**D 2.86** Figure P2.86 shows a circuit that performs a low-pass STC function. Such a circuit is known as a first-order, low-pass active filter. Derive the transfer function and show that the dc gain is $(-R_2/R_1)$ and the 3-dB frequency $\omega_0 = 1/CR_2$. Design the circuit to obtain an input resistance of 10 kΩ, a dc gain of 40 dB, and a 3-dB frequency of 1 kHz. At what frequency does the magnitude of the transfer function reduce to unity?

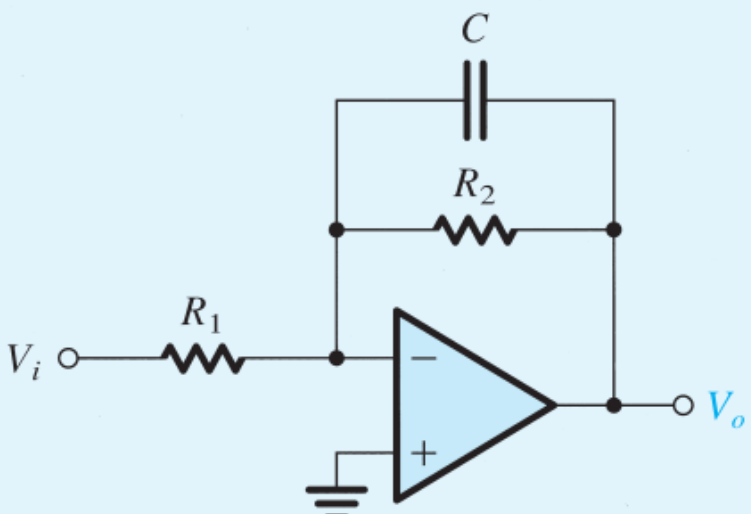
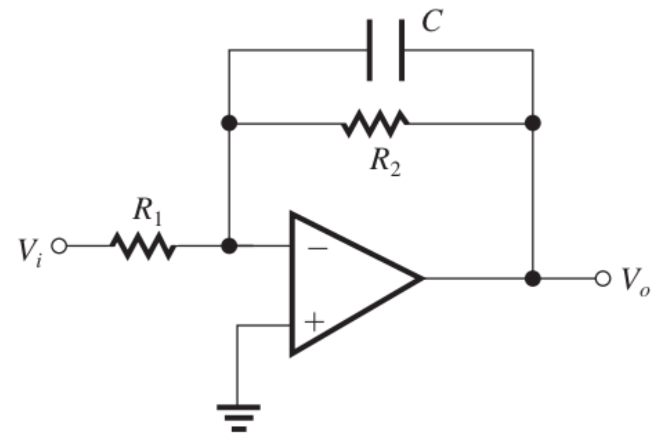



**Figure P2.86**

**2.80**




Let $Z_2 = R_2 \parallel \dfrac{1}{sC}$ and $Z_1 = R_1$

$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{1/R_1}{\dfrac{1}{R_2} + sC}$$

$$= -\frac{(R_2/R_1)}{1 + sCR_2}$$

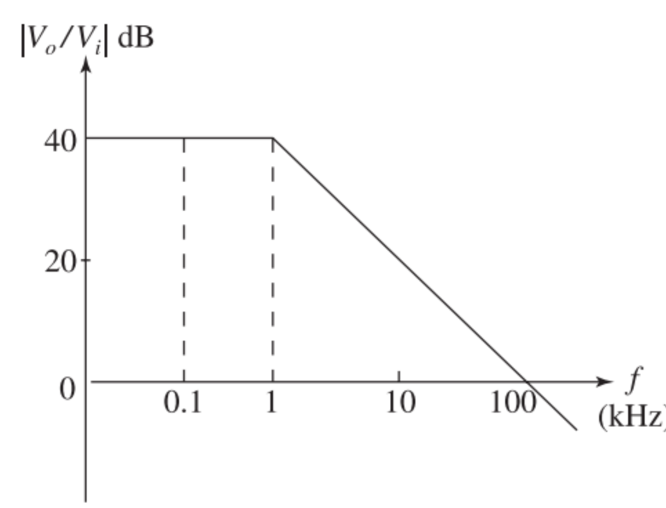
This function is of the STC low-pass type, having a dc gain of $-\dfrac{R_2}{R_1}$ and a 3-dB frequency

$$\omega_0 = \frac{1}{CR_2}$$

$R_{in} = R_1 = 10\ \text{k}\Omega$

dc gain = 40 dB = 100

$$\therefore\ 100 = \frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 1\ \text{M}\Omega$$




3-dB frequency at 1 kHz

$$\therefore \omega_0 = 2\pi \times 1 \times 10^3 = \frac{1}{CR_2}$$

$$C = \frac{1}{2\pi \times 1 \times 10^3 \times 10^6} = 0.16\ \text{nF}$$

From the Bode plot shown in previous column, the unity-gain frequency is 100 kHz.

**D 2.92** Figure P2.92 shows a circuit that performs the high-pass, single-time-constant function. Such a circuit is known as a first-order high-pass active filter. Derive the transfer function and show that the high-frequency gain is $(-R_2/R_1)$ and the 3-dB frequency $\omega_0 = 1/CR_1$. Design the circuit to obtain a high-frequency input resistance of 1 kΩ, a high-frequency gain of 40 dB, and a 3-dB frequency of 2 kHz. At what frequency does the magnitude of the transfer function reduce to unity?

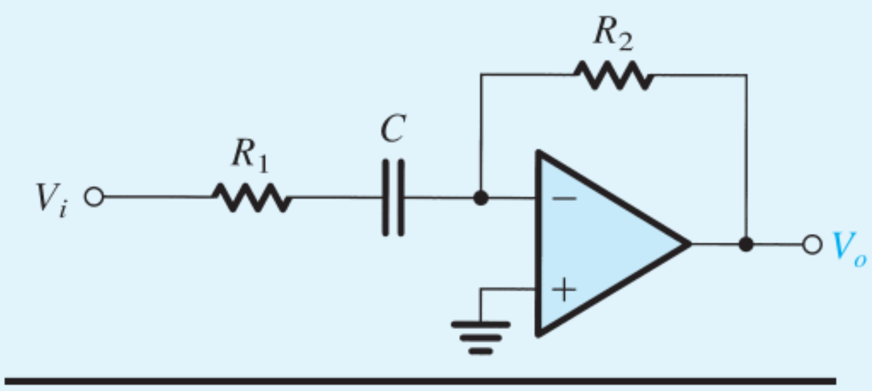
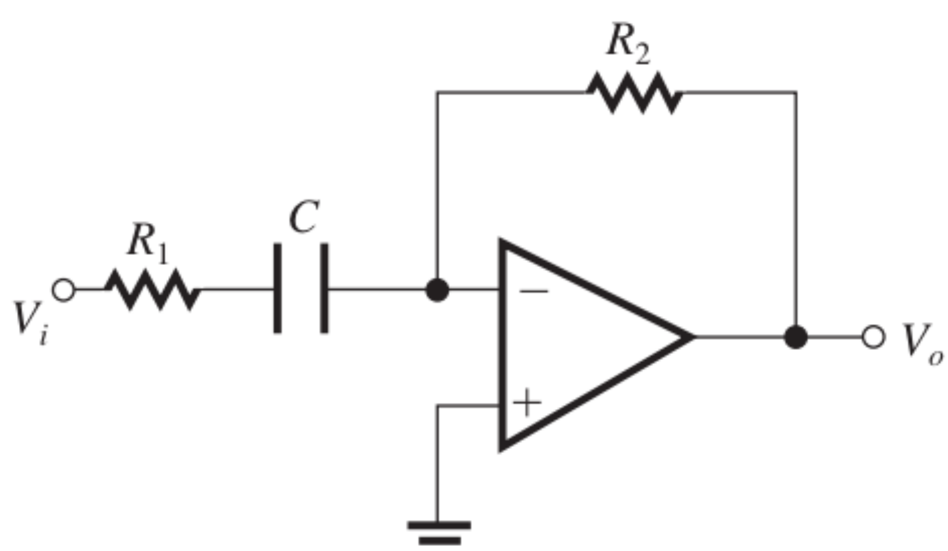



**Figure P2.92**




$$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1 + \dfrac{1}{sC}}$$

Thus,

$$\frac{V_o}{V_i} = -\frac{(R_2/R_1)s}{s + \dfrac{1}{CR_1}}$$

which is that of an STC high-pass type.

$$\text{High-frequency gain } (s \to \infty) = -\frac{R_2}{R_1}$$

$$\text{3-dB frequency } (\omega_{3\text{dB}}) = \frac{1}{CR_1}$$

For a high-frequency input resistance of 1 kΩ, we select $R_1 = 1$ kΩ. For a high-frequency gain of 40 dB,

$$\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 100 \text{ k}\Omega$$

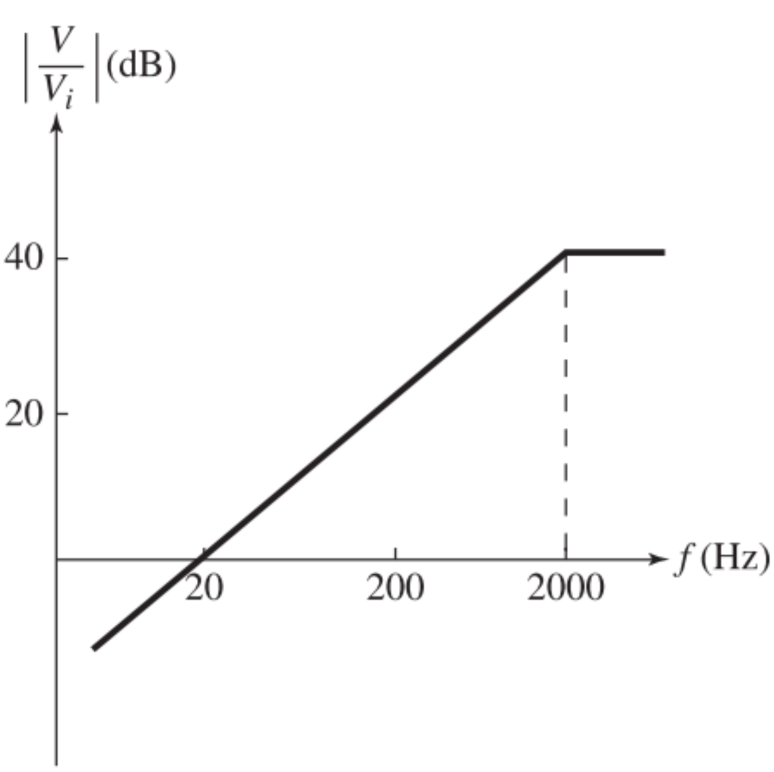
For $f_{3\text{dB}} = 2$ kHz,

$$\frac{1}{2\pi CR_1} = 2 \times 10^3$$

$$\Rightarrow C = 79 \text{ nF}$$

The magnitude of the transfer function reduces from 40 dB to unity (0 dB) in two decades. Thus

$$f \text{ (unity gain)} = \frac{f_{3\text{dB}}}{100} = \frac{2000}{100} = 20 \text{ Hz}$$



**D *2.117** This problem illustrates the use of cascaded closed-loop amplifiers to obtain an overall bandwidth greater than can be achieved using a single-stage amplifier with the same overall gain.

(a) Show that cascading two identical amplifier stages, each having a low-pass STC frequency response with a 3-dB frequency $f_1$, results in an overall amplifier with a 3-dB frequency given by

$$f_{3dB} = \sqrt{\sqrt{2}-1}\,f_1$$

(b) It is required to design a noninverting amplifier with a dc gain of 40 dB utilizing a single internally compensated op amp with $f_t = 2$ MHz. What is the 3-dB frequency obtained?

(c) Redesign the amplifier of (b) by cascading two identical noninverting amplifiers each with a dc gain of 20 dB. What is the 3-dB frequency of the overall amplifier? Compare this to the value obtained in (b) above.

**2.108** (a) Assume two identical stages, each with a gain function:

$$G = \frac{G_0}{1 + j\dfrac{\omega}{\omega_1}} = \frac{G_0}{1 + \dfrac{jf}{f_1}}$$

$$G = \frac{G_0}{\sqrt{1 + \left(\dfrac{f}{f_1}\right)^2}}$$

overall gain of the cascade is $\dfrac{G_0^2}{1 + \left(\dfrac{f}{f_1}\right)^2}$

The gain will drop by 3 dB when

$$1 + \left(\frac{f_{3dB}}{f_1}\right)^2 = \sqrt{2}$$

$$f_{3dB} = f_1\sqrt{\sqrt{2} - 1} \qquad \text{Q.E.D}$$

(b) $40 \text{ dB} = 20\log G_0 \Rightarrow G_0 = 100 = 1 + \dfrac{R_2}{R_1}$

$$f_{3dB} = \frac{f_t}{1 + \dfrac{R_2}{R_1}} = \frac{2 \text{ MHz}}{100} = 20 \text{ kHz}$$

$\boxed{\therefore GB = 1 \cdot f_t = G \cdot f}$

(c) Each stage should have 20-dB gain or $1 + \dfrac{R_2}{R_1} = 10$ and therefore a 3-dB frequency of

$$f_1 = \frac{2 \times 10^6}{10} = 2 \times 10^5 \text{ Hz}$$

The overall $f_{3dB} = 2 \times 10^5\sqrt{\sqrt{2} - 1}$

$= 128.7$ kHz,

which is 6.4 times greater than the bandwidth achieved using a single op amp, as in case (b) above.

