A Practical Algorithm for Topic Modeling with Provable Guarantees

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Information Overload

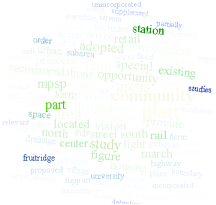
Effective Organisation

Topics


transportation
parking
mobility
system
travel
safe
network
movement
trips
mode
plan
trucks
priority
people
demand
impacts
safety
management
goods
regional
reduce
vehicle
destinations
accessibility
programs
improve
convenient
walking

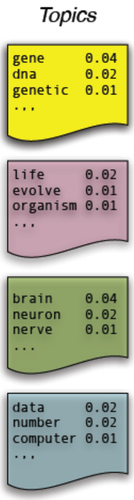

workforce
innovation
education
economy
research
technology
training
systems
clusters
data
work
important
industry
skills
metropolitan
entrepreneurs
industries
improving
institutions
logistics


commercial
industrial
residential
retail
mixed
areas
activity
district
existing
office
surrounding
environmental
concentration
designated
structures
large
area
districts
accommodate
oriented
businesses
adjacent


station
part
mpsp
existing
community
plan
opportunity
special
retail
adopted
study
rail
center
north
fruitridge
studies

1. Introduction
2. **Topic Modelling**
3. Algorithm
4. Efficiently Finding Anchor Words
5. Topic Recovery via Bayes' Rule
6. Experimental Results
7. Conclusion

