TIN093 Algorithms - Assignment 6

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Problem 1

a) There are 99! different ways to order a list of 99 distinct/unique elements.

b) There are $(n-1)!$ edges in a clique/complete graph with n nodes. Why? Because we can draw $(n-1)$ edges from the first node to all other nodes. From the second node to all other nodes, we can draw $(n-2)$ new edges(edges that have not already been added. Note that the edge between the first and second node was already added in the first step).

c) Assume all letters were unique. Since "engineering" has 11 letters this gives us 11! unique ways to order the letters **if they were all unique**. However, we know that the letter "e" occurs 3 times, "n" occurs 3 times, "g" occurs 2 times and "i" 2 times. Why is this significant? Consider any ordering and lets focus on the letters "e". Lets denote the letters e_1, e_2, e_3. We know that for any word we create, they can have 3! different orderings for the same word. How? Simply switch positions between different "e" letters. The word will still be the same, but we switch the position of the "e" letters. The same can be done for all other letters that occur more than 1 time. So we have to account for this when we calculate the number of unique words we can create. It will not be 11! unique words, but rather:

$$\frac{11!}{3!3!2!2!} \tag{1}$$

Problem 2

1. $f_6(n) = n(logn)^2$
2. $f_2(n) = n^{1.5}$
3. $f_4(n) = n^{100}$
4. $f_7(n) = n^{log\ n}$

5. $f_1(n) = 4^n$
6. $f_5(n) = 2^{n^2}$
7. $f_3(n) = 2^{2^n}$

The first growth rate is Quasilinear which grows faster than strict linear growth but is slower than Polynomial growth. The next 2 functions are polynomial, but the exponent in the 3rd function is larger so it has a larger polynomial growth rate. The 4th function is sub-exponential so it grows faster than any polynomial function, but has a slower growth rate than strictly exponential functions such as 4^n. Function 6 and 7 are both "super-exponential". However the exponent of function 7 is an exponential function itself (2^n) compared to function 6 which is a polynomial function (n^2).