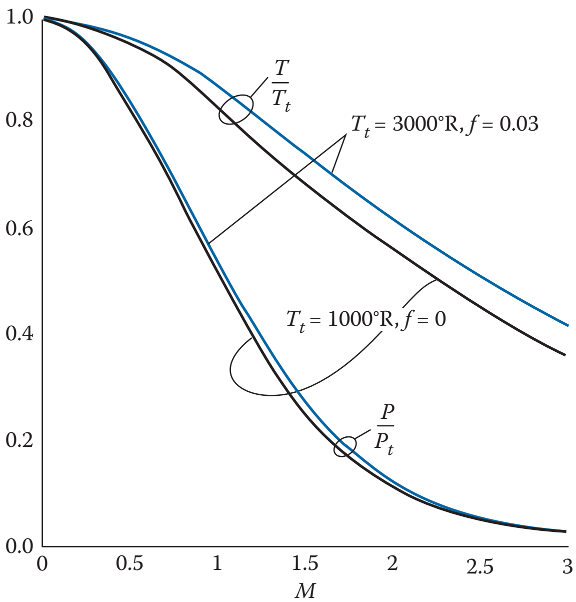

Fig. SM8.2b Variation of P/P_t and T/T_t with Mach number.

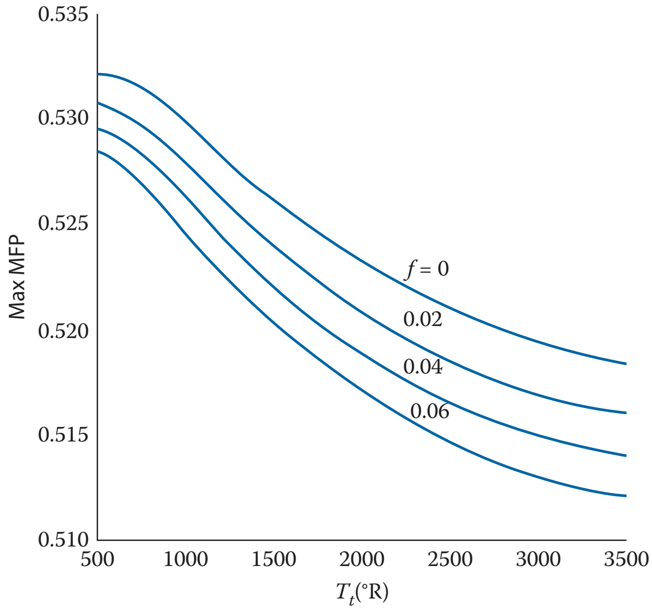

Fig. SM8.3 Variation of maximum MFP with total temperature and fuel/air ratio.

Table SM8.1 Performance Analysis Variables for Afterburning Turbojet Engine with Variable Gas Properties

Component	Variables		
	Independent	Constant or Known	Dependent
Engine	M_0, T_0, P_0		$\dot{m}_0$
Diffuser		$\pi_d = f(M_0)$, τ_0	
Low-pressure compressor		η_{cL}	π_{cL}, τ_{cL}
High-pressure compressor		η_{cH}	π_{cH}, τ_{cH}
Burner	T_{t4}	π_b, η_b	f
High-pressure turbine		η_{tH}, $\dfrac{A_4}{A_{45}}$	π_{tH}, τ_{tH}
Low-pressure turbine		η_{tL}, $\dfrac{A_{4.5}}{A_{8\,\text{dry}}}$	π_{tL}, τ_{tL}
Afterburner	T_{t7}	π_{AB}, η_{AB}	f_{AB}
Nozzle	$\dfrac{P_9}{P_0}$(choked), $\dfrac{A_9}{A_8}$(choked), or $\dfrac{A_8}{A_{8R}}$(unchoked)	π_n, τ_n	M_9
Total number	6		12

Section 6.12 of the textbook develops some of the basics for component performance with variable gas properties. The efficiency relationships between actual and ideal total enthalpy changes are given for the compressor and turbine in textbook Eqs. (6.49) and (6.57), respectively. The relationships between static and total properties are developed and given for the inlet and nozzle in Eqs. (6.42), (6.60), and (6.61). The energy balance of the combustor gives Eq. (6.52) for the fuel/air ratio. These relationships and others will be used in the following section to develop a system of equations that predict the performance of a gas turbine engine with variable gas properties.

8.8.1.SM Turbine Characteristics

We analyze the turbine performance in a gas turbine engine by first considering the high-pressure turbine. Typically, the flow entering both the high-pressure turbine and low-pressure turbine is choked. However, we will consider the general case when the Mach number at stations 4 and 4.5 can be any known value.

Because the mass flows entering and leaving the high-pressure turbine are equal, we can write

$$\dot{m}_4 = \dot{m}_{4.5}$$

Writing this in terms of the mass flow parameter at stations 4 and 4.5 gives

$$\begin{aligned}\mathrm{MFP}_4 = \frac{\dot{m}_4\sqrt{T_{t4}}}{P_{t4}A_4} &= \frac{\dot{m}_{4.5}\sqrt{T_{t4.5}}}{P_{t4.5}A_{4.5}}\frac{P_{t4.5}}{P_{t4}}\frac{A_{4.5}}{A_4}\sqrt{\frac{T_{t4}}{T_{t4.5}}} \\ &= \mathrm{MFP}_{4.5}\frac{P_{t4.5}}{P_{t4}}\frac{A_{4.5}}{A_4}\sqrt{\frac{T_{t4}}{T_{t4.5}}}\end{aligned}$$

Because the fuel/air ratio f is constant for the gas flowing through the high-pressure turbine, this equation can be rewritten as

$$\frac{\pi_{tH}}{\sqrt{T_{t4.5}/T_{t4}}}\frac{\mathrm{MFP}_{4.5}}{\mathrm{MFP}_4} = \frac{A_4}{A_{4.5}} = \mathrm{const} \tag{SM8.8}$$

where the area ratio $A_4/A_{4.5}$ is constant and the mass flow parameters MFP_4 and $\mathrm{MFP}_{4.5}$ are functions of T_{t4}, $T_{t4.5}$, the fuel/air ratio f, and Mach numbers M_4 and $M_{4.5}$.

From Eq. (6.57), the efficiency of the high-pressure turbine is given by

$$\eta_{tH} = \frac{h_{t4} - h_{t4.5}}{h_{t4} - h_{t4.5i}} \tag{SM8.9}$$

Solving this equation for the high-pressure turbine's ideal exit enthalpy $h_{t4.5i}$ gives

$$h_{t4.5i} = h_{t4} - \frac{h_{t4} - h_{t4.5}}{\eta_{tH}} \tag{SM8.10}$$

where the total enthalpies are functions of T_{t4}, $T_{t4.5}$, and the fuel/air ratio f. During normal engine operation, the efficiency of the high-pressure turbine η_{tH} does not vary much. Thus we will consider η_{tH} to be constant in our analysis.

The pressure ratio of the high-pressure turbine π_{tH} is a function of T_{t4}, $T_{t4.5i}$, and the fuel/air ratio f, which can be written in terms of the reduced pressure as

$$\pi_{tH} = \frac{P_{t4.5}}{P_{t4}} = \frac{P_r@T_{t4.5i}}{P_r@T_{t4}} = \frac{P_{rt4.5i}}{P_{rt4}} \tag{SM8.11}$$

In summary, the performance of the high-pressure turbine is a function of the variables T_{t4} and f, known values for the Mach numbers M_4 and $M_{4.5}$, the area ratio $A_4/A_{4.5}$, and the turbine efficiency η_{tH}. Equations (SM8.8), (SM8.11), and (SM8.10) can be written in their functional forms as

$$\begin{aligned} \pi_{tH} &= f_1\left(f,\, T_{t4},\, T_{4.5},\, M_4,\, M_{4.5},\, \frac{A_4}{A_{4.5}}\right) \\ T_{t4.5i} &= f_2(f,\, T_{t4},\, \pi_{tH}) \\ h_{t4.5} &= f_3(f,\, h_{t4},\, h_{t4.5i},\, \eta_{tH}) \end{aligned}$$

For known values of M_4, $M_{4.5}$, $A_4/A_{4.5}$, and η_{tH}, the preceding three functional relationships can be solved for values of T_{t4} and f by using functional iteration [this is the same iteration procedure used to solve Eqs. (8.12a) and (8.12b)].

The performance of the low-pressure turbine depends on the same set of equations as the high-pressure turbine with the subscripts 4, 4.5, and H replaced by 4.5, 5, and L, respectively. Performance calculations for both the high- and low-pressure turbines can be done by using a single subroutine that we call TURB. A flowchart of subroutine TURB is shown in Fig. SM8.4, which uses the general subscripts 4 and 5 for the turbine inlet and exit, respectively. A starting value for the total temperature leaving the turbine T_{t5} is required to initiate calculations. A good starting value for T_{t5} is its value at the reference conditions T_{t5R} or a previously calculated value. The starting value is designated by the variable T_{t5S} in subroutine TURB.

