Classical Diffusion-Inspired Models for Mitigating Fidelity Loss in Approximate Qubit Cloning

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***Abstract*—Approximate cloning remains a fundamental obstacle for quantum machine learning, as the no-cloning theorem guarantees that any attempt to duplicate an unknown quantum state introduces irreducible distortion [1]. This project investigates whether a classical learning model can partially invert the systematic errors introduced by the Bužek–Hillery (BH) universal quantum cloning machine (UQCM) [2], whose optimal output fidelity is bounded at 5/6 [3]. Beginning with a dataset of simulated mixed-state qubits, I first demonstrate that a simple multilayer perceptron (MLP) can significantly improve clone fidelity when trained on Bloch-vector representations. Building on this result, the project explores a diffusion-inspired framework in which repeated applications of the UQCM serve as a forward corruption process and a learned reverse model seeks to undo this degradation. Practical and theoretical challenges, including non-Gaussian corruption, rapid convergence toward the maximally mixed state, and instability in Bloch-space representations, motivated a final architectural insight: decomposing the UQCM into its constituent reversible operations and treating them as inner "substeps" within each diffusion timestep. While this hierarchical substep model was not fully implemented, it provides a promising direction for constructing stable, physically consistent diffusion processes tailored to quantum operations. The combined results demonstrate both the feasibility and the limitations of classical models for mitigating quantum cloning errors, and they lay conceptual groundwork for future hybrid quantum–classical approaches to reconstructing quantum states beyond standard fidelity bounds.**

***Index Terms*—quantum cloning, diffusion models, hybrid quantum-classical, no-cloning theorem, Bužek-Hillery**

I. Introduction

Quantum computing is widely regarded as the next major technological leap, with the potential to push computational capabilities into ranges often associated with science fiction. This promise stems from the fundamental advantages of quantum information processing and the distinctive properties of the quantum bit (qubit). Unlike a classical bit, which can exist only in the state 0 or 1 at any given moment, a qubit can represent complex combinations of both values simultaneously. As additional qubits are added, the size of the representable state space expands exponentially, scaling as 2^n for a system of n qubits. This exponential growth arises from three core principles of quantum mechanics: superposition, entanglement, and measurement.

Superposition allows a qubit to exist in a weighted combination of 0 and 1, dramatically increasing the amount of information a single qubit can encode. Entanglement introduces correlations between qubits such that the state of one cannot be described independently of the others, enabling quantum systems to encode rich, high-dimensional structure. However, measurement collapses a qubit from its superposition into a definite classical state, discarding the information it previously represented. While quantum computers remain in early development due to qubit instability and noise, the measurement problem poses an additional, fundamental challenge for quantum AI and machine learning. Classical ML workflows depend on comparing predicted values to original inputs, a process that requires readable, stable data. Because a qubit irreversibly collapses when measured (and because that collapse also perturbs entangled partners) its pre-measurement state cannot be recovered. This makes direct evaluation or learning from quantum data extremely difficult.

One proposed workaround is to clone a qubit so that one copy may be measured while the original remains untouched. However, Wootters and Zurek [1] proved in their seminal no-cloning theorem that a quantum state cannot be perfectly copied. Subsequent work by Bužek and Hillery [2] introduced the Universal Quantum Cloning Machine (UQCM), which produces approximate clones through specific linear transformations. Later research demonstrated that the theoretical maximum fidelity of any universal approximate clone is $5/6$, meaning that at least $1/6$ of the original information is inevitably lost during cloning [3]. Although this loss may seem modest, it compounds rapidly across many qubits and across iterative layers within a quantum machine-learning model, resulting in significant degradation of accuracy. To address this accumulating error, I propose a hybrid quantum–classical, diffusion-inspired model designed to learn and reverse the noise introduced by approximate cloning. The goal is not to circumvent quantum mechanics, but rather to use classical learning to reconstruct higher-fidelity representations of cloned qubits, potentially surpassing the effective 5/6 limit, while remaining physically consistent. This forms the conceptual foundation for the work that follows.

II. Model Inspiration

The objective of this work is to take a qubit produced by the Bužek–Hillery (BH) 1→2 universal approximate cloning machine (UQCM) (whose output fidelity with respect to the input is upper-bounded by $5/6$) and train a classical model to map this "noisy clone" back toward its original quantum state. The guiding intuition mirrors that of classical denoising diffusion models, in which a learned reverse process progressively removes a known corruption.

