TIN093 Algorithms - Assignment 3

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Problem 5

a) This greedy rule is easily fooled. Consider the simple example:

30	50	40

Here the job with highest payment is the middle job. If we select it we must delete the 2 jobs surrounding it and we get a total payment of 50. However, a better(and optimal) strategy would be to select the job on the first day and the job on the third day. The payout would be $30 + 40 = 70$.

b) A simple counterexample can be constructed here also. Assume we have the jobs:

100	20	30	70

Here the optimal strategy would be to select the job on the 1st and the job on the 4th day. This would give a total payout of $100 + 70 = 170$. If we worked on all even days we would get a total payout of $20 + 70 = 90$ and if we worked on all odd days we would get a total payout of $100 + 30 = 130$.

Problem 6

a) A fast algorithm would start by sorting the n items by their prices in descending order. We could use mergesort which has a time complexity of $O(n\ log\ n)$. We would then group them intro triples by considering the items from left to right and making each disjoint group of 3 items a triple.

b) Consider a grouping/solution done using the algorithm proposed in a). Why is it optimal? Lets prove that we can not rearrange the items in any way to get a cheaper solution. So consider any pair of triplets. We denote the first triplet as $x = (x_1, x_2, x_3)$ and the second triplet as $y = (y_1, y_2, y_3)$. Assume x is a triplet that comes before y in the grouping. Then we must have that $x_1 \geq x_2 \geq x_3 \geq y_1 \geq y_2 \geq y_3$. The discount from these 2 triples is $x_{discount} = x_3$ and $y_{discount} = y_3$ which gives a total discount of $x_3 + y_3$. Now select any item from y and exchange it with any item from x. What can happen? We know that the least expensive item in x will become the item in y we exchanged with. And we also know that the least expensive item in y will be either y_3 (if we did not exchange it) or y_2 if we exchanged

y_3. So the new discount in x will be the item in y that we exchanged and the new discount in y will be y_3 or y_2. Hence we have in the case we did not exchange y_3 and either exchanged y_2 or y_1 we will get the following total discount relationship:

$$y_2 + y_3 \leq y_1 + y_3 \leq x_3 + y_3 \tag{1}$$

In the case we exchanged y_3, we will have the following total discount relationship:

$$y_3 + y_2 \leq x_3 + y_3 \tag{2}$$

In either case, we see that the discount will have been reduced no matter how we exchange the items. Hence there is no way to improve the total discount once we group them intro triples in descending order.

Question to examiner: Just because we did not manage to improve the total discount with 1 exchange of values between 2 triples, does that mean that it is not possible to use a series of exchanges to get a more optimal value? Is the proof above correct and robust or missing this aspect? The hint speaks of an exchange argument between the possible discount values F. Is this the correct exchange argument?