The performance calculations for the low-pressure turbine use engine station 8 (the throat of the exhaust nozzle) to represent the exit conditions from the low-pressure turbine for several reasons. First, the Mach number and flow area at the exit of the low-pressure turbine (M_5 and A_5) are usually unknown because they depend on a more detailed design of the turbine. Second, the Mach number and flow area at the throat of the

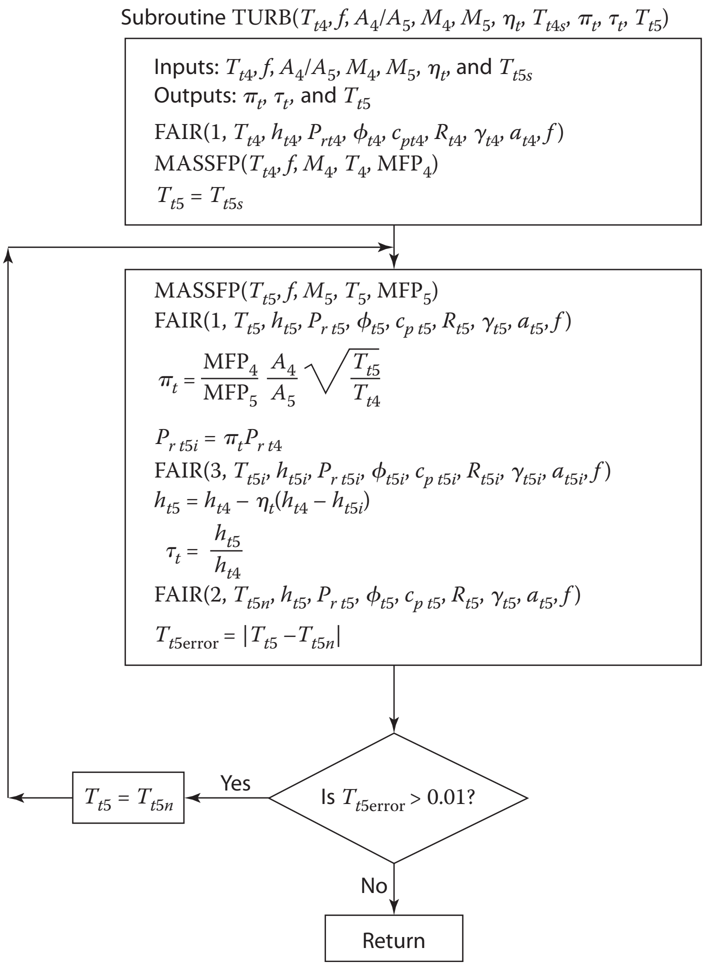

Fig. SM8.4 Flowchart of subroutine TURB.

exhaust nozzle (M_8 and A_8) are directly related to those at station 5. Finally, M_8 and A_8 are known or easily determined. For an afterburning engine, the flow area at the throat of the exhaust nozzle is varied during afterburner operation to maintain the same flow conditions at the exit of the low-pressure turbine. Thus only variations at the throat of the exhaust nozzle when the afterburner is off (designated by *dry*) will affect the operation of the low-pressure turbine. From conservation of mass between engine station 5 and engine station 8 with the afterburner off, we have

$$\frac{\dot{m}_{8\,\mathrm{dry}}}{\dot{m}_5} = 1 = \pi_{\mathrm{AB\,dry}}\sqrt{\frac{T_{t5}}{T_{t8\,\mathrm{dry}}}}\frac{\mathrm{MFP}_{8\,\mathrm{dry}}}{\mathrm{MFP}_5}\frac{A_{8\,\mathrm{dry}}}{A_5}$$

Because the flow from stations 5 to 8 is adiabatic when the afterburner is off, $T_{t5} = T_{t8\,\text{dry}}$ and the above equation gives

$$\text{MFP}_5 A_5 = \pi_{\text{AB dry}} \text{MFP}_{8\,\text{dry}} A_{8\,\text{dry}} \tag{SM8.12}$$

From Eq. (SM8.12), the flow conditions at engine station 8 can be used for those at the exit of the low-pressure turbine by replacing M_5 with M_8 and A_5 with $(\pi_{\text{AB}} A_8)_{\text{dry}}$.

8.8.2.SM Afterburner Area Variation

Using the mass flow parameter at engine station 8, one can easily show that the flow conditions at the throat of the exhaust nozzle, with the afterburner operating (designated by *wet*) and without afterburner (designated by *dry*) are related by

$$\frac{\dot{m}_{8\,\text{wet}}}{\dot{m}_{8\,\text{dry}}} = \frac{1+f+f_{\text{AB}}}{1+f} = \frac{\tau_{\text{AB dry}}}{\pi_{\text{AB wet}}} \sqrt{\frac{T_{t8\,\text{wet}}}{T_{t8\,\text{dry}}}} \frac{\text{MFP}_{8\,\text{dry}}}{\text{MFP}_{8\,\text{wet}}} \frac{A_{8\,\text{dry}}}{A_{8\,\text{wet}}}$$

or

$$\frac{A_{8\,\text{wet}}}{A_{8\,\text{dry}}} = \frac{1+f}{1+f+f_{\text{AB}}} \frac{\pi_{\text{AB dry}}}{\pi_{\text{AB wet}}} \sqrt{\frac{T_{t8\,\text{wet}}}{T_{t8\,\text{dry}}}} \frac{\text{MFP}_{8\,\text{dry}}}{\text{MFP}_{8\,\text{wet}}} \tag{SM8.13}$$

Example SM8.1

We consider the performance of a high-pressure turbine with choked flow at both inlet and exit ($M_4 = 1$ and $M_{4.5} = 1$) as our first example of turbine performance. The reference values are

$$T_{t4\,R} = 3000^\circ\text{R}, \quad T_{t4.5\,R} = 2600^\circ\text{R}, \quad \eta_{tH} = 0.9,$$
$$f_R = 0.03, \quad \frac{A_4}{A_{4.5}} = 0.5183883$$

With subroutine TURB, calculation of the high-pressure turbine performance over the range of T_{t4} from 2000 to 3000°R for fuel/air ratios f of 0.03 and 0.04 gives the results shown in Fig. SM8.5. One can see that τ_{tH}, π_{tH}, and $T_{t4.5}/T_{t4}$ vary little over the range of input data. The changes in τ_{tH}, π_{tH}, and $T_{t4.5}/T_{t4}$ for a change in T_{t4}/f from 2000°R/0.03 to 3000°R/0.04 are 1.22, 0.82, and 1.40%, respectively. This is a very small change, which helps justify the assumption of constant τ_{tH} and π_{tH} used in earlier sections of this chapter.

(*Continued*)

Example SM8.1 (*Continued*)

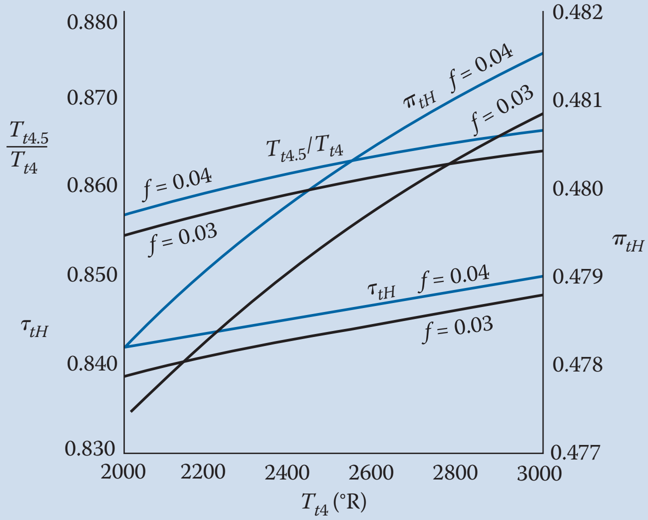

Fig. SM8.5 Variation of high-pressure turbine performance with total temperature.

Example SM8.2

We now consider the performance of a low-pressure turbine with choked flow at its inlet ($M_{4.5} = 1$). Two cases are considered: The Mach number at the throat of the exhaust nozzle M_8 is varied with the area A_8 held constant; and the throat area A_8 is increased with the throat choked ($M_8 = 1$). The reference values are

$$T_{t4.5\,R} = 2600^\circ\text{R}, \quad T_{t5\,R} = 2080^\circ\text{R}, \quad \eta_{tL} = 0.9, \quad f_R = 0.04,$$

$$\pi_{\text{AB dry}} = 1, \quad \frac{A_{4.5}}{A_8} = 0.3686362$$

With subroutine TURB, calculation of the low-pressure turbine performance over the range of exhaust nozzle throat Mach numbers M_8 from 0.6 and 1.0 at fuel/air ratios f of 0.03 and 0.04 gives the results shown in Fig. SM8.6a. Reducing M_8 increases τ_{tL} and π_{tL}. These are the same trends as shown in Fig. 8.7b of the textbook for our basic engine performance model.