To situate this analogy, recall the formulation of the Denoising Diffusion Probabilistic Model (DDPM) of Ho, Jain, and Abbeel [4]. A diffusion model introduces a sequence of latent variables $x_1, \ldots, x_T$ with a Markovian generative structure

$$p_\theta(x_{t-1}|x_t) = \mathcal{N}(x_{t-1}; \mu_\theta(x_t, t), \Sigma_\theta(x_t, t)),$$

where the reverse transitions are Gaussian, and the prior is a standard Gaussian. The forward process is fixed and gradually corrupts data through a noise schedule:

$$q(x_t|x_{t-1}) = \mathcal{N}(x_t; \sqrt{1-\beta_t}x_{t-1}, \beta_t I);$$

as $t \to T$, the data distribution approaches pure Gaussian noise. Training consists of learning the reverse transitions p_θ so that iterative denoising can recover samples resembling x_0.

In the present setting, I treat the BH UQCM as an analogue of this forward-corruption operator. Because the UQCM is optimal and achieves fidelity $5/6$, each application injects a minimum deviation of $1/6$ from the original state. Thus, a single UQCM action can be regarded as a discrete diffusion step, transforming a clean density matrix ρ_0 into a corrupted version ρ_1. My initial plan was to iterate this transformation, feeding the clone at step t into the UQCM again to produce ρ_{t+1}, thereby accumulating noise until the state becomes nearly maximally mixed. A classical model trained on such trajectories would then learn a reverse mapping, eventually reconstructing an approximation of the original.

What distinguishes this approach is that it does not attempt to violate or evade the quantum no-cloning theorem. The classical model never produces a quantum clone: rather, given a clone produced by the UQCM, it outputs a classical representation of a state closer to the original. Since the learned reverse process consists only of classical, linear transformations acting on real-valued encodings of density matrices, it remains fully consistent with quantum mechanics. In this sense, the method leverages a diffusion-like learning paradigm to partially invert a physically valid (but fidelity-limited) quantum operation. The hope is that by learning the structure of the UQCM's systematic distortion, a classical model can recover information that the cloning mechanism itself cannot, while still respecting the fundamental constraints of quantum theory.

III. Mathematical Representations of Qubits

In quantum computing, a qubit exists in a two-dimensional Hilbert space with computational basis states:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Unlike classical bits, qubits can exist in superposition, represented as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

where $|\alpha|^2 + |\beta|^2 = 1$. Here, $|\alpha|^2$ and $|\beta|^2$ correspond to the probabilities of measuring 0 and 1, respectively. Two common representations are used for computation and analysis:

1) Density Matrix: A qubit's density matrix ρ is defined as the outer product of the state vector with its conjugate transpose:

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \beta\alpha^* & |\beta|^2 \end{pmatrix},$$

where the diagonal entries are the probabilities of measuring their respective base state and the off-diagonals describe the superposition and relative phase between the base states.

2) Bloch Vector: A qubit can equivalently be represented on the Bloch sphere:

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle,$$

with polar angle θ and azimuthal angle ϕ. The corresponding density matrix can be written as a linear combination of the Pauli matrices:

$$\rho = \frac{1}{2}(I + \mathbf{r} \cdot \boldsymbol{\sigma}),$$

where $\mathbf{r}$ is the Bloch vector and

$$\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z).$$

These mathematical equivalences are essential for simulating and manipulating qubits in Python. The ability to interconvert between state vectors, density matrices, and Bloch vectors without information loss underpins the construction of training data and the implementation of diffusion-based models in this work.