**2.126** For an amplifier having a slew rate of 40 V/μs, what is the highest frequency at which a 20-V peak-to-peak sine wave can be produced at the output?

**2.114** Slope of the triangle wave $= \dfrac{10\text{ V}}{T/2} = \text{SR}$

Thus $\dfrac{10}{T} \times 2 = 20\text{ V}/\mu\text{s}$

$\Rightarrow T = 1\ \mu\text{s}$ or $f = \dfrac{1}{T} = 1\text{ MHz}$

For a sine wave $v_O = \hat{V}_o \sin(2\pi \times 1 \times 10^6 t)$

$$\left.\frac{dv_O}{dt}\right|_{\max} = 2\pi \times 1 \times 10^6\, \hat{V}_o = \text{SR}$$

$$\Rightarrow \hat{V}_o = \frac{20 \times 10^6}{2\pi \times 10^6 \times 1} = 3.18\text{ V}$$

**11.2** Consider the op-amp circuit shown in Fig. P11.2, where the op amp has infinite input resistance and zero output resistance but finite open-loop gain $A$.

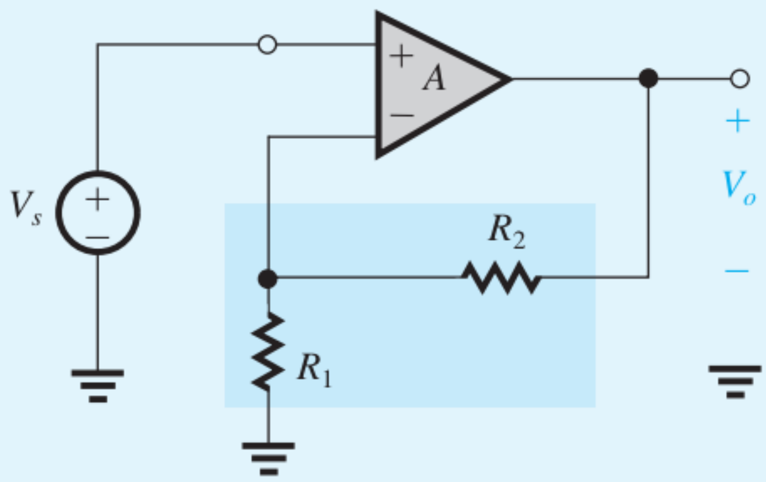



**Figure P11.2**

(a) Convince yourself that $\beta = R_1/(R_1+R_2)$.

(b) If $R_1 = 10\ \text{k}\Omega$, find $R_2$ that results in $A_f = 10$ V/V for the following three cases: (i) $A = 1000$ V/V; (ii) $A = 200$ V/V; (iii) $A = 15$ V/V.

(c) For each of the three cases in (b), find the percentage change in $A_f$ that results when $A$ decreases by 20%. Comment on the results.

**10.2** (a) Because of the infinite input resistance of the op amp, the fraction of the output voltage $V_o$ that is fed back and subtracted from $V_s$ is determined by the voltage divider $(R_1, R_2)$, thus

$$\beta = \frac{R_1}{R_1+R_2}$$

(b) (i) $A = 1000$ V/V

$$A_f = \frac{A}{1+A\beta}$$

$$10 = \frac{1000}{1+1000\beta}$$

$$\Rightarrow \beta = 0.099 \text{ V/V}$$

$$\frac{R_1}{R_1+R_2} = 0.099$$

$$1+\frac{R_2}{R_1} = \frac{1}{0.099}$$

$$R_2 = R_1\left(\frac{1}{0.099}-1\right)$$

$$= 10\left(\frac{1}{0.099}-1\right) = 91\ \text{k}\Omega$$

(ii) $A = 200$ V/V

$$10 = \frac{200}{1+200\beta}$$

$$\Rightarrow \beta = 0.095 \text{ V/V}$$

$$R_2 = R_1\left(\frac{1}{0.095}-1\right)$$

$$= 10\left(\frac{1}{0.095}-1\right) = 95.3\ \text{k}\Omega$$

(iii) $A = 15$ V/V

$$10 = \frac{15}{1+15\beta}$$

$$\Rightarrow \beta = 0.033 \text{ V/V}$$

$$R_2 = 10\left(\frac{1}{0.033}-1\right)$$

$$= 290\ \text{k}\Omega$$

(c) (i) $A = 1000(1-0.2) = 800$ V/V

$$A_f = \frac{800}{1+800\times 0.099}$$

$$= 9.975 \text{ V/V}$$

Thus, $A_f$ changes by

$$= \frac{9.975-10}{10}\times 100 = -0.25\%$$

(ii) $A = 200(1-0.2) = 160$ V/V

$$A_f = \frac{160}{1+160\times 0.095} = 9.877 \text{ V/V}$$

Thus, $A_f$ changes by

$$= \frac{9.877-10}{10}\times 100 = -1.23\%$$

(iii) $A = 15(1-0.2) = 12$ V/V

$$A_f = \frac{12}{1+12\times 0.033} = 8.574$$

Thus, $A_f$ changes by

$$= \frac{8.575-10}{10}\times 100 = -14.3\%$$

We conclude that as $A$ becomes smaller and hence the amount of feedback $(1+A\beta)$ is lower, the desensitivity of the feedback amplifier to changes in $A$ decreases. In other words, the negative feedback becomes less effective as $(1+A\beta)$ decreases.

**11.18** Consider an amplifier having a midband gain $A_M$ and a low-frequency response characterized by a pole at $s = -\omega_L$ and a zero at $s = 0$. Let the amplifier be connected in a negative-feedback loop with a feedback factor $\beta$. Find an expression for the midband gain and the lower 3-dB frequency of the closed-loop amplifier. By what factor have both changed?

**10.16** $A = A_M \dfrac{s}{s + \omega_L}$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{A_M s/(s + \omega_L)}{1 + A_M \beta s/(s + \omega_L)}$$

$$= \frac{A_M s}{s + \omega_L + s A_M \beta}$$

$$= \frac{A_M s}{s(1 + A_M \beta) + \omega_L}$$

$$= \frac{A_M}{1 + A_M \beta} \, \frac{s}{s + \omega_L/(1 + A_M \beta)}$$

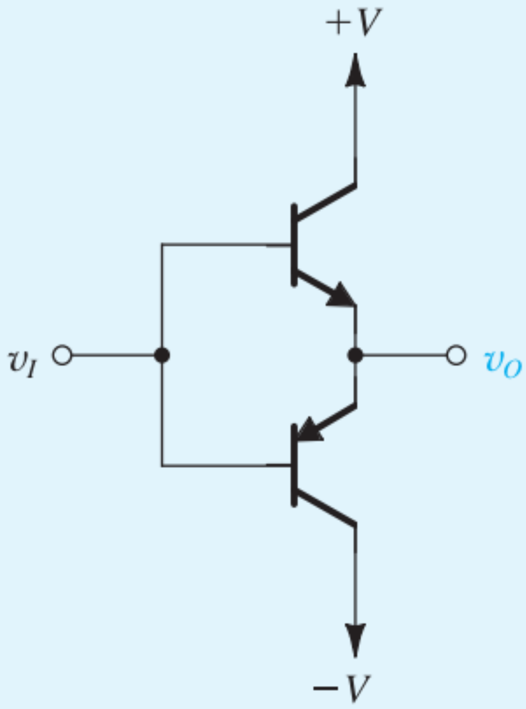
Thus,

$$A_{Mf} = \frac{A_M}{1 + A_M \beta}$$

$$\omega_{Lf} = \frac{\omega_L}{1 + A_M \beta}$$

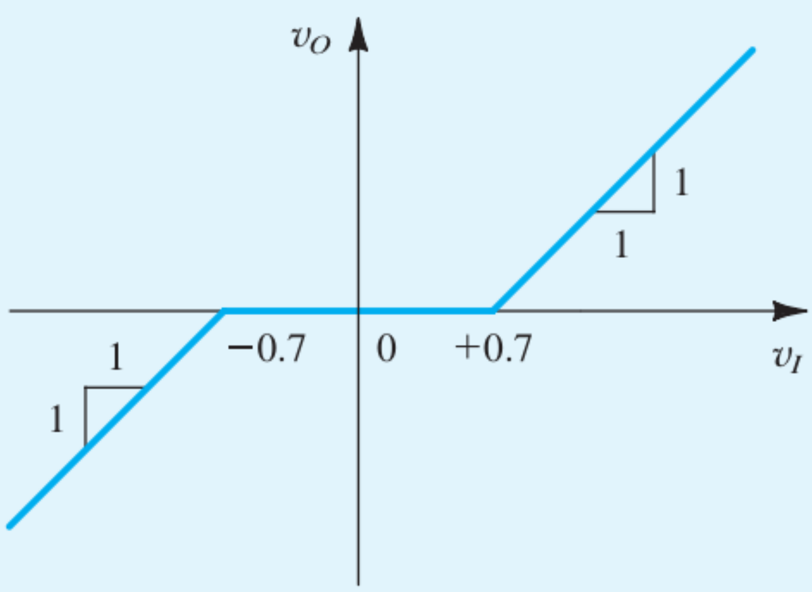
Thus, both the midband gain and the 3-dB frequency are lowered by the amount of feedback, $(1 + A_M \beta)$.

***11.24** The complementary BJT follower shown in Fig. P11.24(a) has the approximate transfer characteristic shown in Fig. P11.24(b). Observe that for $-0.7\text{ V} \le v_I \le +0.7\text{ V}$, the output is zero. This "dead band" leads to crossover distortion (see Section 12.3). Consider this follower to be driven by the output of a differential amplifier of gain 100




(a)

**Figure P11.24**




(b)

**Figure P11.24** *continued*

whose positive-input terminal is connected to the input signal source $v_S$ and whose negative-input terminal is connected to the emitters of the follower. Sketch the transfer characteristic $v_O$ versus $v_S$ of the resulting feedback amplifier. What are the limits of the dead band, and what are the gains outside the dead band?

**10.22**

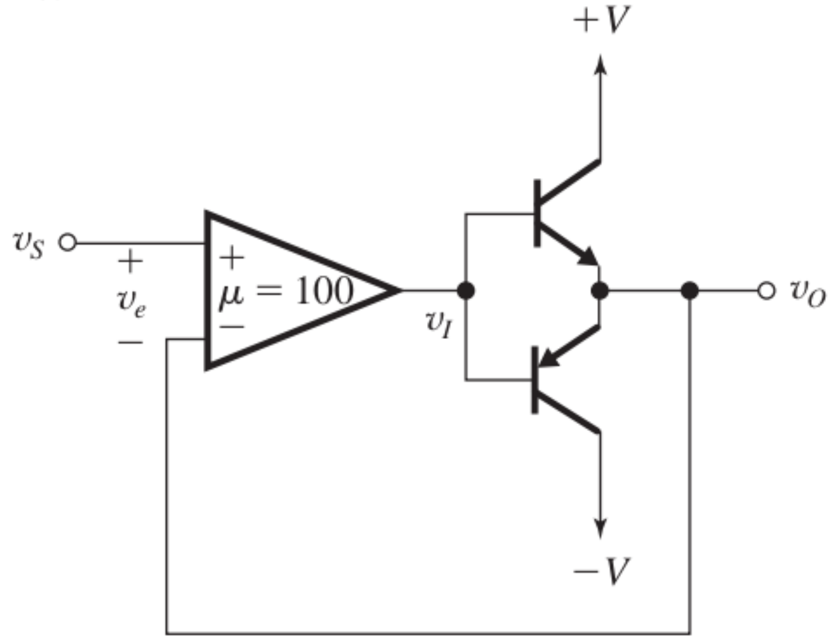



Figure 1

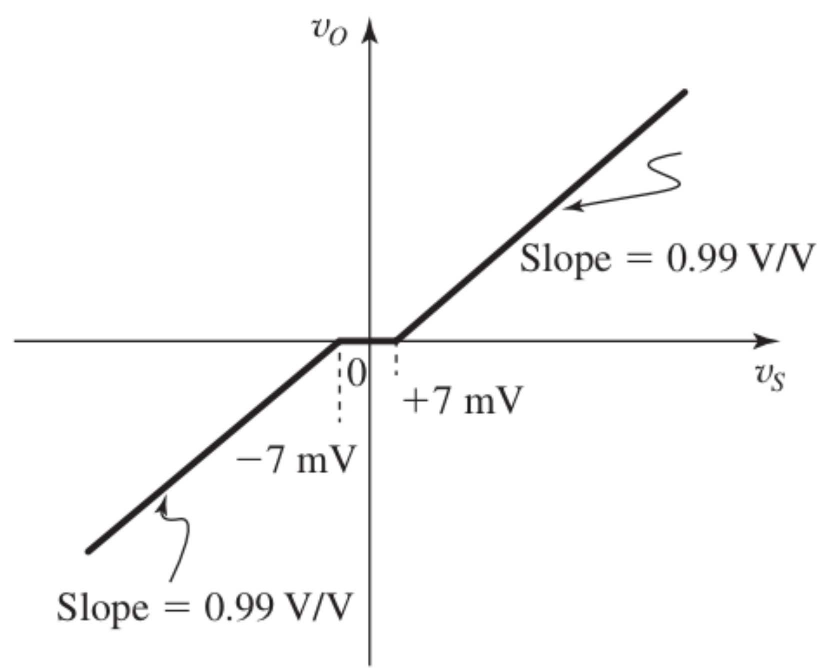



Figure 2

Refer to Fig. 1. For $v_I = +0.7$ V, we have $v_O = 0$ and

$$v_e = \frac{v_I}{\mu} = \frac{+0.7}{100} = +7\text{ mV}$$

Similarly, for $v_I = -0.7$ V, we obtain $v_O = 0$ and

$$v_e = \frac{v_I}{\mu} = \frac{-0.7}{100} = -7\text{ mV}$$

Thus, the limits of the deadband are now ±7 mV. Outside the deadband, the gain of the feedback amplifier, that is, $v_O/v_S$, can be determined by noting that the open-loop gain $A \equiv v_O/v_e = 100$ V/V and the feedback factor $\beta = 1$, thus

$$A_f \equiv \frac{v_O}{v_S} = \frac{A}{1 + A\beta}$$

$$= \frac{100}{1 + 100 \times 1} = 0.99\text{ V/V}$$

The transfer characteristic is depicted in Fig. 2.