Calculation of the low-pressure turbine performance over the range of exhaust nozzle throat areas A_8/A_{8R} from 1.0 to 1.2 at fuel/air ratios f of

(*Continued*)

Example SM8.2 (*Continued*)

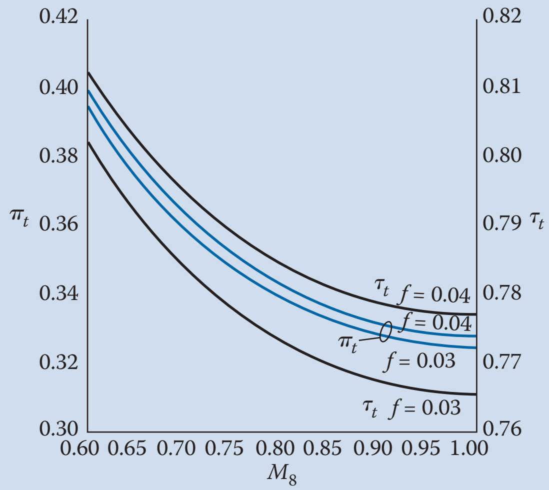

Fig. SM8.6a Variation of low-pressure turbine performance with exhaust nozzle Mach number.

0.03 and 0.04 gives the results shown in Fig. SM8.6b. Increasing A_8 increases τ_{tL} and π_{tL}. Again, these are the same trends shown in Fig. 8.7c of the textbook for our basic engine performance model.

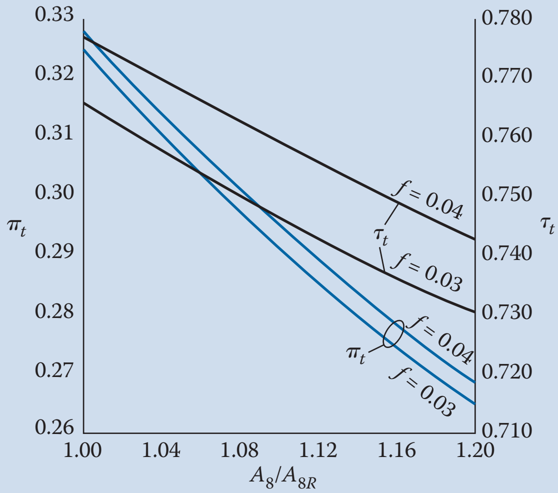

Fig. SM8.6b Variation of low-pressure turbine performance with exhaust nozzle area.

8.8.3.SM Gas Generator Pumping Characteristics

The gas generator pressure ratio P_{t6}/P_{t2} is obtained by multiplying the ratios of total pressure from gas generator inlet to exit, which yields

$$\frac{P_{t6}}{P_{t2}} = \pi_{cL}\pi_{cH}\pi_b\pi_{tH}\pi_{tL} \tag{SM8.14}$$

Similarly, the total temperature ratio of the gas generator T_{t6}/T_{t2} is obtained by multiplying the total temperature ratios, which yields

$$\frac{T_{t6}}{T_{t2}} = \frac{T_{t4}}{T_{t2}}\frac{T_{t4.5}}{T_{t4}}\frac{T_{t5}}{T_{t4.5}} \tag{SM8.15}$$

We consider the case where the inlet flows to the high- and low-pressure turbines are choked ($M_4 = 1$ and $M_{4.5} = 1$) and the flow areas are constant at stations 4, 4.5, 5, 6, and 8. From our previous analysis, the total pressure and total temperature ratios for the high- and low-pressure turbines (π_{tH}, $T_{t4}/T_{t4.5}$, π_{tL}, and $T_{t4.5}/T_{t5}$) are dependent on T_{t4}, f, and exhaust nozzle Mach number M_8.

As shall be shown, the total pressure ratio of the low-pressure compressor π_{cL} and that of the high-pressure compressor π_{cH} are dependent on T_{t2}, T_{t4}, f, and the enthalpy changes across the turbines. We start the compressor analysis with the low-pressure compressor.

8.8.3.1.SM Low-Pressure Compressor

Application of the first law of thermodynamics to the low-pressure compressor and low-pressure turbine gives the exit enthalpy of the low-pressure compressor. Equating the required compressor power to the net output power from the turbine, we have

$$\dot{W}_{cL} = \eta_{mL}\dot{W}_{tL}$$

Rewriting in terms of mass flow rates and total enthalpies gives

$$\dot{m}_0(h_{t2.5} - h_{t2}) = \eta_{mL}(\dot{m}_0 + \dot{m}_f)(h_{t4.5} - h_{t5}) = \eta_{mL}\dot{m}_0(1+f)(h_{t4.5} - h_{t5})$$

Solving for $h_{t2.5}$ gives

$$h_{t2.5} = h_{t2} - \eta_{mL}(1+f)(h_{t4.5} - h_{t5}) \tag{SM8.16}$$

The total pressure ratio of the low-pressure compressor π_{cL} is equal to the ratio of the reduced pressure at the ideal exit state $P_{rt2.5i}$ to the reduced pressure at the inlet P_{rt2}, or

$$\pi_{cL} = \frac{P_{rt2.5i}}{P_{rt2}} \tag{SM8.17}$$

The reduced pressure at the ideal exit of the low-pressure compressor $P_{rt2.5i}$ is obtained by first solving for the ideal exit total enthalpy $h_{t2.5i}$, using the compressor efficiency and the total enthalpies entering and leaving the compressor. Rewriting Eq. (6.49) gives

$$h_{t2.5i} = h_{t2} + \eta_{cL}(h_{t2.5} - h_{t2}) \tag{SM8.18}$$

The reduced pressure for the ideal exit of the low-pressure compressor $P_{rt2.5i}$ follows directly from the value of $h_{t2.5i}$ and the subroutine FAIR.

8.8.3.2.SM High-Pressure Compressor

Application of the first law of thermodynamics to the high-pressure compressor and high-pressure turbine gives the exit enthalpy of the high-pressure compressor, or

$$h_{t3} = h_{t2.5} - \eta_{mH}(1 + f)(h_{t4} - h_{t4.5}) \tag{SM8.19}$$

The total pressure ratio of the high-pressure compressor π_{cH} is equal to the ratio of the reduced pressure at the ideal exit state P_{rt3i} to the reduced pressure at the inlet $P_{rt2.5}$, or

$$\pi_{cH} = \frac{P_{rt3i}}{P_{rt2.5}} \tag{SM8.20}$$

The reduced pressure at the ideal exit of the high-pressure compressor P_{rt3i} is obtained by first solving for the ideal exit total enthalpy h_{t3i}, using the compressor efficiency and the total enthalpies entering and leaving the compressor. Rewriting Eq. (6.49) gives

$$h_{t3i} = h_{t2.5} + \eta_{cH}(h_{t3} - h_{t2.5}) \tag{SM8.21}$$

The reduced pressure for the ideal exit of the high-pressure compressor P_{rt3i} follows directly from the value of h_{t3i} and the subroutine FAIR.

8.8.3.3.SM Main Burner

The fuel/air ratio for the main burner f is given by textbook Eq. (6.36), where h_{t4} is a function of f:

$$f = \frac{h_{t4} - h_{t3}}{\eta_b h_{PR} - h_{t4}} \tag{6.52}$$

8.8.3.4.SM Engine Speed

As will be shown in Chapter 9, the change in total enthalpy across a fan or compressor is proportional to the rotational speed N squared. For the low-pressure compressor, we write

$$h_{t2.5} - h_{t2} = K_1 N_{cL}^2$$

where N_{cL} is the speed of the low-pressure spool. By using Eq. (8.4), this equation can be rewritten in terms of the corrected speed N_{cL} based on the total temperature at engine station 2 as

$$h_{t2.5} - h_{t2} = K_1 \frac{T_{t2}}{T_{\text{ref}}} N_{cL}^2$$

Evaluating the constant K_1 at reference conditions and solving the above equation for the corrected speed give

$$\frac{N_{cL}}{N_{cLR}} = \sqrt{\frac{T_{t2R}}{T_{t2}} \frac{h_{t2.5} - h_{t2}}{(h_{t2.5} - h_{t2})_R}} \tag{SM8.22a}$$

Similarly, for the high-pressure compressor, we have

$$\frac{N_{cH}}{N_{cHR}} = \sqrt{\frac{T_{t2.5R}}{T_{t2.5}} \frac{h_{t3} - h_{t2.5}}{(h_{t3} - h_{t2.5})_R}} \tag{SM8.22b}$$

8.8.3.5.SM Solution Procedure

All the equations required to calculate the pumping characteristics of a dual-spool gas generator with variable gas properties have now been developed. Figure SM8.7 gives a flowchart showing the order of calculation of dependent variables to obtain the gas generator pumping characteristics. The subroutine TURB (see Fig. SM8.4) is used to calculate the performance of both the high- and low-pressure turbines. Note that initial values of the fuel/air ratio f, $T_{t4.5}$, and T_{t5} are needed to start the calculations. Calculations continue until successive values of the fuel/air ratio f are within 0.0001 of each other.