IV. Methods & Results

A. Python Implementation: Qubit Generation and Cloning

Originally, I generated pure superposition states (points lying on the surface of the Bloch sphere) for training. However, further research made it clear that such states are exceptionally fragile in current quantum hardware; coherence times are short, and pure states rapidly decohere into mixed states [5]. To ensure that the model would be relevant to the realities of contemporary quantum computing, I shifted to generating a dataset of 10,000 mixed-state qubits for training. Each qubit was first sampled as a pure superposition state:

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle,$$

with polar angle θ and azimuthal angle ϕ drawn uniformly at random. My initial approach was to convert these into mixed states by deterministically blending with $|0\rangle\langle 0|$: I later realized this introduced a directional bias, every state was being pushed toward $|0\rangle$. To correct this, I introduced a branching condition that mixed with $|0\rangle\langle 0|$ half of the time and with $|1\rangle\langle 1|$ the other half, yielding a distribution of mixed states with no preferred pole on the Bloch sphere. After generating these

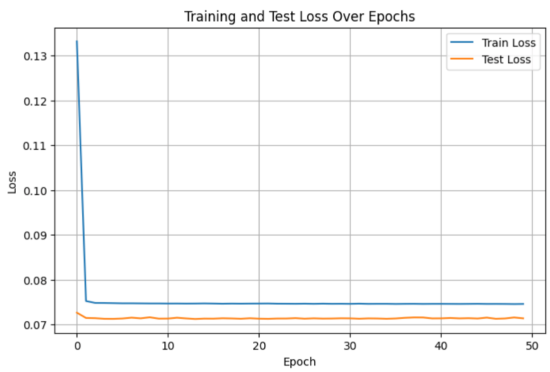

Fig. 1. Training and Test Loss Over Epochs for the Initial MLP Model.

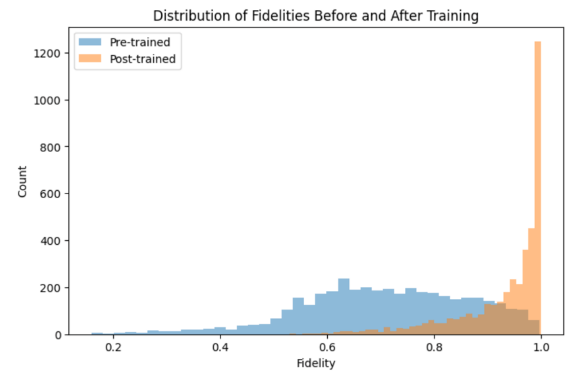

Fig. 2. Fidelity Distributions Between Pre- and Post-Trained MLP Samples.

mixed states, I applied the 1→2 UQCM. Each state ρ was tensored with an ancilla qubit in $|0\rangle\langle 0|$: The cloning unitary was applied and reduced density matrices for each clone were extracted via partial trace. These matrices were then converted into Bloch vectors for neural network input. At this stage, the first challenge arose: density matrices contain complex numbers, which cannot be directly processed by most classical ML architectures. Transforming to Bloch vectors solved this issue, providing three real-valued features per qubit, while maintaining the accuracy of its represented state.

B. Initial Multi-Layer Perceptron Model

With the dataset prepared, I implemented a three-layer feed-forward neural network (MLP) with two ReLU activation blocks, trained to minimize mean-squared error between the predicted and target Bloch vectors. This architecture functioned as a proof-of-concept, demonstrating that a classical neural network can learn to partially invert the distortion introduced by the UQCM. The model was trained for 50 epochs with a learning rate of 0.001. The training results are shown in “Fig. 1” above.

Although the network successfully reduced the fidelity loss between the cloned and original states on the test set, its convergence behavior revealed inherent limitations: training plateaued relatively early, indicating that this simple architecture was unlikely to capture the full structure of the UQCM’s corruption. Despite this, the model achieved a substantial improvement in average fidelity, from $5/6$ (raw UQCM clones) to 0.92 after correction, providing strong evidence that the central idea is viable. A distribution plot of these results are shown in “Fig. 2”. This performance motivated the transition toward the more expressive, diffusion-like model proposed in the next stage of the project.

C. Diffusion-Inspired Model Development

The next phase sought to extend the MLP into a diffusion-like framework, analogous to DDPMs. In this formulation, each forward timestep t was intended to represent one application of the UQCM, and the learned reverse process would invert the corresponding error. However, early experiments revealed limitations in the Bloch-vector representation used previously. Increasing the fidelity of a cloned state toward the original corresponds, in Bloch space, to increasing the vector’s magnitude, a geometric operation far too simplistic to capture the true structure of quantum state degradation under the UQCM. To obtain a more expressive and physically meaningful representation while remaining compatible with classical neural networks, I transitioned to encoding density matrices by separating their real and imaginary parts into real-valued vectors. This allowed a classical model to access the full quantum state information without violating the constraints of real-valued learning architectures.