**D 11.31** Figure P11.31 shows a series–shunt feedback amplifier known as a "feedback triple." All three MOSFETs are biased to operate at $g_m = 4$ mA/V. You may neglect their $r_o$'s.

(a) Select a value for $R_F$ that results in a closed-loop gain that is ideally 10 V/V.

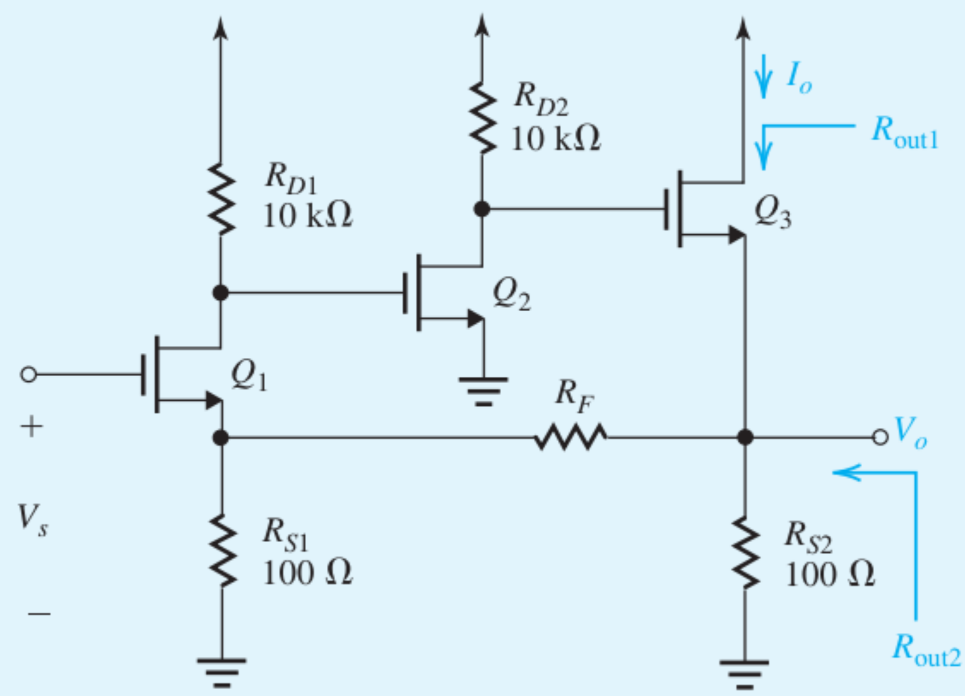



**Figure P11.31**

(b) Determine the loop gain $A\beta$ and hence the value of $A_f$. By what percentage does $A_f$ differ from the ideal value you designed for? How can you adjust the circuit to make $A_f$ equal to 10?

**10.28** (a) The feedback network consists of the voltage divider $(R_F, R_{S1})$. Thus,

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

and the ideal value of the closed-loop gain is

$$A_f = \frac{1}{\beta} = 1 + \frac{R_F}{R_{S1}}$$

$$10 = 1 + \frac{R_F}{0.1}$$

$$\Rightarrow R_F = 0.9\ \text{k}\Omega$$

(b)

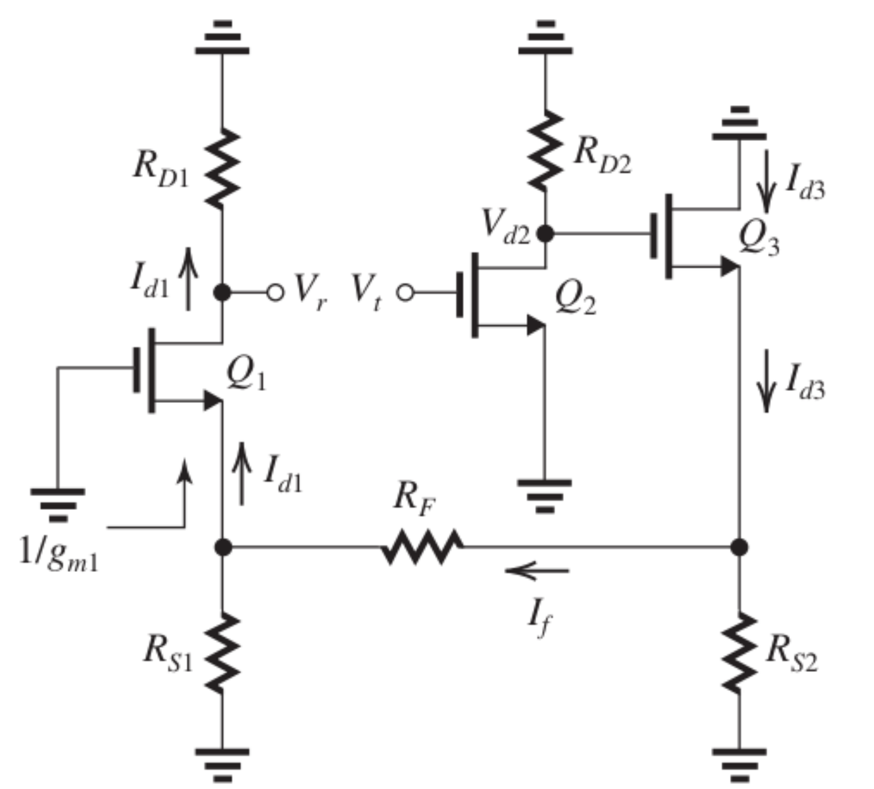



Figure 1

Figure 1 shows the circuit for determining the loop gain. Observe that we have broken the loop at the gate of $Q_2$ where the input resistance is infinite, obviating the need for adding a termination resistance. Also, observe that as usual we have set $V_s = 0$. To determine the loop gain

$$A\beta \equiv -\frac{V_r}{V_t}$$

we write the following equations:

$$V_{d2} = -g_{m2} R_{D2} V_t \tag{1}$$

$$I_{d3} = \frac{V_{d2}}{\dfrac{1}{g_{m3}} + \left\{R_{S2} \parallel \left[R_F + \left(R_{S1} \parallel \dfrac{1}{g_{m1}}\right)\right]\right\}} \tag{2}$$

$$I_f = I_{d3} \frac{R_{S2}}{\left[R_F + \left(R_{S1} \parallel \frac{1}{g_{m1}}\right)\right] + R_{S2}} \tag{3}$$

$$I_{d1} = I_f \frac{R_{S1}}{R_{S1} + \frac{1}{g_{m1}}} \tag{4}$$

$$V_r = I_{d1} R_{D1} \tag{5}$$

Substituting the numerical values in (1)–(5), we obtain

$$V_{d2} = -4 \times 10 V_t = -40 V_t \tag{6}$$

$$I_{d3} = \frac{V_{d2}}{\frac{1}{4} + \left\{0.1 \parallel \left[0.9 + \left(0.1 \parallel \frac{1}{4}\right)\right]\right\}}$$

$$I_{d3} = 2.935 V_{d2} \tag{7}$$

$$I_f = I_{d3} \frac{0.1}{\left[0.9 + \left(0.1 \parallel \frac{1}{4}\right)\right] + 0.1}$$

$$I_f = 0.0933 I_{d3} \tag{8}$$

$$I_{d1} = I_f \frac{0.1}{0.1 + \frac{1}{4}} = 0.286 I_f \tag{9}$$

$$V_r = 10 I_{d1} \tag{10}$$

Combining (6)–(10) gives

$$V_r = -31.33 V_t$$

$$\Rightarrow A\beta = 31.33$$

$$A = \frac{A\beta}{\beta} = \frac{31.33}{0.1} = 313.3 \text{ V/V}$$

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{313.3}{1 + 31.33} = 9.7 \text{ V/V}$$

Thus, $A_f$ is 0.3 V/V lower than the ideal value of 10 V/V, a difference of $-3\%$. The circuit could be adjusted to make $A_f$ exactly 10 by changing $\beta$ through varying $R_F$. Specifically,

$$10 = \frac{313.3}{1 + 313.3\beta}$$

$$\Rightarrow \beta = 0.0968$$

But,

$$\beta = \frac{R_{S1}}{R_{S1} + R_F}$$

$$0.0968 = \frac{0.1}{0.1 + R_F}$$

$$\Rightarrow R_F = 933\ \Omega$$

(an increase of 33 $\Omega$).

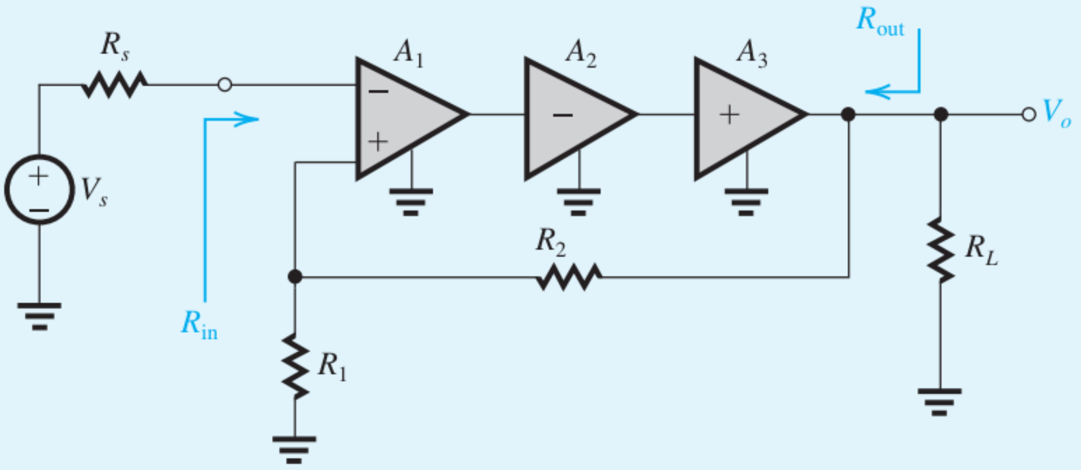



**Figure P11.45**

**D *11.45** Figure P11.45 shows a three-stage feedback amplifier:

$A_1$ has an 82-kΩ differential input resistance, a 20-V/V open-circuit differential voltage gain, and a 3.2-kΩ output resistance.

$A_2$ has a 5-kΩ input resistance, a 20-mA/V short-circuit transconductance, and a 20-kΩ output resistance.

$A_3$ has a 20-kΩ input resistance, unity open-circuit voltage gain, and a 1-kΩ output resistance.

The feedback amplifier feeds a 1-kΩ load resistance and is fed by a signal source with a 9-kΩ resistance.

(a) Show that the feedback is negative.
(b) If $R_1 = 20$ kΩ, find the value of $R_2$ that results in a closed-loop gain $V_o/V_s$ that is ideally 5 V/V.
(c) Supply the small-signal equivalent circuit.
(d) Sketch the $A$ circuit and determine $A$.
(e) Find $\beta$ and the amount of feedback.
(f) Find the closed-loop gain $A_f \equiv V_o/V_s$.
(g) Find the feedback amplifier's input resistance $R_{in}$.
(h) Find the feedback amplifier's output resistance $R_{out}$.
(i) If the high-frequency response of the open-loop gain $A$ is dominated by a pole at 100 Hz, what is the upper 3-dB frequency of the closed-loop gain?
(j) If for some reason $A_1$ drops to half its nominal value, what is the percentage change in $A_f$?

**10.40** (a) Refer to Fig. P10.40. If $V_s$ increases, the output of $A_1$ will decrease and this will cause the output of $A_2$ to increase. This, in turn, causes the output of $A_3$, which is $V_o$, to increase. A portion of the positive increment in $V_o$ is fed back to the positive input terminal of $A_1$ through the voltage divider $(R_2, R_1)$, The increased voltage at the positive input terminal of $A_1$ counteracts the originally assumed increase at the negative input terminal, verifying that the feedback is negative.