Inputs: T_{t4}, T_{t2}, and A_8/A_{8R}

Constants: η_{cL}, η_{cH}, π_b, η_b, η_{tH}, M_4, $A_4/A_{4.5}$, η_{tL}, $M_{4.5}$, and h_{PR}

Reference: T_{t4}, T_{t2}, $T_{t4.5}$, T_{t5}, π_{cL}, π_{cH}, $A_{4.5}/A_8$, M_8, and f

Outputs: $\dfrac{\pi_{cL}}{\pi_{cLR}}, \dfrac{N_{cL}}{N_{cLR}}, \dfrac{\pi_{cH}}{\pi_{cHR}}, \dfrac{N_{cH}}{N_{cHR}}, \dfrac{\pi_c}{\pi_{cR}}, \dfrac{\dot m_{c2}}{\dot m_{c2\,R}}, \dfrac{\dot m_{cf}}{\dot m_{cf\,R}}, \dfrac{T_{t4}}{T_{t2}}, \dfrac{P_{t6}}{P_{t2}}$, and $\dfrac{T_{t6}}{T_{t2}}$

Calculations:

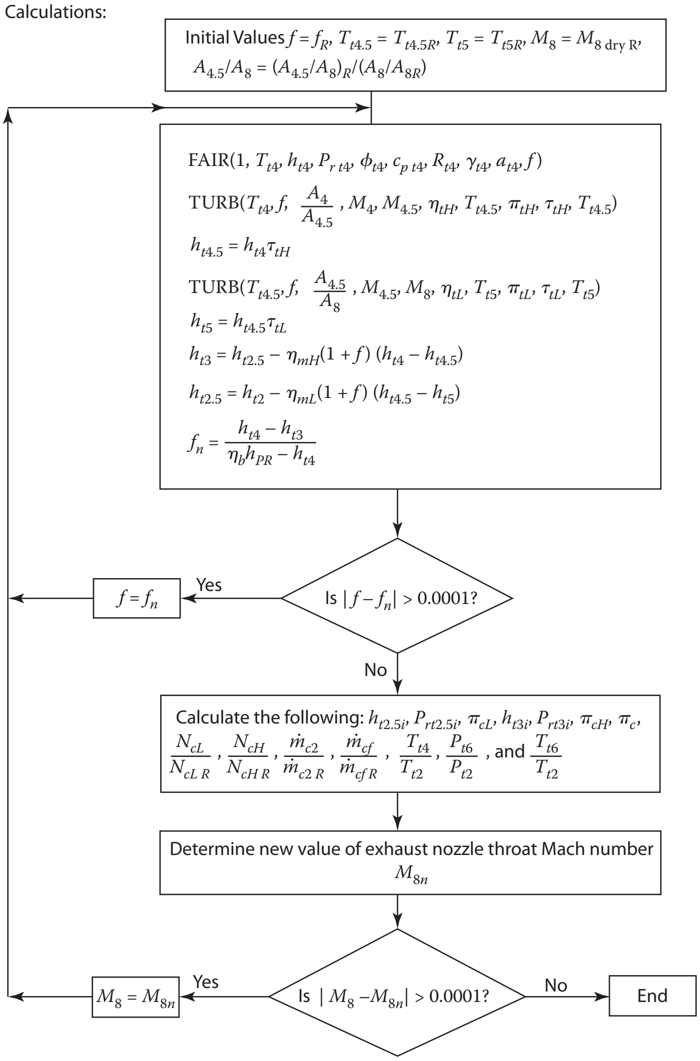

Fig. SM8.7 Flow chart of calculations for gas generator performance.

Example SM8.3

We now consider a dual-spool gas generator with the following reference conditions:

$$T_{t2} = 518.7°\text{R}, \quad T_{t4} = 3200°\text{R}, \quad f = 0.03272, \quad M_4 = 1,$$
$$\pi_{cL} = 5, \quad \eta_{cL} = 0.875724, \quad \pi_{cH} = 4, \quad \eta_{cH} = 0.880186,$$
$$\tau_{tH} = 0.876058, \quad \pi_{tH} = 0.553469, \quad \tau_{tL} = 0.896517, \quad \pi_{tL} = 0.617798,$$
$$\frac{A_4}{A_{4.5}} = 0.587805, \quad M_{4.5} = 1, \quad \left(\frac{\pi_{AB}A_{4.5}}{A_8}\right)_{dry} = 0.587805, \quad M_8 = 1$$

The performance of this gas generator was calculated by using the procedure flowcharted in Fig. SM8.7 for the following three cases:

1. Varied T_{t4} with T_{t2} and $(\pi_{AB}A_{4.5}/A_8)_{dry}$ equal to their reference values
2. Varied T_{t2} with T_{t4} and $(\pi_{AB}A_{4.5}/A_8)_{dry}$ equal to their reference values
3. Varied T_{t4} with T_{t2} equal to its reference values and $A_{8\,dry} = 1.4A_{8\,dry R}$

The pumping characteristics of the gas generator are plotted in Fig. SM8.8 vs the corrected speed of the low-pressure spool for cases *a* and *b* with blue

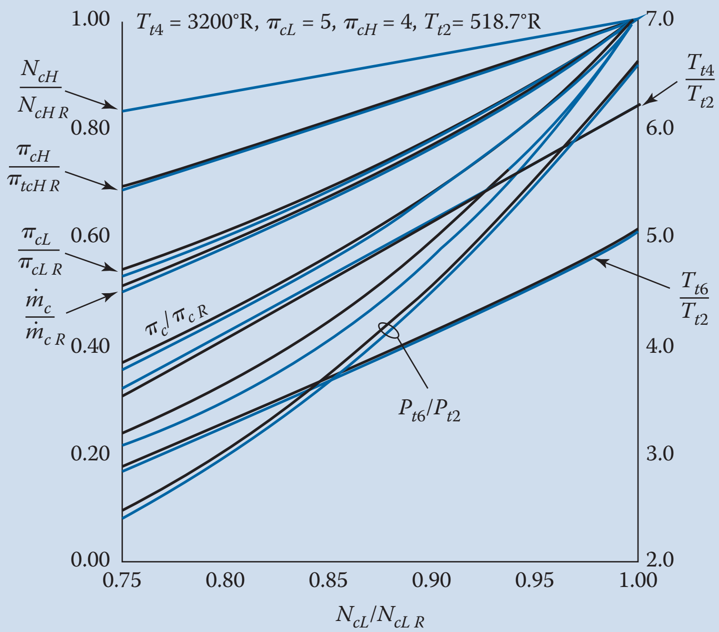

Fig. SM8.8 Dual-spool gas generator pumping characteristics.

(Continued)

Example SM8.3 (*Continued*)

and black lines, respectively. Note that at reduced corrected speeds of the low-pressure spool, all the quantities plotted are less than their maximum values. Also note that the variations of the high-pressure spool's corrected quantities are much less than those of the low-pressure spool. These general trends (see Fig. SM8.8) are the same as those we obtained with the basic performance model (see Fig. 8.13 in the textbook). Only the corrected fuel flow rate, T_{t4}/T_{t2}, and P_{t6}/P_{t2} seem to be impacted by which variable is varied—T_{t2} or T_{t4}. Even for engine cycle performance calculations as complex as these, *there is essentially a one-to-one correspondence between the temperature ratio* T_{t4}/T_{t2} *and the gas generator's pumping characteristics between station 2 and station* 6.

Figure SM8.9 presents the variation of the compressor pressure ratio π_c vs the corrected mass flow rate for cases *a* and *c*. With T_{t4}/T_{t2} held constant, increasing A_8 will increase both the compressor pressure ratio and the corrected mass flow rate, which shifts the compressor operating line to the right and up. These are the same trends that we obtained with the basic performance model (see Fig. 8.8 in the textbook).

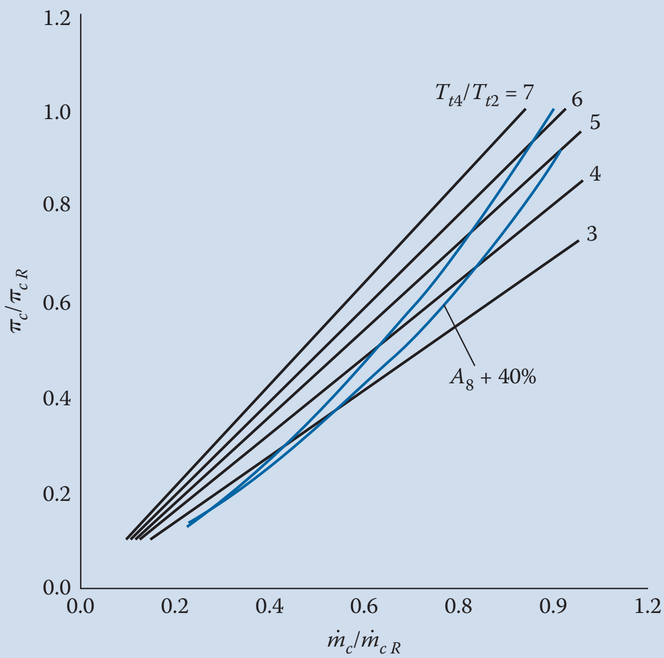

Fig. SM8.9 Compressor map with operating lines and lines of constant T_{t4}/T_{t2}.