Significant challenges emerged immediately. Iteratively converting between density matrices and their real-valued vector encodings at every timestep proved computationally expensive. Attempts to approximate the UQCM directly in Bloch space, by simply shrinking vector magnitudes, failed to reproduce physically realistic behavior (as mentioned above). Moreover, training exhibited fidelity oscillations, and produced outputs exceeding physical density-matrix constraints, indicating that the forward process was poorly constructed for a valid diffusion model, as shown in “Fig. 3”. Compounding this, the fixed 1/6 fidelity loss per UQCM application meant that after only about six iterations the state approached maximal mixing, leaving too few meaningful timesteps for a diffusion process to learn a stable reverse dynamic.

A deeper issue also emerged: unlike the Gaussian corruption assumed in DDPMs, the distribution of states produced by repeated UQCM applications is distinctly non-Gaussian. This ruled out the direct use of standard diffusion machinery and required constructing a bespoke diffusion framework tailored to the algebraic structure of quantum operations. Finally, while the UQCM is typically applied as a single unitary transformation, I found that it can be decomposed into nine reversible operations. This decomposition forms the basis for the theoretical framework developed in the next section.

D. Hierarchical Substep Approach

Insights from the previous challenges motivated a new direction: decomposing the UQCM into its constituent operations and treating each as an inner substep within an outer timestep. In this formulation, the outer timestep corresponds

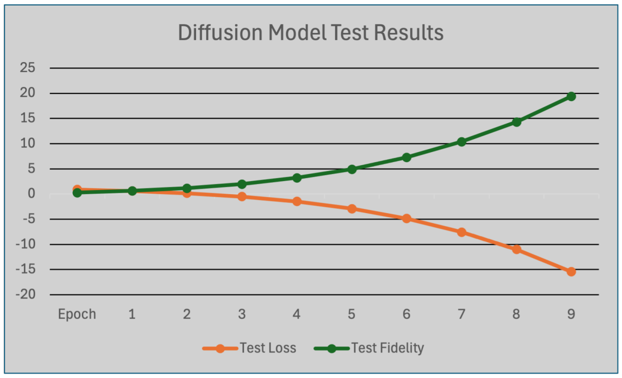

Fig. 3. Diffusion Test Loss and Fidelity over Epochs.

to a full cloning operation, while each inner substep applies one of the reversible unitary components of the UQCM. This hierarchical structure substantially increases the effective number of timesteps available for learning, while remaining mathematically sound, each sub-operation is itself reversible, providing a well-conditioned target for a learned reverse process. The finer temporal resolution also offers richer learning signals, addressing many of the stability issues encountered when attempting to learn the inverse of the entire UQCM in a single step.

Although time constraints prevented full implementation of this substep-based diffusion model, the conceptual framework represents a promising direction for future work. Importantly, it emerged organically from the iterative trial-and-error process: by closely examining the points where classical diffusion techniques fail in quantum settings, it became possible to identify a more appropriate, theoretically grounded architecture tailored to the structure of quantum operations.

V. Discussion and Conclusion

This work demonstrates a hybrid quantum–classical strategy for mitigating fidelity loss in approximate qubit cloning. Beginning with simulated mixed-state qubits, I developed a workflow that incorporated UQCM cloning, Bloch-vector representation, MLP-based denoising, and ultimately a diffusion-inspired forward process. At each stage, new computational and conceptual challenges emerged, from generating unbiased mixed states, to representing quantum states in a form compatible with classical architectures, to constructing a timestep-conditioned model that remained faithful to quantum mechanical constraints.

A central insight arising from this process is the hierarchical substep concept: by decomposing the UQCM into its smaller reversible operations, one can dramatically increase the resolution of the forward process while preserving physical validity. Although this substep-based diffusion framework was not fully implemented within the project timeframe, it provides a promising pathway toward models capable of learning more effective inverse transformations. Such an approach may enable future quantum–AI hybrid systems to surpass classical fidelity limitations without violating the principles of quantum mechanics.

In summary, this project offers both a proof-of-concept implementation and a detailed examination of the obstacles involved in integrating quantum simulation with classical learning frameworks. The workflow, observations, and proposed hierarchical structure establish a foundation for ongoing exploration of hybrid models aimed at improving fidelity in approximate qubit cloning.

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