(b) $A_f\big|_{\text{ideal}} = \dfrac{1}{\beta}$

where

$$\beta = \frac{R_1}{R_1 + R_2}$$

Thus, to obtain an ideal closed-loop gain of 5 V/V we need $\beta = 0.2$:

$$0.2 = \frac{20}{20 + R_2}$$

$$\Rightarrow R_2 = 80 \text{ k}\Omega$$

(c) Figure 1 on the next page shows the small-signal equivalent circuit of the feedback amplifier.

(d) Figure 2 on the next page shows the $A$ circuit and the $\beta$ circuit together with the determination of its loading effects, $R_{11}$, and $R_{22}$. We can write

$$\frac{V_1}{V_i} = -\frac{82}{82 + 9 + 16} = -0.766 \text{ V/V}$$

$$V_2 = 20V_1 \times \frac{5}{3.2 + 5} = 12.195V_1$$

$$V_3 = -20V_2(20 \parallel 20) = -200V_2$$

This figure belongs to Problem 10.40, part (c).

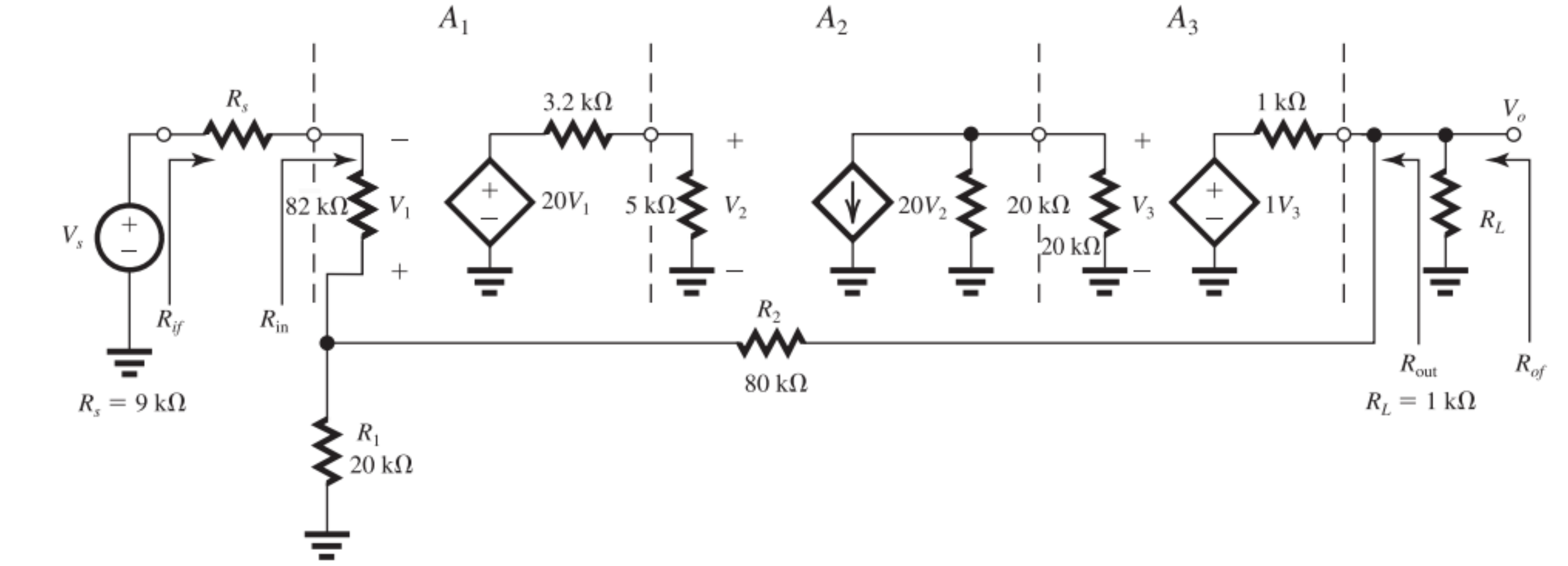



Figure 1

This figure belongs to Problem 10.40, part (d).

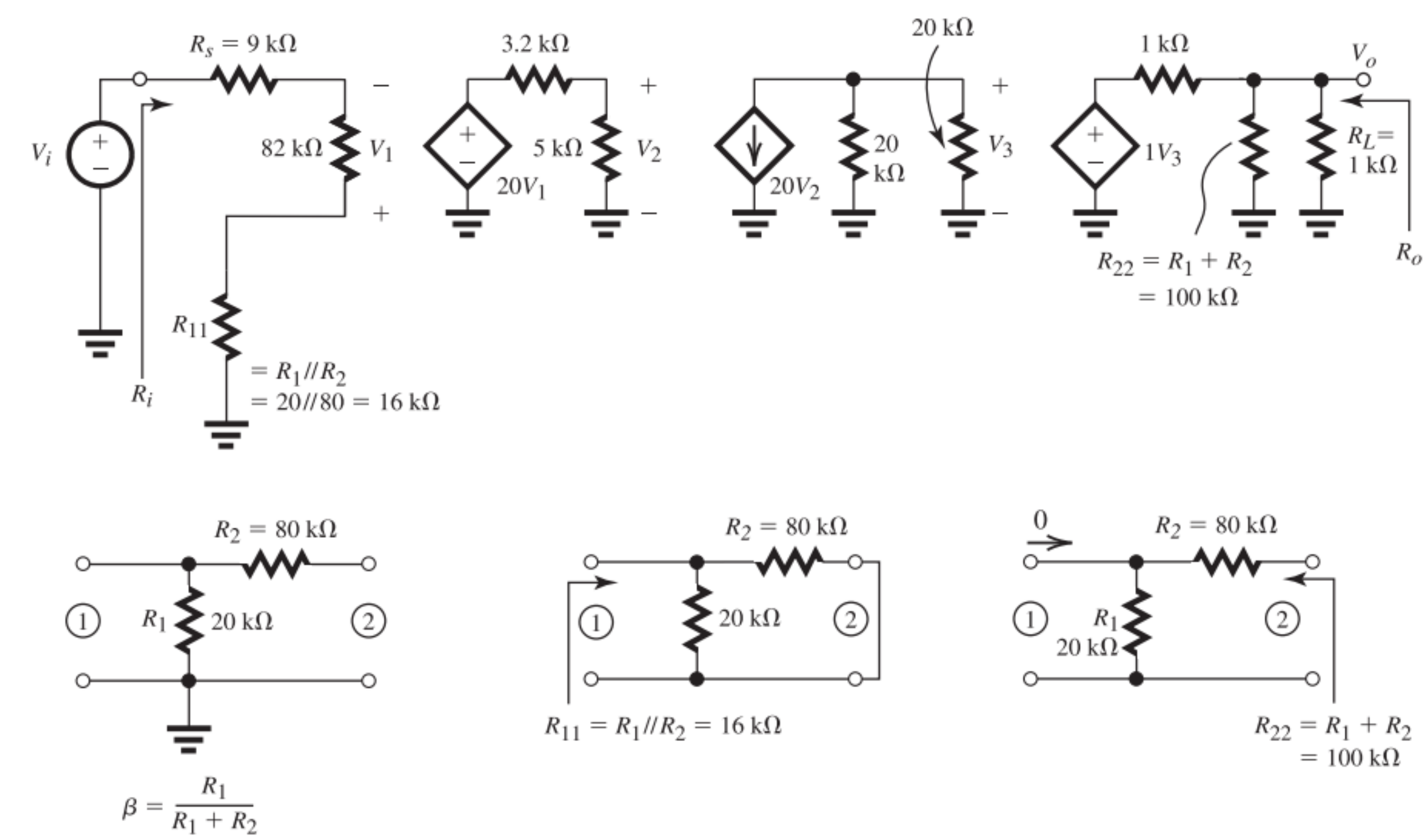



Figure 2

$$V_o = V_3 \frac{1 \parallel 100}{(1 \parallel 100) + 1} = 0.497 V_3$$

Thus,

$$A \equiv \frac{V_o}{V_i} = 0.497 \times -200 \times 12.195 \times -0.766$$

$$= 928.5 \text{ V/V}$$

(e) $\beta = \dfrac{20}{20 + 80} = 0.2$ V/V

$$1 + A\beta = 1 + 928.5 \times 0.2 = 186.7$$

(f) $A_f \equiv \dfrac{V_o}{V_s} = \dfrac{A}{1 + A\beta}$

$$= \frac{928.5}{186.7} = 4.97 \text{ V/V}$$

which is nearly equal to the ideal value of 5 V/V.

(g) From the $A$ circuit,

$$R_i = 9 + 82 + 16 = 107 \text{ k}\Omega$$

$$R_{if} = R_i(1 + A\beta)$$

$$= 107 \times 186.7 = 19.98 \text{ M}\Omega$$

$$R_{\text{in}} = R_{if} - R_s \simeq 19.98 \text{ M}\Omega$$

(h) From the $A$ circuit,

$$R_o = R_L \parallel R_{22} \parallel 1 \text{ k}\Omega$$

$$= 1 \parallel 100 \parallel 1 = 497.5 \ \Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{497.5}{186.7} = 2.66 \ \Omega$$

$R_{\text{out}} \parallel R_L = R_{of}$

$R_{\text{out}} \parallel 1000 = 2.66\ \Omega$

$R_{\text{out}} \simeq 2.66\ \Omega$

(i) $f_{Hf} = f_H(1 + A\beta)$

$= 100 \times 186.7$

$= 18.67$ kHz

(j) If $A_1$ drops to half its nominal value, $A$ will drop to half its nominal value:

$$A = \frac{1}{2} \times 928.5 = 464.25$$

and $A_f$ becomes

$$A_f = \frac{464.25}{1 + 464.25 \times 0.2} = 4.947 \text{ V/V}$$

Thus, the percentage change in $A_f$ is

$$= \frac{4.947 - 4.97}{4.97} = -0.47\%$$

**D 11.53** The transconductance amplifier in Fig. P11.53 utilizes a differential amplifier with gain $\mu$ and a very high input resistance. The differential amplifier drives a transistor $Q$ characterized by its $g_m$ and $r_o$. A resistor $R_F$ senses the output current $I_o$.

(a) For $A\beta \gg 1$, find an approximate expression for the closed-loop transconductance $A_f \equiv I_o/V_s$. Hence, select a value for $R_F$ that results in $A_f \simeq 5$ mA/V.

(b) Find the $A$ circuit and derive an expression for $A$. Evaluate $A$ for the case $\mu = 1000$ V/V, $g_m = 2$ mA/V, $r_o = 20$ kΩ, and the value of $R_F$ you selected in (a).

(c) Give an expression for $A\beta$ and evaluate its value and that of $1 + A\beta$.

(d) Find the closed-loop gain $A_f$ and compare to the value you designed for in (a) above.

(e) Find expressions and values for $R_o$ and $R_{of}$. [*Hint*: The resistance looking into the drain of a MOSFET with a resistance $R_s$ in its source is $(r_o + R_s + g_m r_o R_s)$.]

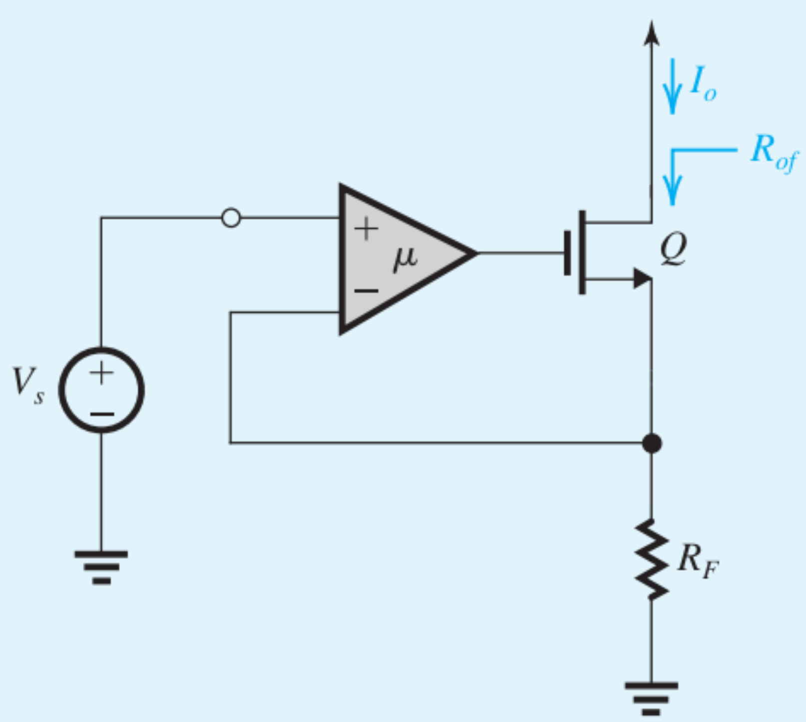



**Figure P11.53**

**10.49** (a) The $\beta$ circuit is shown in Fig. 1:

$\beta = R_F$

For $A\beta \gg 1$, $A_f \equiv I_o/V_s$ approaches the ideal value

$$A_f\big|_{\text{ideal}} = \frac{1}{\beta} = \frac{1}{R_F}$$

To obtain $A_f \simeq 5$ mA/V, we use

$$R_F = \frac{1}{5} = 0.2\text{ k}\Omega = 200\ \Omega$$

(b) Determining the loading effects of the $\beta$ network is illustrated in Fig. 1:

$R_{11} = R_{22} = R_F$

Figure 2 (next page) shows the $A$ circuit. An expression for $A \equiv I_o/V_i$ can be derived as follows:

$$V_1 = V_i \tag{1}$$

$$V_{gs} = \mu V_1 - I_o R_F \tag{2}$$

$$I_o = g_m V_{gs} \frac{r_o}{r_o + R_F} \tag{3}$$

Combining Eqs. (1)–(3) yields

$$A \equiv \frac{I_o}{V_i} = \frac{\mu g_m r_o}{r_o + R_F + g_m r_o R_F}$$

For $\mu = 1000$ V/V, $g_m = 2$ mA/V, $r_o = 20$ kΩ, and $R_F = 0.2$ kΩ, we have

$$A = \frac{1000 \times 2 \times 20}{20 + 0.2 + 2 \times 20 \times 0.2}$$

$= 1418.4$ mA/V

(c) $$A\beta = \frac{\mu g_m r_o R_F}{r_o + R_F + g_m r_o R_F}$$

$A\beta = 283.7$

$1 + A\beta = 284.7$

(d) $$A_f \equiv \frac{I_o}{V_s} = \frac{A}{1 + A\beta}$$

$$= \frac{1418.4}{284.7} = 4.982\text{ mA/V}$$

which is very close to the ideal value of 5 mA/V.