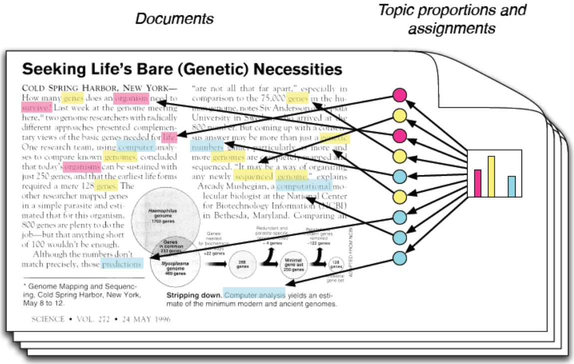
Topics



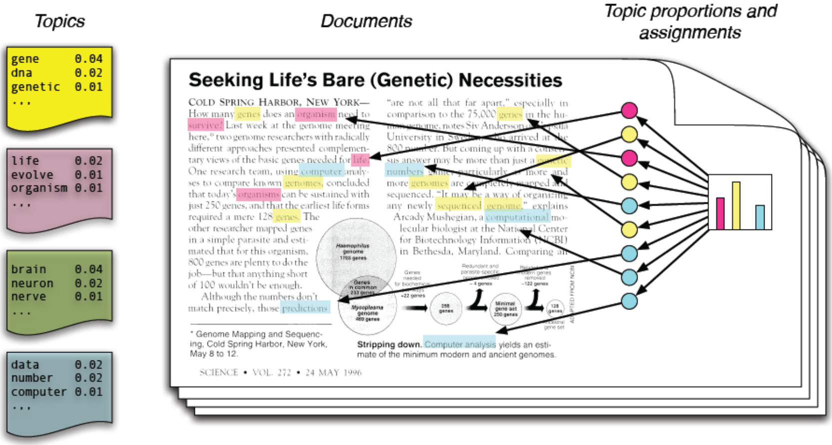
Topics are distributions over words

Documents

Documents have distribution of topics

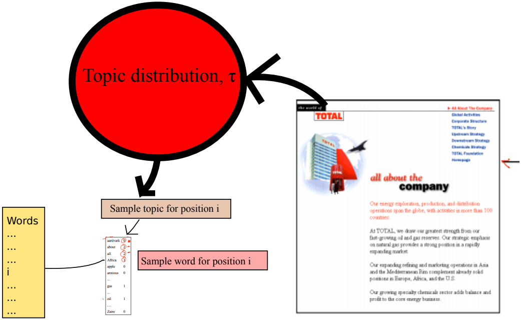


Topic Modelling



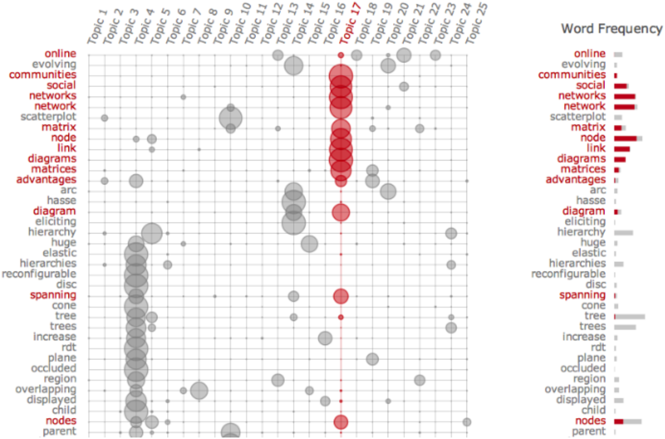
Task

- Assume documents are generated by probabilistic model with unknown variables
- Infer hidden structure onto document
- Situate new document into model



Word-topic Matrix

Extracted: Word-topic matrix



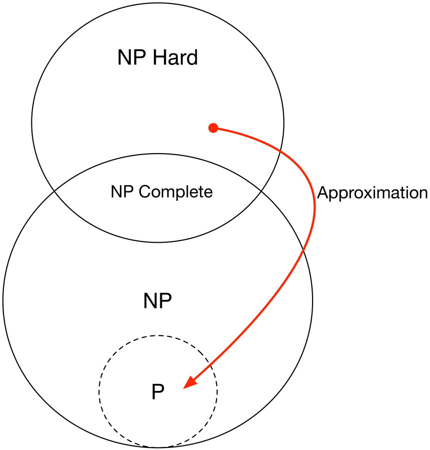
Aim: Find document-topic matrix

Anchor Words

- Word-topic distributions are separable
- There is a word unique to each topic
- Indicates document is partially about that topic
- Can learn parameters in polynomial time provided there is a large enough number of documents

Approximate Inference & Provable Guarantees

- Document-topic inference:
    - NP-hard
- Approximate techniques
- Provably polynomial-time?



Algorithm

Input: Corpus $\mathcal{D}$, Number of topics K

Output: Word-topic matrix A, topic-topic matrix R

1. Compute word-word co-occurrence matrix
2. Normalize the matrix
3. Find anchor words
4. Recover topics

Assumptions:

- Topics may be correlated
- Word-topic distributions are separable

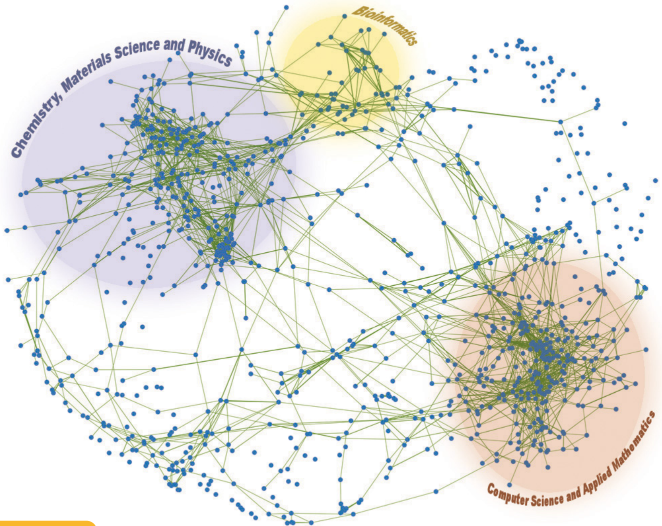
Contributions

1. Anchor Selection
   - Combinatorial rather than ILP
   - Stable in the presence of noise
   - polynomial sample complexity
2. Recovery step
   - Previous matrix-inversion approach sensitive to noise
   - Replaced with Gradient-based inference
3. Empirical comparison of algorithms