Performance Analysis: Dual-Spool Afterburning Turbojet Engine

Figure SM8.10 shows a cross-sectional drawing of the dual-spool afterburning turbojet engine and its station numbering. The uninstalled thrust is given by

$$F = \frac{\dot{m}_0 a_0}{g_c}\left[(1+f_0)\frac{V_9}{a_0} - M_0 + (1+f_0)\frac{R_9}{R_0}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_0}\right] \tag{SM8.23}$$

The exit velocity V_9 is determined from the total and static enthalpies at station 9 by using the following from Chapter 6 in the textbook:

$$V_9 = \sqrt{2g_c(h_{t9} - h_9)} \tag{6.60}$$

The total enthalpy at station 9 is obtained from application of the first law to the engine and tracking the changes in energy from the engine's inlet to its exit. The static state at station 9 (h_9, T_9, etc.) is obtained by using the following relationship between the reduced pressure at the static state P_{r9}, the reduced pressure at the total state P_{rt9}, and the nozzle pressure ratio P_{t9}/P_9 from Chapter 6 in the textbook:

$$P_{r9} = \frac{P_{rt9}}{P_{t9}/P_9} \tag{6.61}$$

The nozzle pressure ratio P_{t9}/P_9 is obtained by multiplying the ratios of pressure from engine inlet to exit, which yields

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r\pi_d\pi_{cL}\pi_{cH}\pi_b\pi_{tH}\pi_{tL}\pi_{AB}\pi_n \tag{SM8.24}$$

As previously shown, the total pressure ratio for the high-pressure turbine π_{tH} and the total pressure ratio for the low-pressure turbine π_{tL}

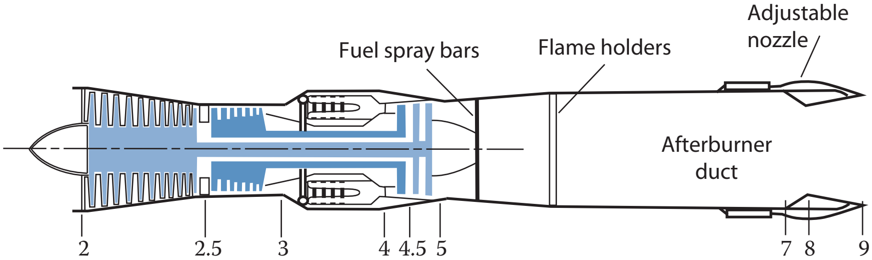

Fig. SM8.10 Station numbering for dual-spool turbojet engine.

are dependent on T_{t4}, f, and the exhaust nozzle Mach number M_8. The pressure ratio of the low-pressure compressor π_{cL} and the pressure ratio of the high-pressure compressor π_{cH} are dependent on T_{t2},T_{t4}, f, and the enthalpy changes across the turbines.

The flow in an operating afterburner can be modeled as a combination of Fanno (simple friction) and Rayleigh (simple heating) flows. When the afterburner is off (dry operation with $T_{t7} = T_{t5}$), only Fanno losses occur and the total pressure losses are about 50% of the losses when it is operating. When the afterburner is on (wet operation), the Rayleigh losses are proportional to the rise in total temperature across the afterburner $(T_{t7} - T_{t5})$. Thus we approximate the total pressure ratio of an afterburner as

$$\pi_{AB} = 1 - 0.5\left[1 + \frac{T_{t7} - T_{t5}}{(T_{t7} - T_{t5})_R}\right](1 - \pi_{ABR}) \tag{SM8.25}$$

The static pressure ratio P_0/P_9 is required to obtain the uninstalled thrust equation [Eq. (SM8.23)] and the exhaust nozzle pressure ratio [Eq. (SM8.24)]. The exhaust nozzle area ratio A_9/A_8 is an alternative input to the static pressure ratio P_0/P_9. And A_9/A_8 is directly related to the mass flow parameter at station 9 by

$$\text{MFP}_9 = \frac{\text{MFP}_8}{\pi_n A_9/A_8} \tag{SM8.26}$$

For a given value of the mass flow parameter and total temperature T_t, there are two Mach numbers: a subsonic one and a supersonic one. The total/static pressure ratio determines which Mach number is appropriate. The Mach number M at a station can be obtained by a computer subroutine from the corresponding values of the mass flow parameter, total temperature, and fuel/air ratio at that station. The subroutine MACH was written to do just this, and its flowchart is sketched in Fig. SM8.11. An initial Mach number M_i is input and used to indicate which solution is desired ($M_i > 1$ for supersonic, $M_i < 1$ for subsonic). This subroutine uses a modified Newtonian iteration and the subroutine MASSFP (see Fig. SM8.7) to obtain successive values of the Mach number and mass flow parameter, respectively. This process is repeated until the calculated value of the mass flow parameter is within 0.00001 of the specified value. The subroutine MACH also gives the static temperature T, which, together with the total temperature T_t, defines the total/static pressure ratio P_t/P.

Given the exhaust nozzle area ratio A_9/A_8 and Mach region, Eqs. (SM8.26) and (SM8.24) and subroutines MACH, MASSFP, and FAIR will give the exit Mach number M_9 and the static pressure ratio P_0/P_9. Thus the exhaust nozzle area ratio A_9/A_8 is an alternative input to P_0/P_9 with the static pressure ratio P_0/P_9 determined by using Eq. (SM8.24), P_{t9}/P_9, and the component π's.

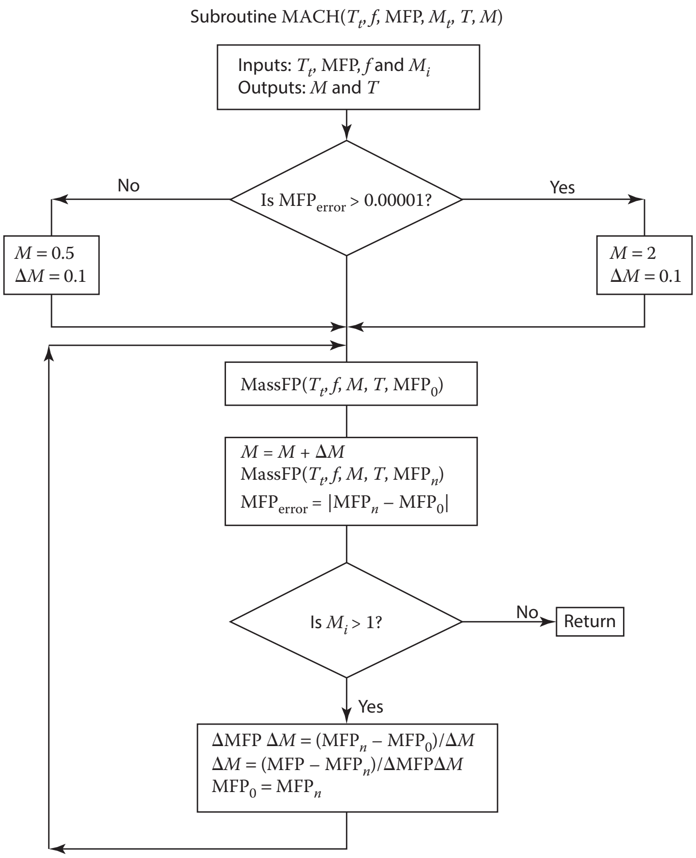

Fig. SM8.11 Flow chart of subroutine MACH.