This figure belongs to Problem 10.49, part (b).

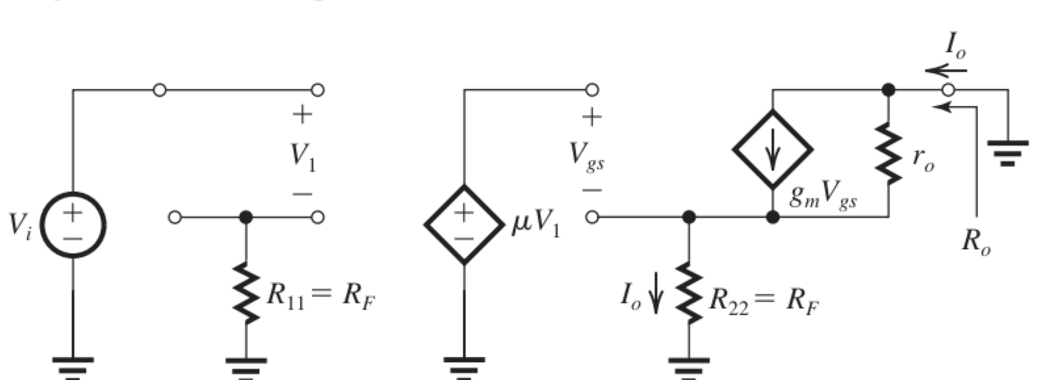



Figure 2

(e) From the $A$ circuit in Fig. 2, we have

$$R_o = r_o + R_F + g_m r_o R_F$$

$$1 + A\beta = 1 + \frac{\mu g_m r_o R_F}{r_o + R_F + g_m r_o R_F}$$

$$R_{of} = (1 + A\beta)R_o$$

$$= r_o + R_F + g_m r_o R_F + \mu g_m r_o R_F$$

$$= r_o + R_F + (\mu + 1) g_m r_o R_F$$

$$\simeq \mu g_m r_o R_F$$

$$R_o = 20 + 0.2 + 2 \times 20 \times 0.2$$

$$= 28.2\ \text{k}\Omega$$

$$R_{of} = 20 + 0.2 + 1001 \times 2 \times 20 \times 0.2$$

$$= 20 + 0.2 + 8008 = 8028.2\ \text{k}\Omega$$

$$\simeq 8\ \text{M}\Omega$$

***11.54** It is required to show that the output resistance of the BJT circuit in Fig. P11.54 is given by

$$R_o = r_o + [R_e \| (r_\pi + R_b)]\left(1 + g_m r_o \frac{r_\pi}{r_\pi + R_b}\right)$$

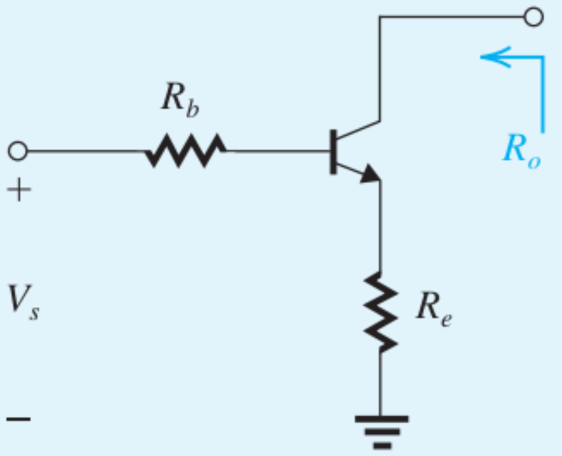



**Figure P11.54**

To derive this expression, set $V_s = 0$, replace the BJT with its small-signal, hybrid-$\pi$ model, apply a test voltage $V_x$ to the collector, and find the current $I_x$ drawn from $V_x$ and hence $R_o$ as $V_x/I_x$. Note that the bias arrangement is not shown. For the case of $R_b = 0$, find the maximum possible value for $R_o$. Note that this theoretical maximum is obtained when $R_e$ is so large that the signal current in the emitter is nearly zero. In this case, with $V_x$ applied and $V_s = 0$, what is the current in the base, in the $g_m V_\pi$ generator, and in $r_o$, all in terms of $I_x$? Show these currents on a sketch of the equivalent circuit with $R_e$ set to $\infty$.

This figure belongs to Problem 10.50.

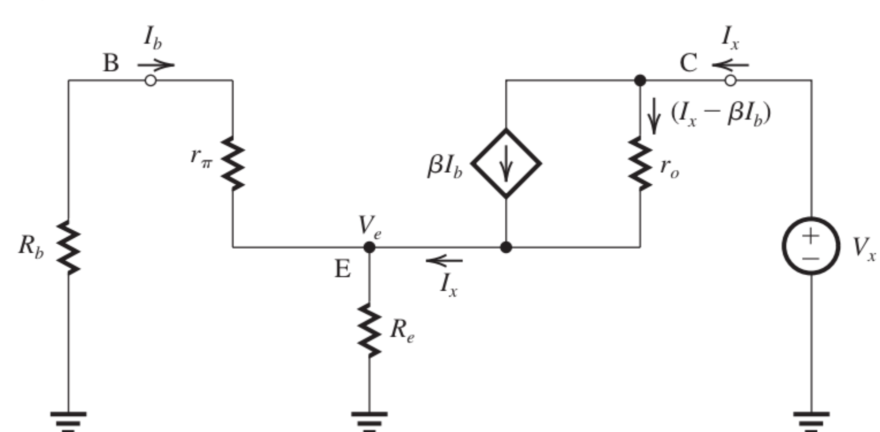



Figure 1

**10.50** Figure 1 on the next page shows the equivalent circuit with $V_s = 0$ and a voltage $V_x$ applied to the collector for the purpose of determining the output resistance $R_o$,

$$R_o \equiv \frac{V_x}{I_x}$$

Some of the analysis is displayed on the circuit diagram. Since the current entering the emitter node is equal to $I_x$, we can write for the emitter voltage

$$V_e = I_x[R_e \parallel (r_\pi + R_b)] \qquad (1)$$

The base current can be obtained using the current-divider rule applied to $R_e$ and $(r_\pi + R_b)$ as

$$I_b = -I_x \frac{R_e}{R_e + r_\pi + R_b} \qquad (2)$$

The voltage from collector to ground is equal to $V_x$ and can be expressed as the sum of the voltage drop across $r_o$ and $V_e$,

$$V_x = (I_x - \beta I_b) r_o + V_e$$

Substituting for $V_e$ from (1) and for $I_b$ from (2), we obtain

$$R_o = \frac{V_x}{I_x} = r_o + [R_e \parallel (r_\pi + R_b)]$$

$$+ \frac{R_e \beta r_o}{R_e + r_\pi + R_b}$$

$$= r_o + [R_e \parallel (r_\pi + R_b)]\left[1 + r_o \frac{\beta}{r_\pi + R_b}\right]$$

Since $\beta = g_m r_\pi$, we obtain

$$R_o = r_o + [R_e \parallel (r_\pi + R_b)]\left[1 + g_m r_o \frac{r_\pi}{r_\pi + R_b}\right] \qquad \text{Q.E.D.}$$

For $R_b = 0$,

$$R_o = r_o + (R_e \parallel r_\pi)(1 + g_m r_o)$$

The maximum value of $R_o$ will be obtained when $R_e \gg r_\pi$. If $R_e$ approaches infinity (zero signal current in the emitter), $R_o$ approaches the theoretical maximum:

$$\begin{aligned} R_{o\max} &= r_o + r_\pi(1 + g_m r_o) \\ &= r_o + r_\pi + \beta r_o \qquad (3) \\ &\simeq \beta r_o \end{aligned}$$

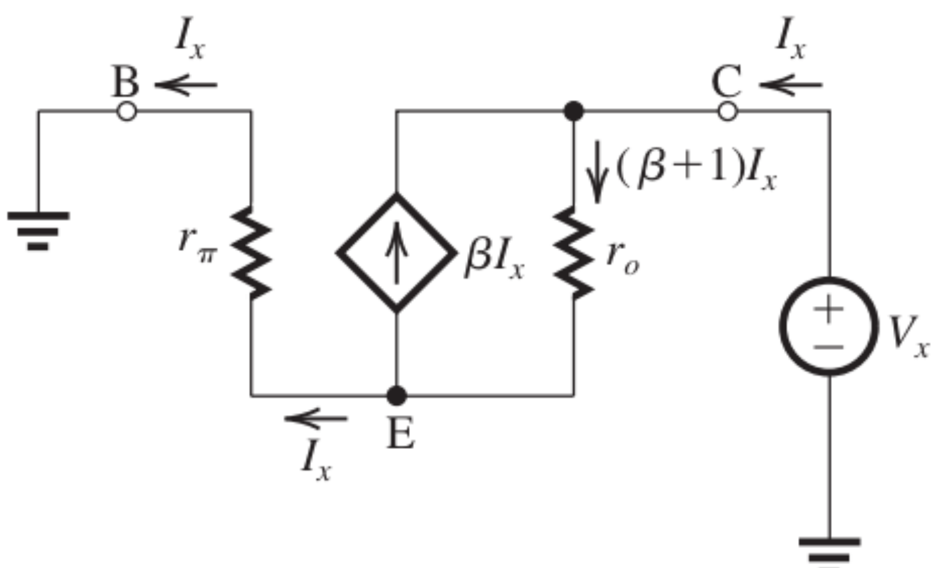



Figure 2

The situation that pertains in the circuit when $R_e = \infty$ is illustrated in Fig. 2. Observe that since the signal current in the emitter is zero, the base current will be equal to the collector current ($I_x$) and in the direction indicated. The controlled-source current will be $\beta I_x$, and this current adds to $I_x$ to provide a current $(\beta + 1)I_x$ in the output resistance $r_o$. A loop equation takes the form

$$V_x = (\beta + 1)I_x r_o + I_x r_\pi$$

and thus

$$R_o \equiv \frac{V_x}{I_x} = r_\pi + (\beta + 1)r_o$$

which is identical to the result in Eq. (3).

**11.65** The feedback transresistance amplifier in Fig. P11.65 utilizes two identical MOSFETs biased by ideal current sources $I = 0.4$ mA. The MOSFETs are sized to operate at $V_{OV} = 0.2$ V and have $V_t = 0.5$ V and $V_A = 16$ V. The feedback resistance $R_F = 10$ kΩ.

(a) If $I_s$ has a zero dc component, find the dc voltage at the input, at the drain of $Q_1$, and at the output.

(b) Find $g_m$ and $r_o$ of $Q_1$ and $Q_2$.

(c) Provide the $A$ circuit and derive an expression for $A$ in terms of $g_{m1}$, $r_{o1}$, $g_{m2}$, $r_{o2}$, and $R_F$.

(d) What is $\beta$? Give an expression for the loop gain $A\beta$ and the amount of feedback $(1 + A\beta)$.

(e) Derive an expression for $A_f$.

(f) Derive expressions for $R_i$, $R_{in}$, $R_o$, and $R_{out}$.

(g) Evaluate $A$, $\beta$, $A\beta$, $A_f$, $R_i$, $R_o$, $R_{in}$, and $R_{out}$ for the component values given.