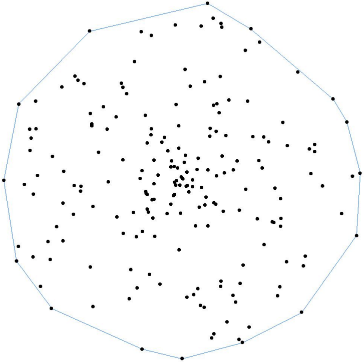
Word-word co-occurrence matrix

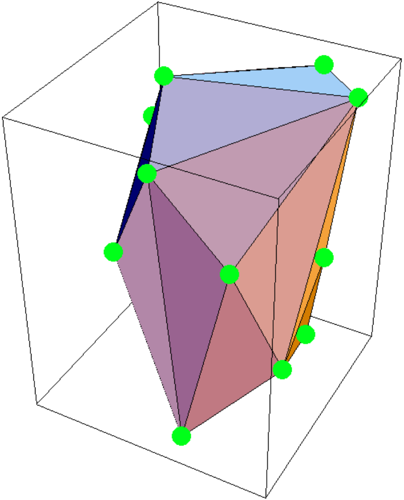
	bank	California	Canada	career	careers	employers	employment	federal	human	job	jobs	listings	openings	opportunities	positions	recruiters	resources	resume	resumes	retirement	search	state	texas	unemployment	work
bank	--																								
California		--																							
Canada	1		--																						
career	3	3		--																					
careers			2	9	--																				
employers		2		11	7	--																			
employment	3	26	22	66	10	16	--																		
federal	1	1	5		1		11	--																	
human		4	12	1	1			4	--																
job	34	14	2	49	8	13	92	13	2	--															
jobs		18	6	62	11	27	204	19	2	74	--														
listings		4	2	15	4	9	68	2	55	44		--													
openings		4		7	2	9	28			49	30		--												
opportunities	4	8	3	51	9	13	181	9		84	106	25	19	--											
positions		1		8	2	10	19			16	20	9	13	21	--										
recruiters				10	4	3	9			5	4	2	2	5	2	--									
resources		4	12		1			4	74	3	2						--								
resume		4	3	5		2	3	1	1	10	3			1	2	1		--							
resumes				8	3	3	11			5	16	1		8	5			15	--						
retirement		1	1						3			2		1						--					
search			3	4	6			10			18	6		6	2			3	1		--				
state			4	1			18		1	12	7	6		3			1			2		--			
texas	2			1			18			12	6		1	2						9			--		
unemployment																						2	2	--	
work			2	1		3	3	2		2	8	2	4	7	5				1			1	2		--

Words as vertices



Convex Hull





Computing Convex Hull

- Efficient for 2 dimensions - O($n \log n$)
- Inefficient for $n > 2$ dimensions
- Complexity depends on method and approximation used
- Previous method: ILP
- New method: Recursive greedy
  1. Compute subspace span of current convex hull
  2. Find point furthest from this sub-span
  3. Add point to convex hull
  4. Repeat until K points found

Topic Recovery Task

- Recovers the topics
- Represented as topic-word distributions
- Topic uniquely identified by anchor word

Previous method

1. Discard rows not containing anchor words from word-word co-occurrence matrix (Q)
2. Permute matrix into a diagonal matrix
3. We know that $Q = ARA^T$ where A is the word-topic matrix and R is the topic-topic matrix
4. Solve $Q = ARA^T$ using matrix inversion

Word-word co-occurrence probability matrix

For an anchor word (row) in the co-ocurrence matrix

$$Q_{s_k,j} = \sum_{k'} p(z_1 = k' | w_1 = s_k) p(w_2 = j | z_1 = k')$$

= 1 because of the anchor word property

$$= p(w_2 = j | z_1 = k) = C_{i,k}$$

For any other row

$$\bar{Q}_{i,j} = \sum_{k} p(z_1 = k | w_1 = i) p(w_2 = j | z_1 = k)$$

But this is clearly a convex combination of anchor words

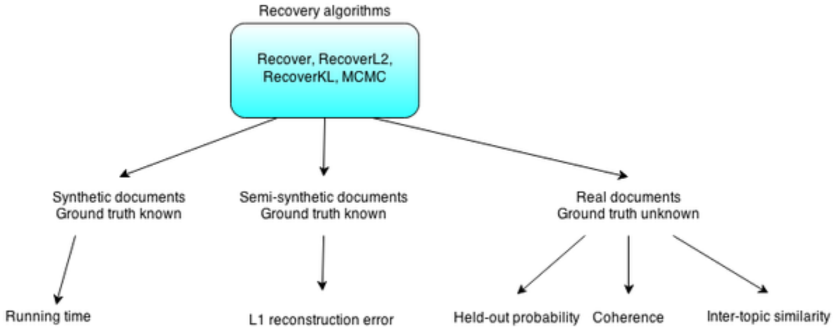
$$\bar{Q}_{i,j} = \sum_{k} C_{i,k} \bar{Q}_{s,k}$$

