JPEG Example

- 8×8 input image matrix (from LENA image)

$$s_{x,y} = \begin{bmatrix} 139 & 144 & 149 & 153 & 155 & 155 & 155 & 155 \\ 144 & 151 & 153 & 156 & 159 & 156 & 156 & 156 \\ 150 & 155 & 160 & 163 & 158 & 156 & 156 & 156 \\ 159 & 161 & 162 & 160 & 160 & 159 & 159 & 159 \\ 159 & 160 & 161 & 162 & 162 & 155 & 155 & 155 \\ 161 & 161 & 161 & 161 & 160 & 157 & 157 & 157 \\ 162 & 162 & 161 & 163 & 162 & 157 & 157 & 157 \\ 162 & 162 & 161 & 161 & 163 & 158 & 158 & 158 \end{bmatrix}$$

- DCT transformation $\rightarrow$ 64 coefficients

$$S_{u,v} = \begin{bmatrix} 1260 & -1 & -12 & -5 & 2 & -2 & -3 & 1 \\ -23 & -17 & -6 & -3 & -3 & 0 & 0 & -1 \\ -11 & -9 & -2 & 2 & 0 & -1 & -1 & 0 \\ -7 & -2 & 0 & 1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 2 & 0 & -1 & 1 & 1 \\ 2 & 0 & 2 & 0 & -1 & 1 & 1 & -1 \\ -1 & 0 & 0 & -1 & 0 & 2 & 1 & -1 \\ -3 & 2 & -4 & -2 & 2 & 1 & -1 & 0 \end{bmatrix}$$

- a typical quantization matrix for JPEG

$$Q_{u,v} = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 113 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

- quantization $\rightarrow$

$$F_{u,v} = \begin{bmatrix} 79 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- zig-zag scan

  79 0 -2 -1 -1 -1 0 0 -1 EOB

- entropy coding
  - Huffman code table

Zero Run	Category	Code Length	Codeword
0	1	2	00
0	2	2	01
0	3	3	100
0	4	4	1011
0	5	5	11010
...	...	...	...
1	1	4	1100
1	2	6	111001
1	3	7	1111001
...	...	...	...
2	1	5	11011
2	2	8	11111000
...	...	...	...
3	1	6	111010
...	...	...	...
4	1	6	111011
...	...	...	...
EOB		4	1010

- AC coefficient grouping, range for group i is $\pm(2^{i-1}..2^i - 1)$

category	AC Coefficient Range
1	-1, 1
2	-3,-2, 2,3
3	-7..-4, 4..7
4	-15..-8, 8..15
...	.....

- codeword (exclude DC-coefficient):

```
11100101   000   000   000   110110   1010
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1-zero run, (-2) = category 2 -> (01)

0-zerorun (-1)=category 1 -> 0

2-zero run, (-1) = category 1 -> (0)

  - each code is extended with one sign bit and possibly one magitute bit indicating the rank within its group
  - 0: negative, 1: positiive
  - no extension: smallest in the group, 1: second smallest, ...

- both of the tables should be used for the entire picture

- reverse the process and reconstruct the image

$$S'_{x,y} = \begin{bmatrix} 144 & 146 & 149 & 152 & 154 & 156 & 156 & 156 \\ 148 & 150 & 152 & 154 & 156 & 156 & 156 & 156 \\ 155 & 156 & 157 & 158 & 158 & 157 & 156 & 155 \\ 160 & 161 & 161 & 162 & 161 & 159 & 157 & 155 \\ 163 & 163 & 164 & 163 & 162 & 160 & 158 & 156 \\ 163 & 163 & 164 & 164 & 162 & 160 & 158 & 157 \\ 160 & 161 & 162 & 162 & 162 & 161 & 159 & 158 \\ 158 & 159 & 161 & 161 & 162 & 161 & 159 & 158 \end{bmatrix}$$

- error matrix $E_{x,y} = S_{x,y} - S'_{x,y}$

$$E_{x,y} = \begin{bmatrix} -5 & -2 & 0 & 1 & 1 & -1 & -1 & -1 \\ -4 & 1 & 1 & 2 & 3 & 0 & 0 & 0 \\ -5 & -1 & 3 & 5 & 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & -2 & -1 & 0 & 2 & 4 \\ -4 & -3 & -3 & -1 & 0 & -5 & -3 & -1 \\ -2 & -2 & -3 & -3 & -2 & -3 & -1 & 0 \\ 2 & 1 & -1 & 1 & 0 & -4 & -2 & -1 \\ 4 & 3 & 0 & 0 & 1 & -3 & -1 & 0 \end{bmatrix}$$

- RMSE (root mean squared error)

$$\sqrt{\frac{1}{64}\sum_{x=0}^{8}\sum_{y=0}^{8}(S_{x,y}-S'_{x,y})^2}$$

- PSNR

$$20\log_{10}\left(\frac{255}{RMSE}\right)$$

  - typically range between 20dB and 40dB
  - it is believed that an improvement of 0.5dB magtitute can be visible