Project

Devendra Singh

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With the FEM method we are going to solve the equation,

$$2\partial_x(\eta\partial_x v_x) + \frac{1}{2}\partial_z(\eta\partial_z v_x) = \rho g \partial_x h \tag{1}$$

$$\partial_x v_z = -\partial_x v_x \tag{2}$$

$$\partial_t h + v_x|_{z=s}\partial_x h = v_z \tag{3}$$

1. Order of equation :

The Equation 1, is **second order** for horizontal velocity.

2. Type of equation :

For constant viscosity, the Equation 1 can be written as,

$$\eta(2\partial_{xx} + \frac{1}{2}\partial_{zz})v_x = \rho g \partial_x h \tag{4}$$

When we compare this with the discriminat, will be negative as B is zero, suggesting that its an **elliptic** PDE of second order.

3. FEM formulism :

To solve Equation 1 using FEM, we first need to convert this PDE into weak(Variational) form. For that, we define a function space V_h over our mesh Ω. In this function space we find $u_x \in V_h$ such that $u_x = 0$ on the bed(Homogeneous Dirichlet BC).

$$V_h = \{v \in H^1(\Omega) : v|_{\Gamma_{bed}} = 0\} \tag{5}$$

The weak form to find $u V_h $ for all $v V_h $ has ,

the bilinear form

$$a(u, v) = \int_\Omega (2\eta\partial_x u \partial_x v + \eta\partial_z u \partial_z v)\; d\Omega \tag{6}$$

and the linear form,

$$L(v) = -\int_\Omega \rho g \partial_x h v \, d\Omega \tag{7}$$

The boundary condition:

At the bed surface(Γ_{bed}): $u_x = 0$

At the ice/atmosphere interface($\Gamma_{surface}$):

$$\eta(2\partial_x v_x \partial_x h - \frac{1}{2}\partial_z v_x) = 0 \tag{8}$$