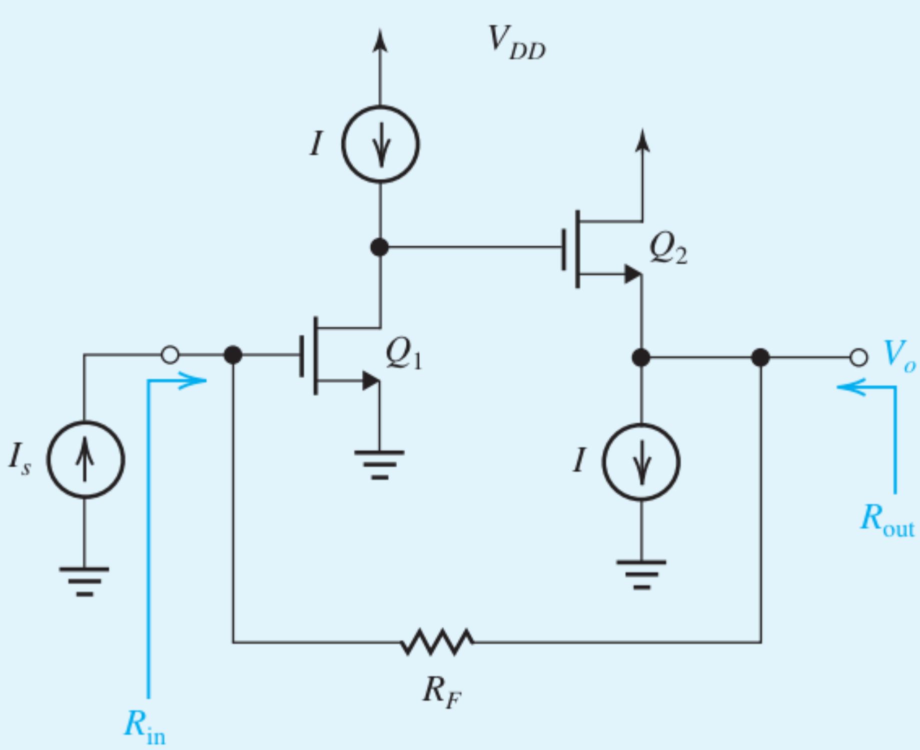



**Figure P11.65**

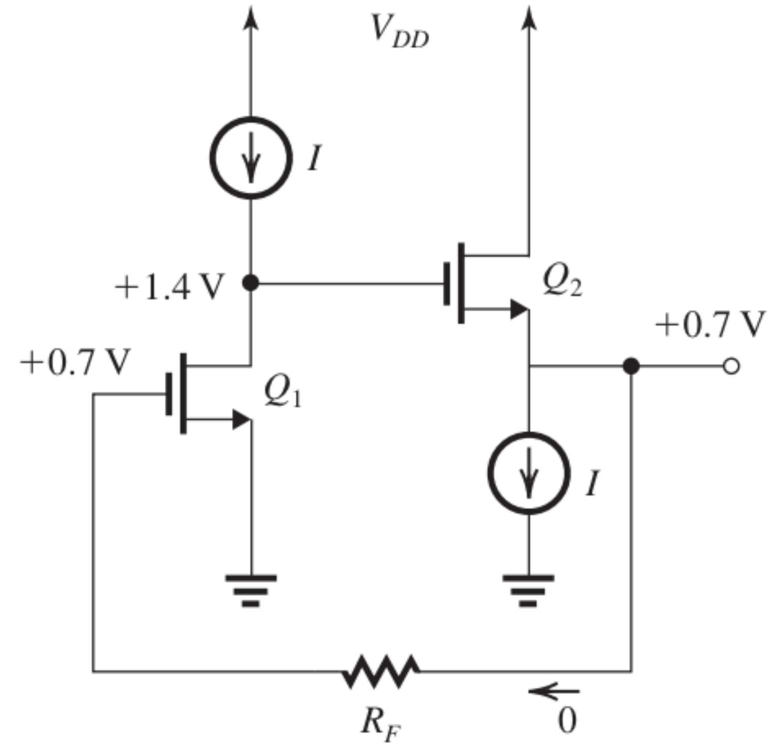



Figure 1

(a) See Figure 1.

$$V_{G1} = V_{GS1} = V_t + V_{OV}$$

$$= 0.5 + 0.2 = +0.7 \text{ V}$$

(because the dc voltage across $R_F$ is zero)

$$V_O = V_{G1}$$

$$V_O = +0.7 \text{ V}$$

$$V_{D1} = V_O + V_{GS2}$$

$$= 0.7 + 0.5 + 0.2$$

$$= +1.4 \text{ V}$$

(b) $$g_{m1,2} = \frac{2I}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$$

$$r_{o1,2} = \frac{V_A}{I} = \frac{16 \text{ V}}{0.4 \text{ mA}} = 40 \text{ k}\Omega$$

(c) Figure 2 on the next page shows the $\beta$ circuit and the determination of its loading effects,

$$R_{11} = R_{22} = R_F$$

Figure 2 shows also the $A$ circuit. We can write

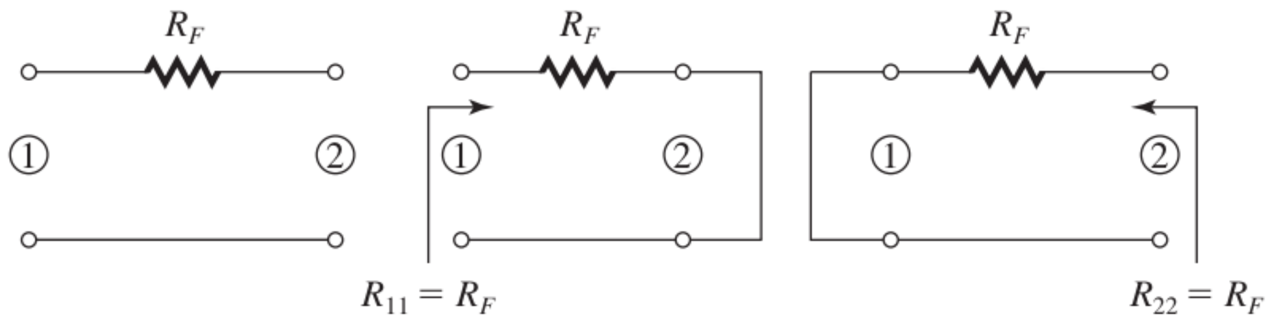
$$V_{g1} = I_i R_i \tag{1}$$

where

$$R_i = R_{11} = R_F \tag{2}$$

$$V_{d1} = -g_{m1} r_{o1} V_{g1} \tag{3}$$

This figure belongs to Problem 10.58, part (c).



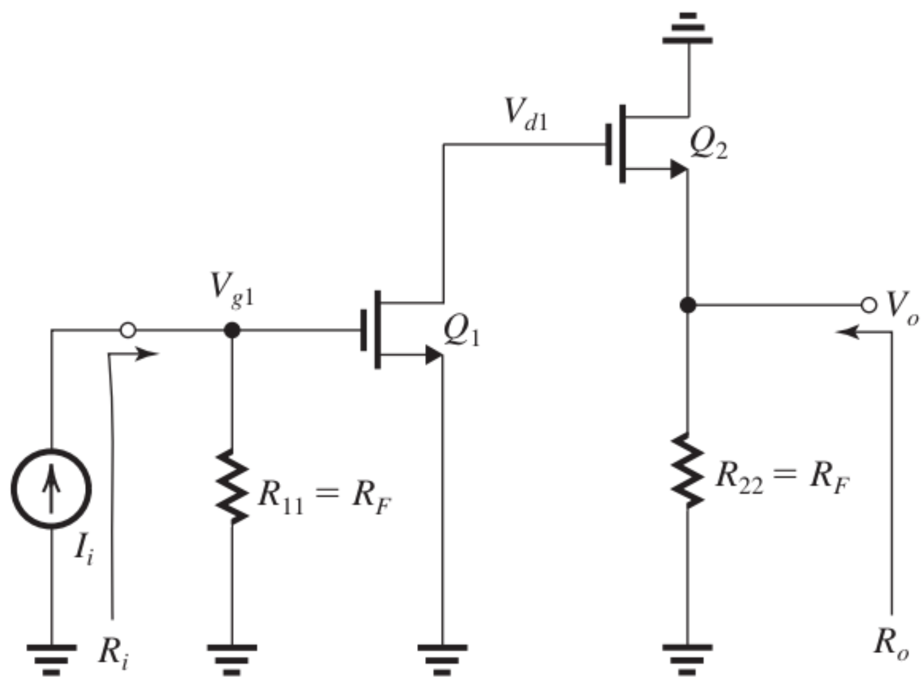



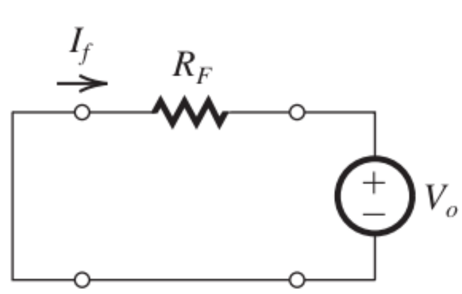

Figure 2

$$\frac{V_o}{V_{d1}} = \frac{R_{22} \parallel r_{o2}}{(R_{22} \parallel r_{o2}) + \dfrac{1}{g_{m2}}} \qquad (4)$$

Combining Eqs. (1)–(4) results in

$$A \equiv \frac{V_o}{I_i} = -g_{m1} r_{o1} R_F \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

(d)




$$\beta \equiv \frac{I_f}{V_o} = -\frac{1}{R_F}$$

Figure 3

From Fig. 3 we see that

$$\beta = -\frac{1}{R_F}$$

$$A\beta = g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

$$1 + A\beta = 1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}$$

(e) $A_f \equiv \dfrac{V_o}{I_s} = \dfrac{A}{1 + A\beta}$

$$= -\frac{g_{m1} r_{o1} R_F (R_F \parallel r_{o2})}{(R_F \parallel r_{o2}) + 1/g_{m2} + (g_{m1} r_{o1})(R_F \parallel r_{o2})}$$

(f) $R_i = R_F$

$$R_{\text{in}} = R_{if} = R_i/(1 + A\beta)$$

$$= R_F \Bigg/ \left[1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}\right]$$

$$R_{\text{out}} = R_{of} = R_o/(1 + A\beta)$$

where from the $A$ circuit we have

$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

$$R_{\text{out}} = \left(R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}\right) \Bigg/ \left[1 + g_{m1} r_{o1} \frac{R_F \parallel r_{o2}}{(R_F \parallel r_{o2}) + 1/g_{m2}}\right]$$

(g) $A = -4 \times 40 \times 10 \dfrac{10 \parallel 40}{(10 \parallel 40) + 0.25}$

$= -1551.5\ \text{k}\Omega$

$$\beta = -\frac{1}{R_F} = -\frac{1}{10\ \text{k}\Omega} = -0.1\ \text{mA/V}$$

$$A\beta = 155.15$$

$$1 + A\beta = 156.15$$

$$A_f = -\frac{1551.5}{156.15} = -9.94\ \text{k}\Omega$$

$$R_i = R_F = 10\ \text{k}\Omega$$

$$R_{\text{in}} = R_{if} = \frac{R_F}{1 + A\beta} = \frac{10,000\ \Omega}{156.15} = 64\ \Omega$$

$$R_o = R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

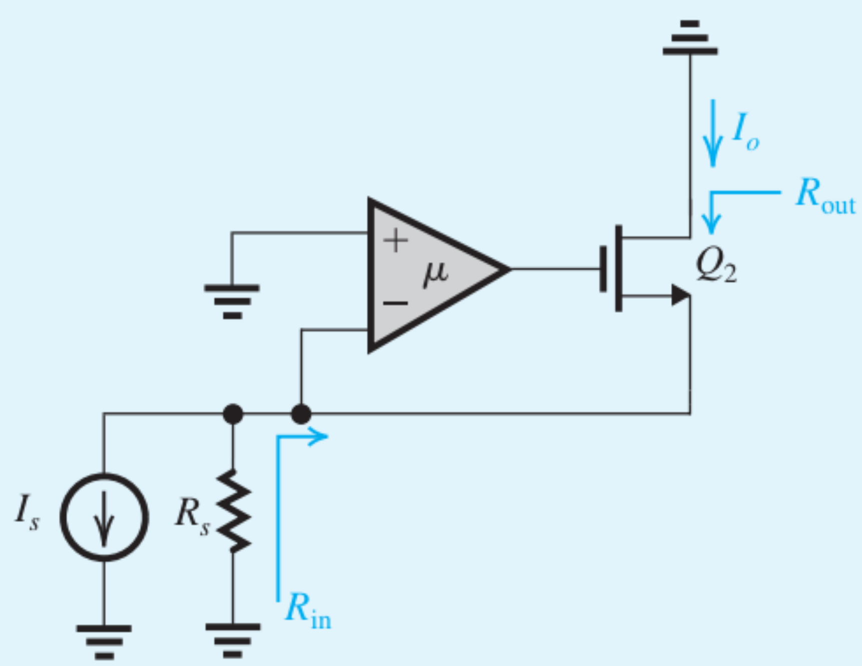
$$R_o = 10 \parallel 40 \parallel 0.25 = 242\ \Omega$$

$$R_{\text{out}} = R_{of} = \frac{R_o}{1 + A\beta}$$

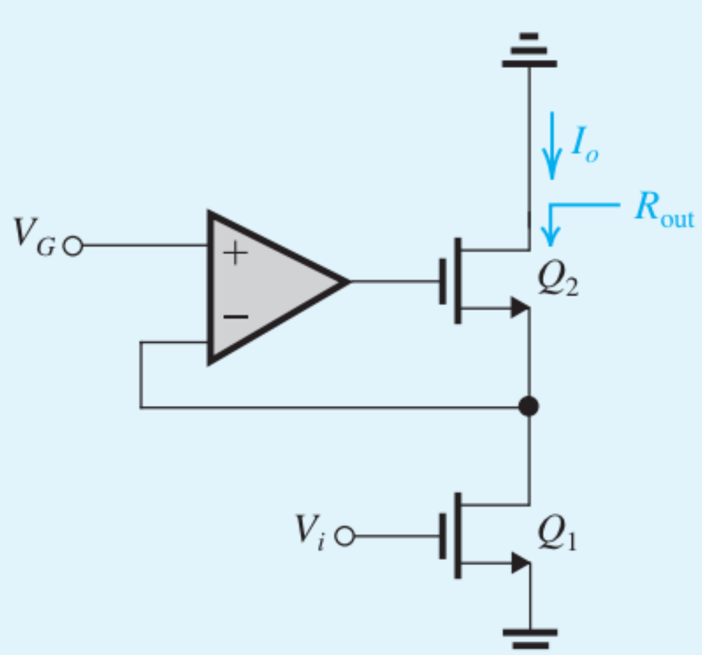
$$= \frac{242}{156.15} = 1.55\ \Omega$$

***11.78** The feedback current amplifier in Fig. P11.78(a) can be thought of as a "super" CG transistor. Note that rather than connecting the gate of $Q_2$ to signal ground, an amplifier is placed between source and gate.