Summary of Equations: Dual-Spool Afterburning Turbojet Engine with Variable Gas Properties

Inputs:

Choices

Flight parameters:	M_0, T_0 (K, °R), P_0 (kPa, psia)
Throttle setting:	T_{t4} (K, °R), T_{t7} (K, °R)
Exhaust nozzle:	P_0/P_9 or A_9/A_8 (nozzle choked); $A_{8\,\mathrm{dry}}/A_{8\,\mathrm{dry}R}$ (nozzle unchoked)

Design constants

π:	$\pi_{d\max}$, π_b, π_n
η:	η_{cL}, η_{cH}, η_b, η_{tH}, η_{tL}, η_{mL}, η_{mH}, η_{AB}
Fuel:	h_{PR} (kJ/kg, Btu/lbm)
Areas:	A_4, $A_{4.5}$

Reference conditions

Flight parameters:	M_{0R}, T_{0R} (K, °R), P_{0R} (kPa, psia), τ_{rR}, π_{rR}
Throttle setting:	T_{t4R} (K, °R), T_{t7R} (K, °R)
Component behavior:	π_{dR}, π_{cLR}, π_{cHR}, π_{tHR}, τ_{tHR}, π_{tLR}, τ_{tLR}, $\pi_{\mathrm{AB}R}$, $\pi_{\mathrm{AB\,dry}}$, $T_{t4.5R}$, T_{t5R}, f_R, $f_{\mathrm{AB}R}$
Exhaust nozzle:	$A_{8\,\mathrm{dry}}$, M_{8R}, M_{9R}

Outputs:

Overall performance:	F(N, lbf), $\dot{m}_0\left(\frac{\text{kg}}{\text{sec}}, \frac{\text{lbm}}{\text{sec}}\right)$, $S\left(\frac{\text{g/s}}{\text{kN}}, \frac{\text{lbm/h}}{\text{lbf}}\right)$, f_O, η_P, η_{Th}, η_O
Component behavior:	π_{cL}, τ_{cL}, π_{cH}, τ_{cH}, π_{tH}, τ_{tH}, π_{tL}, τ_{tL}, π_{AB}, f, f_{AB}, M_9, $N_{\mathrm{LP}}/N_{\mathrm{LP}R}$, $N_{\mathrm{HP}}/N_{\mathrm{HP}R}$, $\dot{m}_{c0}$, $\dot{m}_{c2}$, $\dot{m}_{fc}$

Equations:

FAIR(1, T_0, h_0, P_{r0}, ϕ_0, c_{p0}, R_0, γ_0, a_0, 0)

$$V_0 = M_0 a_0 \tag{SM8.27a}$$

$$h_{t0} = h_0 + \frac{V_0^2}{2g_c} \tag{SM8.27b}$$

FAIR(2, T_{t0}, h_{t0}, P_{rt0}, ϕ_{t0}, c_{pt0}, R_{t0}, γ_{t0}, a_{t0}, 0)

$$\tau_r = \frac{h_{t0}}{h_0} \tag{SM8.27c}$$

$$\pi_r = \frac{P_{r,t0}}{P_{r0}} \tag{SM8.27d}$$

$$\eta_r = 1 \quad \text{for} \quad M_0 \le 1 \tag{SM8.27e}$$

$$\eta_r = 1 - 0.075(M_0 - 1)^{1.35} \quad \text{for} \quad M_0 > 1 \tag{SM8.27f}$$

$$\pi_d = \pi_{d\,\max}\eta_r \tag{SM8.27g}$$

$$h_{t2} = h_{r0} \tag{SM8.27h}$$

$$P_{rt2} = P_{rt0} \tag{SM8.27i}$$

$$T_{t2} = T_{t0} \tag{SM8.27j}$$

Set initial values:

$$f = f_R, \quad T_{t4.5} = T_{t4.5\,R}, \quad T_{t5} = T_{t5\,R}, \quad M_4 = 1, \quad M_{4.5} = 1, \quad M_8 = M_{8R},$$
$$(\pi_{\mathrm{AB}}A_8)_{\mathrm{dry}} = (\pi_{\mathrm{AB}}A_8)_{\mathrm{dry}R}, \quad M_{9o} = M_{9\,R}$$

A

$$\text{FAIR}(1,\ T_{t4},\ h_{t4},\ P_{rt4},\ \phi_{t4},\ c_{pt4},\ R_{t4},\ \gamma_{t4},\ a_{t4}, f)$$

$$\text{TURB}\left(T_{t4}, f, \frac{A_4}{A_{4.5}}, M_4, M_{4.5}, \eta_{tH}, T_{t4.5}, \pi_{tH}, \tau_{tH}, T_{t4.5}\right)$$

B

$$\text{FAIR}(1,\ T_{t4.5},\ h_{t4.5},\ P_{rt4.5},\ \phi_{t4.5},\ c_{pt4.5},\ R_{t4.5},\ \gamma_{t4.5},\ a_{t4.5}, f)$$

$$\text{TURB}\left(T_{t4.5}, f, \frac{A_{4.5}}{(\pi_{\text{AB}}A_8)_{\text{dry}}}, M_{4.5}, M_8, \eta_{tL}, T_{t5}, \pi_{tL}, \tau_{tL}, T_{t5}\right)$$

$$h_{t2.5} = h_{t2} + (h_{t4.5} - h_{t5})(1+f)\eta_{mL} \tag{SM8.27k}$$

$$h_{t3} = h_{t2.5} + (h_{t4} - h_{t4.5})(1+f)\eta_{mH} \tag{SM8.27l}$$

$$f_n = \frac{h_{t4} - h_{t3}}{\eta_b h_{PR} - h_{t4}} \tag{SM8.27m}$$

If $|f - f_n| > 0.0001$ then $f = f_n$ go to **A**

$$h_{t2.5i} = h_{t2} + (h_{t2.5} - h_{t2})\eta_{cL} \tag{SM8.27n}$$

$$\text{FAIR}(2,\ T_{t2.5i},\ h_{t2.5i},\ P_{rt2.5i},\ \phi_{t2.5ri},\ c_{pt2.5i},\ R_{t2.5i},\ \gamma_{t2.5i},\ a_{t2.5i}, f)$$

$$\pi_{cL} = \frac{P_{rt2.5i}}{P_{rt2}} \tag{SM8.27o}$$

$$\tau_{cL} = \frac{h_{t2.5}}{h_{t2}} \tag{SM8.27p}$$

$$h_{t3i} = h_{t2.5} + (h_{t3} - h_{t2.5})\eta_{cH} \tag{SM8.27q}$$

$$\text{FAIR}(2,\ T_{t3i},\ h_{t3i},\ P_{rt3i},\ \phi_{t3i},\ c_{pt3i},\ R_{t3i},\ \gamma_{t3i},\ a_{t3i}, f)$$

$$\pi_{cH} = \frac{P_{rt3i}}{P_{rt2.5}} \tag{SM8.27r}$$

$$\tau_{cH} = \frac{h_{t3}}{h_{t2.5}} \tag{SM8.27s}$$

$$\pi_c = \pi_{cL}\pi_{cH} \tag{SM8.27t}$$

If the afterburner is off, then

$$T_{t7} = T_{t5} \tag{SM8.27u}$$

$$f_O = f \tag{SM8.27v}$$

Else

$$f_{\text{AB}i} = f_{\text{AB}R} \tag{SM8.27w}$$

C

$$f_O = f + f_{ABi} \tag{SM8.27x}$$

FAIR(1, T_{t7}, h_{t7}, P_{rt7}, ϕ_{t7}, c_{pt7}, R_{t7}, γ_{t7}, a_{t7}, f_O)

$$f_{AB} = \frac{h_{t7} - h_{t5}}{\eta_{AB} h_{PR} - h_{t7}} \tag{SM8.27y}$$

If $|f_{AB} - f_{ABi}| > 0.0001$, then $f_{ABi} = f_{AB}$, go to **C**

End if

$$\pi_{AB} = 1 - 0.5\left[1 + \frac{T_{t7} - T_{t5}}{(T_{t7} - T_{t5})_R}\right](1 - \pi_{ABR}) \tag{SM8.27z}$$

$$T_{t9} = T_{t7} \tag{SM8.27aa}$$

FAIR(1, T_{t9}, h_{t9}, P_{rt9}, ϕ_{t9}, c_{pt9}, R_{t9}, γ_{t9}, f_O)

If P_0/P_9 is given for exhaust nozzle, then

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9} \pi_r \pi_d \pi_{cL} \pi_{cH} \pi_b \pi_{tH} \pi_{tL} \pi_{AB} \pi_n \tag{SM8.27ab}$$

$$P_{r9} = \frac{P_{rt9}}{P_{t9}/P_9} \tag{SM8.27ac}$$

FAIR(3, T_9, h_9, P_{r9}, ϕ_9, c_{p9}, R_9, γ_9, a_9, f_O)

$$V_9 = \sqrt{2gc(h_{t9} - h_9)} \tag{SM8.27ad}$$

$$M_9 = \frac{V_9}{a_9} \tag{SM8.27ae}$$

If $M_9 < 1$ then

$$M_8 = M_9$$

Get value of $A_{8\,\text{dry}}/A_{8\,\text{dry}\,R}$ from user

$$(\pi_{AB} A_8)_{\text{dry}} = (\pi_{AB} A_8)_{\text{dry}R} \left(\frac{A_{8\,\text{dry}}}{A_{8\,\text{dry}R}}\right) \tag{SM8.27af}$$

If $M_8 = M_{8\,R}$, then

$$M_{9o} = M_9$$

Else

If $|M_9 - M_{9o}| > 0.0001$, then go to **B**

End if

Go to **D**

Else

$M_8 = 1$

If $|M_9 - M_{9o}| > 0.0001$, then $M_{9o} = M_9$ go to **B**

End if

MASSFP(T_{t9}, f_O, M_8, MFP$_8$)