New method

- Row normalize Q into $\bar{Q}$
- Recover A and R using Bayes rule

$$p(w_1 = i | z_1 = k) = \frac{p(z_1 = k | w_1 = i) p(w_1 = i)}{\sum_i p(z_1 = k | w_1 = i') p(w_1 = i')}$$

Experiments



Metrics

- Ground truth known
  - Reconstruction error
- Ground truth unknown
  - Held out probability - probability of an unseen document under the learned model
  - Coherence - measure of semantic quality
  - Inter-topic similarity - measure of uniqueness of topics

Results - Efficiency

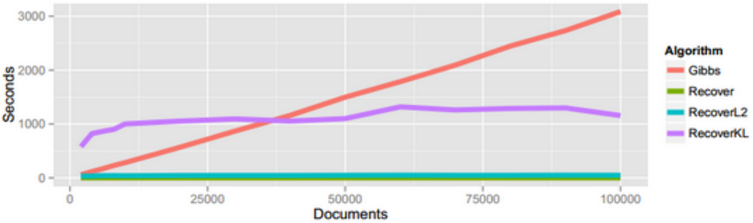


Figure 1. Training time on synthetic NIPS documents.

Results - Reconstruction error

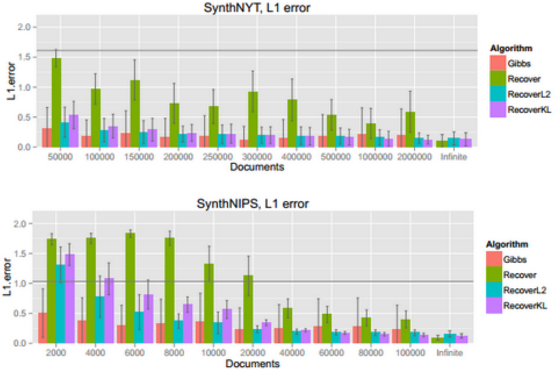


Figure 2. ℓ_1 error for learning semi-synthetic LDA models with $K = 100$ topics (**top**: based on NY Times, **bottom**: based on NIPS abstracts). The horizontal lines indicate the ℓ_1 error of K uniform distributions.

Results - Held out probability, coherence and unique words

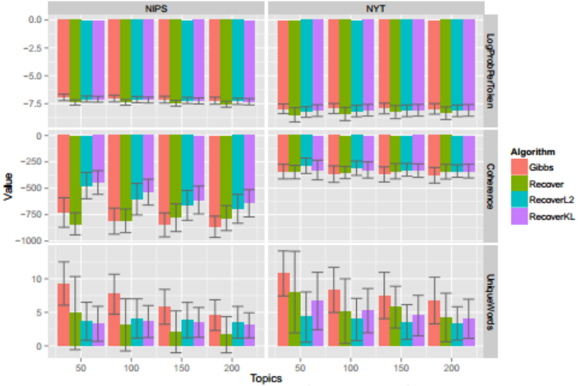


Figure 5. Held-out probability (per token) is similar for RecoverKL, RecoverL2, and Gibbs sampling. RecoverKL and RecoverL2 have better coherence, but fewer unique words than Gibbs. (Up is better for all three metrics.)

Summary

- New algorithms for topic recovery
  - Empirical results comparable to MCMC
  - Anchor word assumption
  - Bayes' rule $\rightarrow$ empirical performance improvements
- Simple to implement, maintains provable guarantees
- Attractive feature: *Running time independent of corpus size*

Future Work

- Use output of algorithms for further optimization
- Parallel implementations

Comments

- Theory is consistent with results: large performance improvements
- No obvious inconsistencies
- Incremental contrbution

Thanks!

Any questions please email either of us:

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