(a) If $\mu$ is very large, what is the signal voltage at the input terminal? What is the input resistance? What is the current gain $I_o/I_s$?
(b) For finite $\mu$ but assuming that the input resistance of the amplifier $\mu$ is very large, find the $A$ circuit and derive expressions for $A$, $R_i$, and $R_o$.
(c) What is the value of $\beta$?
(d) Find $A\beta$ and $A_f$. If $\mu$ is large, what is the value of $A_f$?
(e) Find $R_{in}$ and $R_{out}$ assuming the loop gain is large.
(f) The "super" CG transistor can be utilized in the cascode configuration shown in Fig. P11.78(b), where $V_G$ is a dc bias voltage. Replacing $Q_1$ by its small-signal model, use the analogy of the resulting circuit to that in Fig. P11.78(a) to find $I_o$ and $R_{out}$.




(a)




(b)

**Figure P11.78**

**10.70** (a) If $\mu$ is a very large, a virtual ground will appear at the input terminal. Thus the input resistance $R_{in} = V_-/I_i = 0$. Since no current flows in $R_s$, or into the amplifier input terminal, all the current $I_s$ will flow in the transistor source terminal and hence into the drain, thus

$$I_o = I_s$$

and

$$\frac{I_o}{I_s} = 1$$

(b) This is a shunt-series feedback amplifier in which the feedback circuit consists of a wire, as shown on the next page in Fig. 1. As indicated,

$$R_{11} = \infty$$

$$R_{22} = 0$$

The $A$ circuit is shown in Fig. 2 (next page) for which we can write

$$V_{id} = -I_i(R_s \parallel R_{id})$$

$$\simeq -I_i R_s \tag{1}$$

(since $R_{id}$ is very large)

$$V_{gs} = \mu V_{id} \tag{2}$$

$$I_o = g_{m2} V_{gs} \tag{3}$$

Combining (1)–(3), we obtain

$$A \equiv \frac{I_o}{I_i} = -\mu g_{m2} R_s$$

These figures belong to Problem 10.70, part (b).

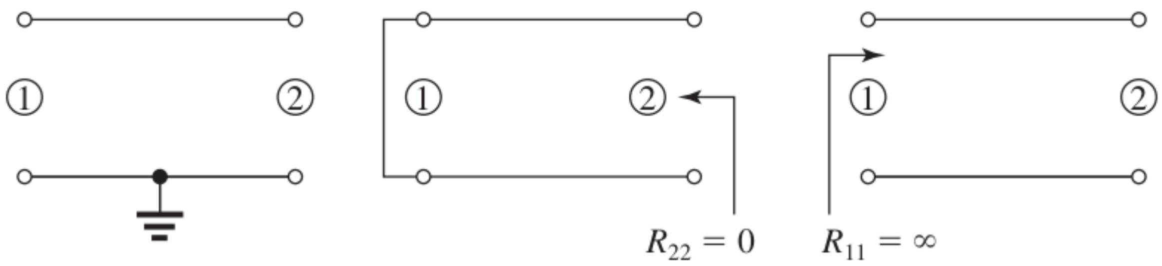



Figure 1

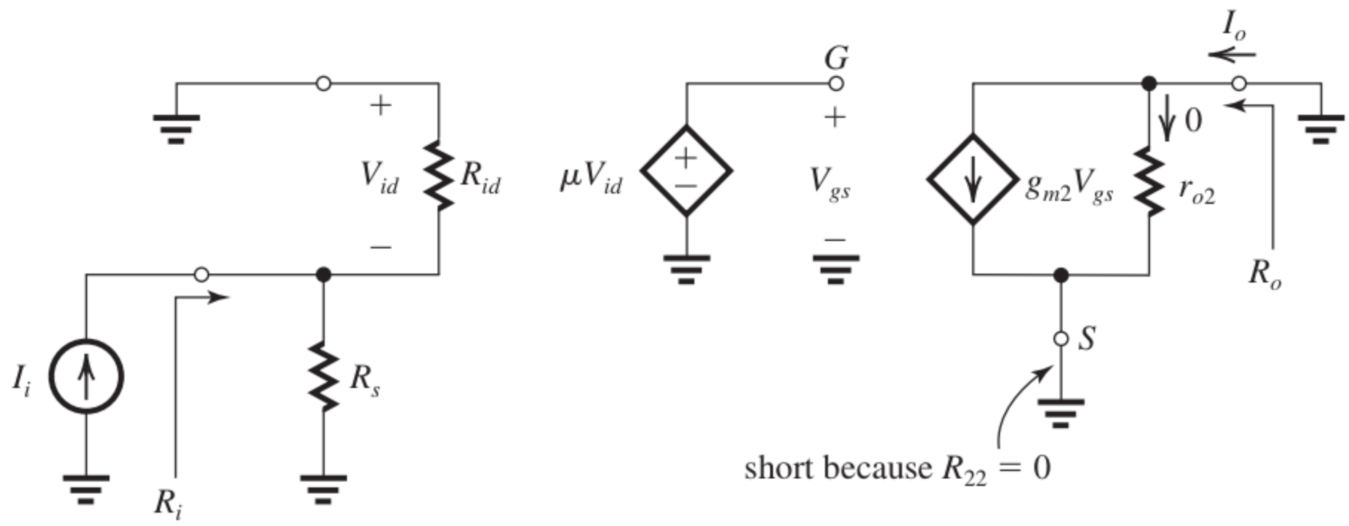



Figure 2

$R_i = R_s$

$R_o = r_{o2}$

(c)

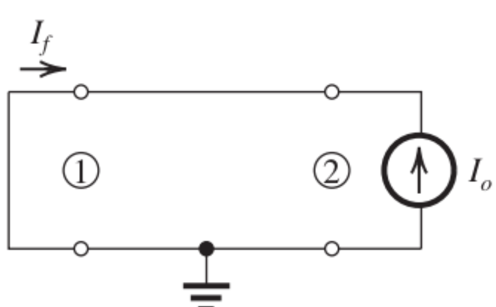



Figure 3

From Fig. 3 we find

$$\beta \equiv \frac{I_f}{I_o} = -1$$

(d) $A\beta = \mu g_{m2} R_s$

$$A_f = \frac{A}{1 + A\beta}$$

$$= -\frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

Note that the negative sign is due to our assumption that $I_s$ flows into the input node (see Fig. 2 for the way $I_i$ is applied). If instead $I_s$ is flowing out of the input node, as indicated in Fig. P10.70, then

$$A_f \equiv \frac{I_o}{I_s} = \frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s}$$

If $\mu$ is large so that $\mu g_{m2} R_s \gg 1$,

$$A_f \simeq 1$$

(e) $R_{if} = \dfrac{R_i}{1 + A\beta}$

$$= \frac{R_s}{\mu g_{m2} R_s}$$

$$R_{if} = R_{\text{in}} \parallel R_s$$

$$\frac{1}{R_{if}} = \frac{1}{R_{\text{in}}} + \frac{1}{R_s}$$

$$\frac{1}{R_s} + \mu g_{m2} = \frac{1}{R_{\text{in}}} + \frac{1}{R_s}$$

$$\Rightarrow R_{\text{in}} = \frac{1}{\mu g_{m2}}$$

$$R_{\text{out}} = R_{of} = R_o(1 + A\beta)$$

$$= r_{o2}(1 + \mu g_{m2} R_s)$$

(f)

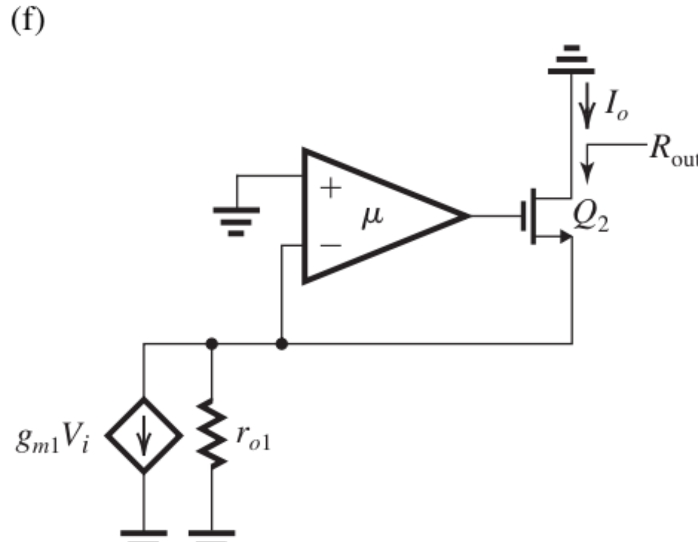



Figure 4

The circuit of the cascode amplifier in Fig. P10.70 with $V_G$ replaced by signal ground, and $Q_1$ replaced by its equivalent circuit at the drain (Fig. 4, previous page) looks identical to that of Fig. 10.70(a). Thus we can write

$$I_o \simeq g_{m1} v_i$$

$$R_{\text{out}} = r_{o2}(1 + \mu g_{m2} r_{o1}) = r_{o2} + \mu(g_{m2} r_{o2}) r_{o1}$$

$$\simeq \mu(g_{m2} r_{o2}) r_{o1}$$

Compared to the case of a regular cascode, we see that while $I_o \simeq g_{m1} V_i$ as in the regular cascode, utilizing the "super" CG transistor results in increasing the output resistance by the additional factor $\mu$!

**11.86** Consider a feedback amplifier for which the open-loop gain $A(s)$ is given by

$$A(s) = \frac{10{,}000}{(1 + s/10^4)(1 + s/10^5)^2}$$

If the feedback factor $\beta$ is independent of frequency, find the frequency at which the phase shift is 180°, and find the critical value of $\beta$ at which oscillation will commence.

**10.77** $A(s) = \dfrac{10^4}{\left(1 + \dfrac{s}{10^4}\right)\left(1 + \dfrac{s}{10^5}\right)^2}$

$$A(j\omega) = \frac{10^4}{\left(1 + j\dfrac{\omega}{10^4}\right)\left(1 + j\dfrac{\omega}{10^5}\right)^2}$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega}{10^5}\right)$$

$$-180 = -\tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) - 2\tan^{-1}\left(\frac{\omega_{180}}{10^5}\right)$$

By trial and error we find

$$\omega_{180} = 1.095 \times 10^5 \text{ rad/s}$$

At this frequency,

$$|A| = \frac{10^4}{\sqrt{1 + 10.95^2}\sqrt{1 + 1.095^2}}$$

$= 413.6$

For stable operation,

$|A|\beta_{cr} < 1$

$\beta_{cr} < 2.42 \times 10^{-3}$

Thus, oscillation will commence for

$\beta \geq 2.42 \times 10^{-3}$

***11.89** An amplifier having a low-frequency gain of $10^4$ and poles at $10^4$ Hz and $10^5$ Hz is operated in a closed negative-feedback loop with a frequency-independent $\beta$.

(a) For what value of $\beta$ do the closed-loop poles become coincident? At what frequency?
(b) What is the low-frequency, closed-loop gain corresponding to the situation in (a)? What is the value of the closed-loop gain at the frequency of the coincident poles?
(c) What is the value of $Q$ corresponding to the situation in (a)?
(d) If $\beta$ is increased by a factor of 10, what are the new pole locations? What is the corresponding pole $Q$?