MASSFP(T_{t9}, f_O, M_9, MFP$_9$)

$$\frac{A_9}{A_8} = \frac{\mathrm{MFP}_8}{\pi_n \mathrm{MFP}_9} \tag{SM8.27ag}$$

End if

If A_9/A_8 is given for exhaust nozzle, then

$$\frac{P_{t9}}{P_0} = \pi_r \pi_d \pi_{cL} \pi_{cH} \pi_b \pi_{tH} \pi_{tL} \pi_{AB} \pi_n \tag{SM8.27ah}$$

$$P_{r9i} = \frac{P_{rt9}}{P_{t9}/P_9} \tag{SM8.27ai}$$

FAIR(3, T_{9i}, h_{9i}, P_{r9i}, ϕ_{9i}, c_{p9i}, R_{9i}, γ_{9i}, a_{9i}, f_O)

$$M_{9i} = \frac{\sqrt{2g_c(h_{t9} - h_{9i})}}{a_{9i}} \tag{SM8.27aj}$$

If $M_{9i} > 1$, then

$M_8 = 1$

Else

$M_8 = M_{9i}$

Get value of $A_{8\,\mathrm{dry}}/A_{8\,\mathrm{dry}\,R}$ from user

$$(\pi_{AB} A_8)_{\mathrm{dry}} = (\pi_{AB} A_8)_{\mathrm{dry}\,R} \frac{A_{8\,\mathrm{dry}}}{A_{8\,\mathrm{dry}\,R}} \tag{SM8.27ak}$$

If $M_8 = M_{8R}$, then

$M_{9o} = M_9$

Else

If $|M_9 - M_{9o}| > 0.0001$, then go to **B**

End if

MASSFP(T_{t9}, f_O, M_8, T_8, MFP$_8$)

$$\mathrm{MFP}_9 = \frac{\mathrm{MFP}_8}{\pi_n (A_9/A_8)} \tag{SM8.27al}$$

MACH(T_{t9}, f_O, MFP$_9$, M_{9i}, T_9, M_9)

FAIR(1, T_9, h_9, P_{r9}, ϕ_9, c_{p9}, R_9, γ_9, a_9, f_O)

$$V_9 = M_9 a_9 \tag{SM8.27am}$$

$$\frac{P_{t9}}{P_9} = \frac{P_{rt9}}{P_{r9}} \tag{SM8.27an}$$

$$\frac{P_0}{P_9} = \frac{P_{t9}/P_9}{P_{t9}/P_0} \tag{SM8.27ao}$$

End if

D

MASSFP(T_{t4}, f, M_4, T_4, MFP$_4$)

$$\dot{m}_0 = \frac{P_0 \pi_r \pi_d \pi_{cL} \pi_{cH} \pi_b A_4 \text{MFP}_4}{(1+f)\sqrt{T_{t4}}} \tag{SM8.27ap}$$

$$F = \frac{\dot{m}_0 a_0}{g_c}\left[(1+f_O)\frac{V_9}{a_0} - M_0 + (1+f_O)\frac{R_9}{R_0}\frac{T_9/T_0}{V_9/a_0}\frac{1-P_0/P_9}{\gamma_0}\right] \tag{SM8.27aq}$$

$$S = \frac{f_O}{F/\dot{m}_0} \tag{SM8.27ar}$$

$$\eta_{Th} = \frac{a_0^2[(1+f_O)(V_9/a_0)^2 - M_0^2]}{2g_c f_o h_{PR}} \tag{SM8.27as}$$

$$\eta_P = \frac{2g_c V_0(F/\dot{m}_0)}{a_0^2[(1+f_O)(V_9/a_0)^2 - M_0^2]} \tag{SM8.27at}$$

$$\eta_O = \eta_P \eta_{Th} \tag{SM8.27au}$$

$$\frac{N_{cL}}{N_{cLR}} = \sqrt{\frac{T_{t2R}}{T_{t2}}\frac{h_{t2.5} - h_{t2}}{(h_{t2.5} - h_{t2})_R}} \tag{SM8.27av}$$

$$\frac{N_{cH}}{N_{cHR}} = \sqrt{\frac{T_{t2.5R}}{T_{t2.5}}\frac{h_{t3} - h_{t2.5}}{(h_{t3} - h_{t2.5})_R}} \tag{SM8.27aw}$$

$$\theta_0 = \frac{T_{t0}}{T_{\text{ref}}} \tag{SM8.27ax}$$

$$\delta_0 = \frac{P_{t0}}{P_{\text{ref}}} \tag{SM8.27ay}$$

$$\delta_2 = \pi_d \delta_0 \tag{SM8.27az}$$

$$\dot{m}_{c0} = \frac{\dot{m}_0\sqrt{\theta_0}}{\delta_0} \tag{SM8.27ba}$$

$$\dot{m}_{c2} = \frac{\dot{m}_0\sqrt{\theta_0}}{\delta_2} \tag{SM8.27bb}$$

$$\dot{m}_{fc} = \frac{f\dot{m}_0}{\sqrt{\theta_0}\delta_2} \tag{SM8.27bc}$$

Example SM8.4

We consider an afterburning turbojet engine with the same input data as those considered for the afterburning turbojet in Example 8.7 in the textbook. Thus we have variable specific heats and the following reference data and operating conditions.

Reference:

$$\text{Sea-level static } (T_0 = 518.7°\text{R},\quad P_0 = 14.696 \text{ psia}),\quad \pi = 20,$$
$$\pi_{cL} = 5,\quad \pi_{cH} = 4,\quad e_{cL} = 0.9,\quad e_{cH} = 0.9,$$
$$e_{tH} = 0.9,\quad e_{tL} = 0.9,\quad \pi_{d\,\max} = 0.98,\quad \pi_b = 0.96,\quad \pi_n = 0.98,$$
$$T_{t4} = 3200°\text{R},\quad \eta_b = 0.995,\quad \eta_{mL} = 0.995,$$
$$\eta_{mH} = 0.995,\quad h_{PR} = 18{,}400 \text{ Btu/lbm},\quad T_{t7} = 3600°\text{R},$$
$$\pi_{\text{AB}} = 0.94,\quad \eta_{\text{AB}} = 0.95,\quad \eta_{cL} = 0.8756,\quad \eta_{cH} = 0.8801,$$
$$\eta_{tH} = 0.9058,\quad \eta_{tL} = 0.9048,\quad \pi_{tH} = 0.5478,\quad \tau_{tH} = 0.8752,$$
$$\pi_{tL} = 0.6135,\quad \tau_{tL} = 0.8959,\quad M_8 = 1,\quad M_9 = 1.815,$$
$$f = 0.03287,\quad f_{\text{AB}} = 0.020304,\quad f_O = 0.05423,\quad F = 25{,}000 \text{ lbf},$$
$$S = 1.4050 \text{ (lbm/h)/lbf},\quad \dot{m}_0 = 178.18 \text{ lbm/s}$$

Operation:

Maximum $T_{t4} = 3200°\text{R}$	Maximum $T_{t7} = 3600°\text{R}$
Maximum $\pi_c = 20$	
Mach number: 0 to 2	Altitudes (kft): 0, 20, and 40

Comparison of these reference data to those of Example 8.7 in the textbook shows the following:

1. The efficiencies of the low- and high-pressure compressors are a little higher and the efficiencies of the high- and low-pressure turbines are a little lower.
2. The fuel/air ratio of the main burner is higher, and that of the afterburner is the same.

(*Continued*)

Example SM8.4 (*Continued*)

3. The engine mass flow rate is about 1% lower due to the higher specific thrust.
4. The thrust specific fuel consumption is about 1% higher.

These results agree with those found in Section 7.7.

To calculate the performance of the afterburning turbojet with variable gas properties, Eqs. (SM8.27a–SM8.27bc) and subroutines FAIR, MASSFP, and TURB were used to write a short computer program. Due to its numerous iteration loops, the calculations of this engine's performance took about 80 times longer than those for the afterburning turbojet with constant gas properties. For some, this larger calculation time is prohibitive, and faster, less accurate results may be appropriate.

The maximum thrust results (wet and dry) for this engine are shown in Fig. SM8.12a. Comparison with the maximum thrust results of Example 8.7 in the textbook (see Fig. 8.31) shows that the maximum thrusts predicted for the two engine models are nearly equal for most operating conditions.

The thrust specific fuel consumption results (wet and dry) for this engine are shown in Fig. SM8.12b. Comparison with the fuel consumption results of Example 8.7 in the textbook (see Fig. 8.32) shows that they are very nearly the same for all flight conditions.