**10.82** (a) The closed-loop poles become coincident when $Q = 0.5$. Using Eq. (10.70), we obtain

$$Q = \frac{\sqrt{(1+A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1}+\omega_{P2}}$$

$$0.5 = \frac{\sqrt{(1+A_0\beta)\omega_{P1}\omega_{P2}}}{\omega_{P1}+\omega_{P2}}$$

$$\Rightarrow 1 + A_0\beta = 0.5^2 \frac{(\omega_{P1}+\omega_{P2})^2}{\omega_{P1}\omega_{P2}}$$

$$= 0.5^2 \times \frac{(2\pi)^2(10^4+10^5)^2}{(2\pi)^2 \times 10^4 \times 10^5}$$

$$= 0.5^2 \times \frac{11^2}{10} = 3.025$$

$$\beta = 2.025 \times 10^{-4}$$

$$\omega_c = \frac{1}{2}(\omega_{P1}+\omega_{P2})$$

$$= \frac{1}{2} \times 2\pi(f_{P1}+f_{P2})$$

$$f_c = \frac{1}{2} \times (10^4+10^5) = 5.5 \times 10^4 \text{ Hz}$$

(b) $A_f(0) = \dfrac{A_0}{1+A_0\beta}$

$$= \frac{10^4}{1+2.205\times 10^{-4}\times 10^4} = 3306 \text{ V/V}$$

$$A_f(s) = \frac{A(s)}{1+A(s)\beta}$$

where

$$A(s) = \frac{A_0}{\left(1+\dfrac{s}{\omega_{P1}}\right)\left(1+\dfrac{s}{\omega_{P2}}\right)}$$

$$A_f(s) = \frac{A_0}{\left(1+\dfrac{s}{\omega_{P1}}\right)\left(1+\dfrac{s}{\omega_{P2}}\right)+A_0\beta}$$

$$= \frac{A_0}{(1+A_0\beta)+s\left(\dfrac{1}{\omega_{P1}}+\dfrac{1}{\omega_{P2}}\right)+\dfrac{s^2}{\omega_{P1}\omega_{P2}}}$$

$$A_f(j\omega) = \frac{A_0}{(1+A_0\beta)+j\left(\dfrac{\omega}{\omega_{P1}}+\dfrac{\omega}{\omega_{P2}}\right)-\left(\dfrac{\omega}{\omega_{P1}}\right)\left(\dfrac{\omega}{\omega_{P2}}\right)}$$

$$A_f(j\omega_c) = \frac{10^4}{3.025+j(5.5+0.55)-5.5\times 0.55}$$

$$A_f(j\omega_c) = \frac{10^4}{j\,6.05}$$

$$|A_f|(j\omega_c) = \frac{10^4}{6.05} = 1653 \text{ V/V}$$

(c) $Q = 0.5$.

(d) If $\beta = 2.025 \times 10^{-3}$ V/V. Using Eq. (10.68), we obtain

$$s = -\frac{1}{2} \times 2\pi(10^4+10^5)$$

$$\pm\frac{1}{2} \times 2\pi\sqrt{(10^4+10^5)^2 - 4(1+10^4\times 2.025\times 10^{-3})\times 10^4\times 10^5}$$

$$\frac{s}{2\pi} = -5.5\times 10^4$$

$$\pm 0.5\sqrt{121\times 10^8 - 4\times 21.25\times 10^9}$$

$$= -5.5\times 10^4 \pm 0.5\times 10^4\sqrt{121-40\times 21.25}$$

$$= -5.5\times 10^4 \pm j0.5\times 10^4\times 27$$

$$= (-5.5 \pm j13.25)\times 10^4 \text{ Hz}$$

Using Eq. (10.70), we obtain

$$Q = \frac{\sqrt{(1+10^4\times 2.025\times 10^{-3})10^4\times 10^5}}{10^4+10^5}$$

$$= 1.325$$

SIM **11.106 The op amp in the circuit of Fig. P11.106 has an open-loop gain of $10^5$ and a single-pole rolloff with $\omega_{3dB} = 10$ rad/s.

(a) Sketch a Bode plot for the loop gain.

(b) Find the frequency at which $|A\beta| = 1$, and find the corresponding phase margin.

(c) Find the closed-loop transfer function, including its zero and poles. Sketch a pole-zero plot. Sketch the magnitude of the transfer function versus frequency, and label the important parameters on your sketch.

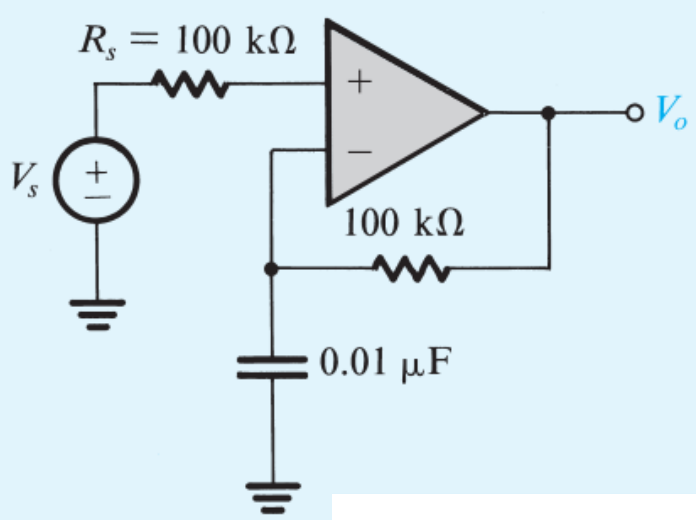



Figure P11.106

$$A(s)\beta(s) = -\frac{V_r}{V_t}$$

$$= A(s)\frac{1/sC}{R + 1/sC}$$

$$A(s)\beta(s) = \frac{10^5}{1+\frac{s}{10}}\;\frac{1}{1+sCR}$$

$$CR = 0.01 \times 10^{-6} \times 100 \times 10^3 = 10^{-3}\text{ s}$$

$$A(s)\beta(s) = \frac{10^5}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^3}\right)}$$

(a) Bode plots for the magnitude and phase of $A\beta$ are shown in Fig. 2. From the magnitude plot we find the frequency $f_1$ at which $|A\beta| = 1$ is

$f_1 = 3.16 \times 10^4$ Hz

(b) From the phase plot we see that the phase at $f_1$ is 180° and thus the phase margin is zero. A more exact value for the phase margin can be obtained as follows:

**10.96**

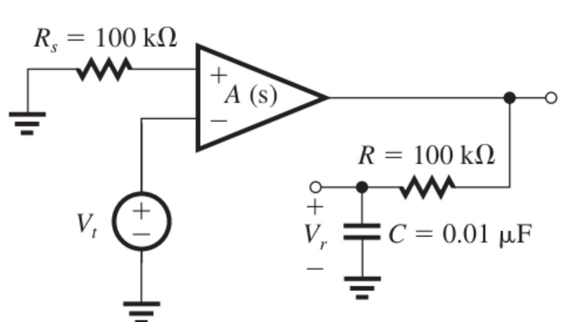



Figure 1

This figure belongs to Problem 10.96, part (a).

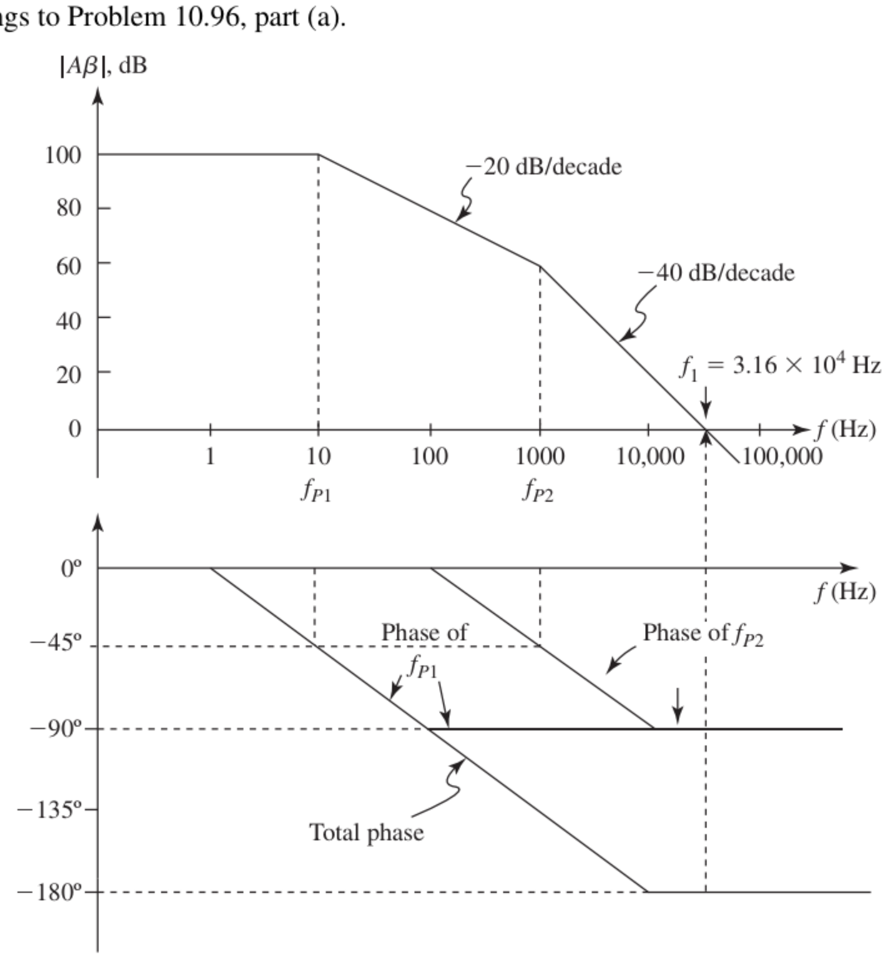



Figure 2

$$\theta(f_1) = -\tan^{-1}\frac{3.16\times 10^4}{10} - \tan^{-1}\frac{3.16\times 10^4}{10^3}$$

$$= -89.98 - 88.19 = -178.2$$

Thus the phase margin is 1.8°.

(c) $$A_f(s) = \frac{A(s)}{1+A(s)\beta(s)}$$

$$= \frac{10^5\Big/\left(1+\frac{s}{10}\right)}{1+\frac{10^5}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^3}\right)}}$$

$$= \frac{10^5\left(1+\frac{s}{10^3}\right)}{10^5+\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^3}\right)}$$

$$= \frac{\left(1+\frac{s}{10^3}\right)}{1+10^{-5}(1+0.101s+0.0001s^2)}$$

At $s = 0$,

$$A_f \simeq 1$$

The transmission zero is

$$s_Z = -10^3 \text{ rad/s}$$

The poles are the roots of

$$10^{-9}s^2 + 1.01\times 10^{-6}s + 1 = 0$$

which are

$$s = (-0.505 \pm j31.62)\times 10^3 \text{ rad/s}$$

The poles and zero are shown in Fig. 3.

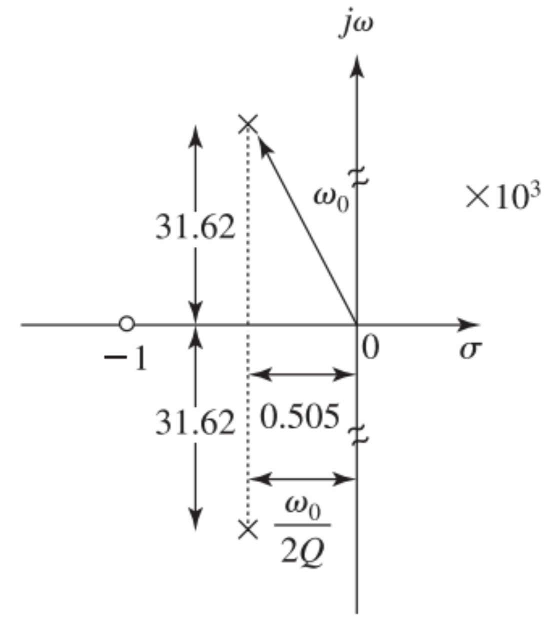



Figure 3

The pair of complex-conjugate poles have

$\omega_0 \simeq 31.62$ krad/s

$Q = 31.3$

Thus, the response is very peaky, as shown in Fig. 4.

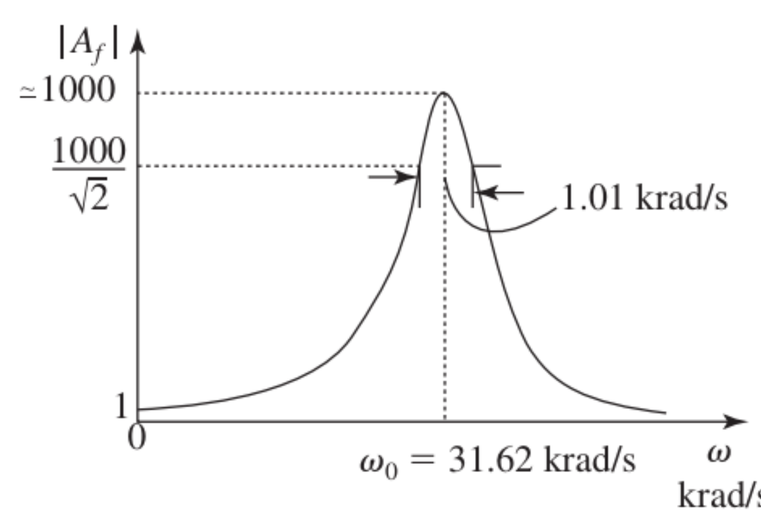



Figure 4