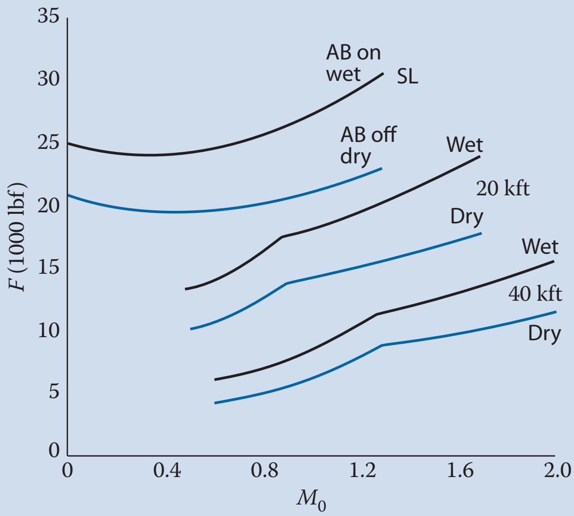

Fig. SM8.12a Variation of thrust with flight Mach number and altitude.

(*Continued*)

Example SM8.4 (*Continued*)

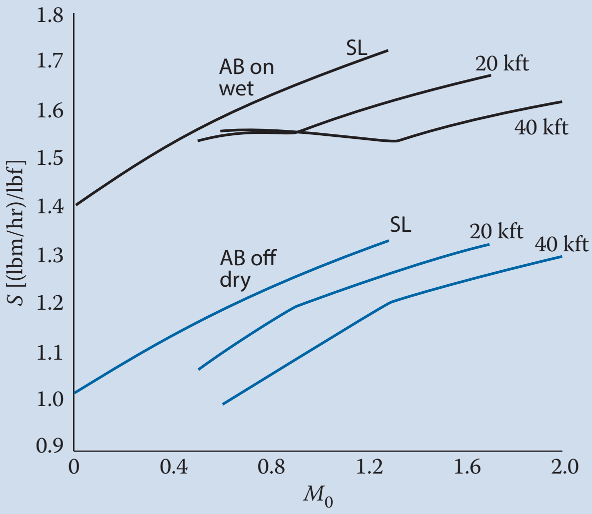

Fig. SM8.12b Variation of thrust specific fuel consumption with flight Mach number and altitude.

Figure SM8.13a shows the variation of maximum corrected thrust (wet and dry) with dimensionless total temperature θ_0. These are the same trends that we obtained with the basic engine performance model (see Fig. 8.24 in the textbook). The change in maximum total temperature leaving the main burner T_{t4} with θ_0 is shown by a solid line in Fig. SM8.13b for the engine model with variable gas properties and by a dashed line for the constant specific heat engine model [Eq. (SM8.5p)].

Figure SM8.13c shows the variations of the maximum compressor corrected mass flow rate and pressure ratio with dimensionless total temperature θ_o. At flight conditions where θ_0 is greater than unity, both the maximum compressor corrected mass flow rate and the pressure ratio fall off with increasing Mach number. At flight conditions where θ_0 is less than unity, the maximum compressor pressure ratio is constant, and the maximum corrected mass flow rate increases slightly. These are the same fundamental trends that we obtained with the basic engine performance model (see Figs. 8.16 and 8.23 in the textbook). The slight increase in the maximum corrected mass flow rate, when $\theta_0 < 1.0$, is primarily caused by the increased corrected mass flow rate at engine station 4 that results from the reductions in both fuel-to-air ratio f and total temperature T_t at this engine station (see Fig. SM8.2a).

(*Continued*)

Example SM8.4 (*Continued*)

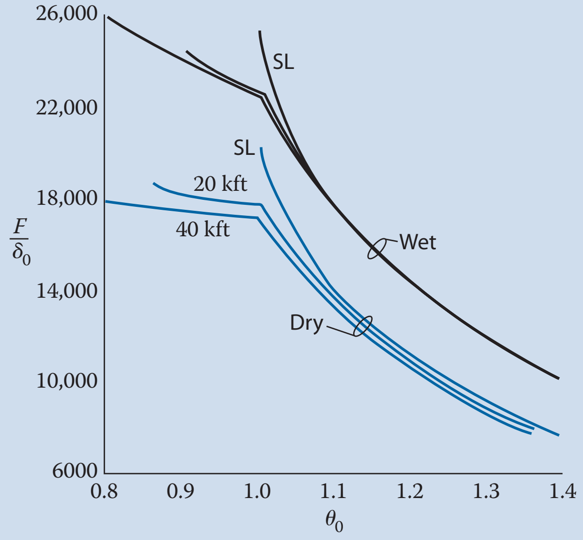

Fig. SM8.13a Variation of corrected thrust with θ_0 and altitude.

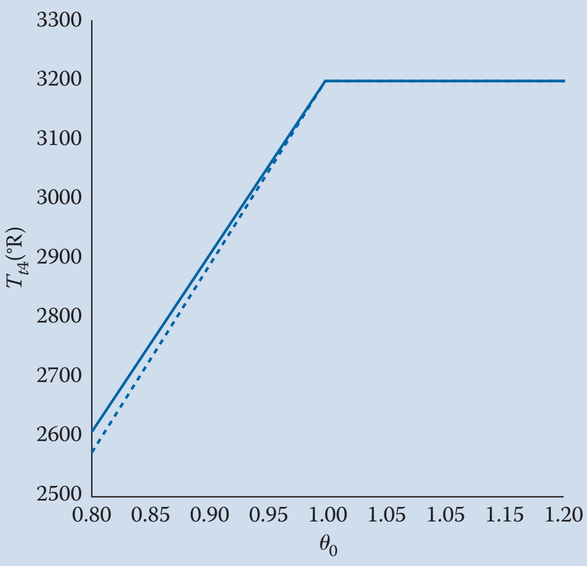

Fig. SM8.13b Variation of maximum T_{t4} with θ_0.

(*Continued*)

Example SM8.4 (*Continued*)

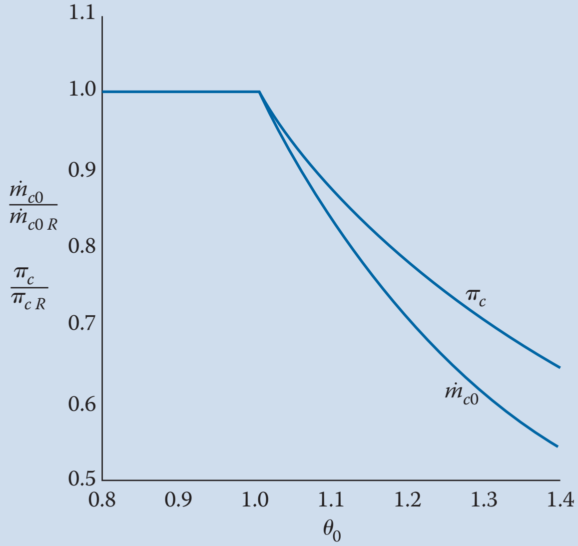

Fig. SM8.13c Variation of corrected mass flow and compressor pressure ratio with θ_0.

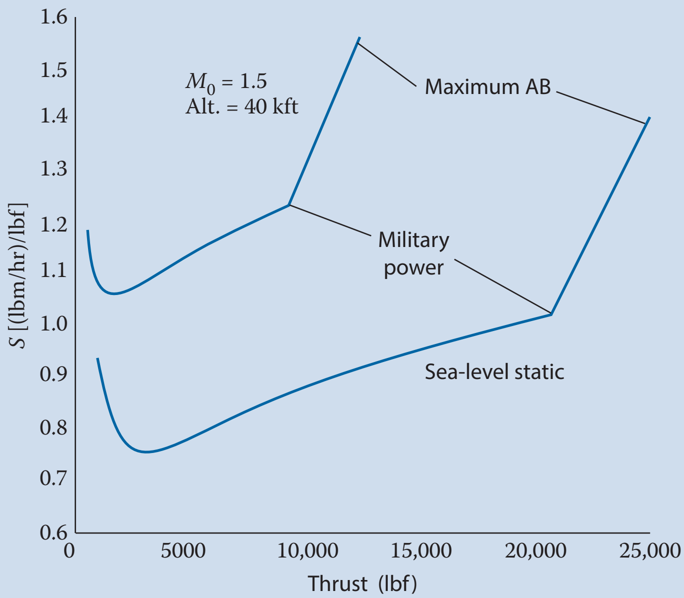

Fig. SM8.14 Partial-throttle performance of afterburning turbojet.

(*Continued*)

Example SM8.4 (*Continued*)

The partial-throttle performance of this afterburning turbojet is shown in Fig. SM8.14 at two operating conditions: sea-level static and Mach 1.5 at 40 kft. Comparison of these partial-throttle curves to those of Example 8.7 in the textbook (see Fig. 8.33) shows the following:

1. The predicted fuel consumptions are the same for afterburner operation.
2. The predicted fuel consumption decreases more at partial throttle for this engine model. The minimum fuel consumptions are about 70% and 75% of their sea-level static dry values for this engine model and the basic engine model, respectively. The large reduction in fuel consumption at partial throttle shown in Fig. SM8.14 is greater than that of an actual engine (see Fig. 1.14c in the textbook) because this engine model assumes constant efficiency of all engine components.