

# Assignment - 3

## Distance measures

### Question 1

$$u = [1, 0.25, 0, 0, 0.5, 0]$$

$$v = [0.75, 0, 0, 0.2, 0.4, 0]$$

$$w = [0, 0.1, 0.75, 0, 0, 1]$$

Cosine similarity for all pairs

$$\cos(x, y) = \frac{x \cdot y}{\sqrt{x^2} \sqrt{y^2}}$$

$$\begin{aligned} a) \cos(u, v) &= \frac{1(0.75) + 0 + 0 + (0.5)(0.4) + 0}{\sqrt{(0.75)^2 + (0.2)^2 + (0.4)^2} \sqrt{1 + (0.25)^2 + (0.5)^2}} \\ &= \frac{0.95}{0.9918} = 0.95 \end{aligned}$$

$$\cos(u, v) = 0.95 \approx 18^\circ //$$

$$b) \cos(v, w) = \frac{0 + 0 + 0 + 0 + 0}{\sqrt{(0.75)^2 + (0.2)^2 + (0.4)^2} \sqrt{(0.1)^2 + (0.75)^2 + 1}}$$

$$\cos(v, w) = 0 = 90^\circ //$$

$$\begin{aligned} c) \cos(u, w) &= \frac{0 + (0.25)(0.1) + 0 + 0 + 0 + 0}{\sqrt{1 + (0.25)^2 + (0.5)^2} \sqrt{(0.1)^2 + (0.75)^2 + 1}} \\ &= \frac{0.025}{1.425} = 0.0175 \end{aligned}$$

$$\cos(u, w) \approx 89^\circ //$$

Question - 2

Question - 2

a =  $\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

b =  $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

c =  $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

d =  $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

e =  $\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

$$JS = \frac{f_{11}}{f_{11} + f_{10} + f_{01}}$$

$$JD = 1 - JS$$

$f_{11} + f_{10} + f_{01}$   
 $i) JS(a, b) = \frac{1}{7} \Rightarrow JD = 1 - JS = 1 - \frac{1}{7} = \frac{6}{7}$   
 $JD = 1 - 0 = 1$

ii)  $JL(a, c) = 0/8 \Rightarrow JD = 1 - 0 = 1$

ii)  $JX(a, c) = 0/8 \Rightarrow JD = 1$   
 iii)  $JY(a, d) = \frac{3}{10} \Rightarrow JD = 1 - 3/10 = 7/10$

iii)  $JS(a, d) = \frac{3}{10} \Rightarrow JD = 1 - 3/10 = 7/10$   
 iv)  $JS(a, e) = 3/10 \Rightarrow 1 - 1/10 = 9/10$

$\gamma_v) JS(a, e) = 3/10 \Rightarrow JD = 1 - 1/7 = 6/7$   
 $v) JS(b, c) = 1/7 \Rightarrow JD = 1 - 4/9 = 5/9$

v)  $J^S(b, c) = 1/7 \Rightarrow JD = 1 - 4/9 = 5/9$   
 vi)  $J^S(b, d) = 4/9 \Rightarrow JD = 1 - 3/10 = 7/10$

vii)  $JS(b, e) = 3/10 \Rightarrow JD = 1 - 3/10 = 7/10$

$$JS(c,d) = 4/9 \Rightarrow JD = 1 - 4/9 = 5/9$$

ix)  $JS(c,e) = 4/q \Rightarrow JD = 1 - 4/q = 5/q$

7) JS (d,e) :  $8/10 \Rightarrow JD = 1 - 8/10 = 2/10$

Question - 3

Manhattan distance ( $L_1$  norm)

a : 1 1 1 1 0 0 0 0 0 0

b : 0 1 0 0 1 0 0 1 0 1

c : 0 0 0 0 0 1 1 1 1 0

d : 0 1 1 1 1 1 1 1 1 1

e : 1 0 1 1 1 1 1 1 1 1

$$\text{Manhattan distance} = d(x, y) = \sum_{i=1}^n |x_i - y_i|$$

$$\begin{aligned} d(a, b) &= |1-0| + |1-1| + |1-0| + |1-0| + |0-1| + |0-0| + |0-0| + |0-1| + |0-0| + |0-1| \\ &= 6 \end{aligned}$$

$$\begin{aligned} d(a, c) &= |1-0| + |1-0| + |1-0| + |1-0| + |0-0| + |0-1| + |0-1| + |0-1| + |0-1| + |0-0| \\ &= 8 \end{aligned}$$

$$\begin{aligned} d(a, e) &= |1-1| + |1-0| + |1-1| + |1-1| + |0-1| + |0-1| + |0-1| + |0-1| + |0-1| + |0-1| \\ &= 7 \end{aligned}$$

$$\begin{aligned} d(b, c) &= |0-0| + |1-1| + |0-1| + |0-1| + |1-1| + |0-1| + |0-1| + |1-1| + |0-1| + |1-0| \\ &= 6 \end{aligned}$$

$$\begin{aligned} d(b, d) &= |0-0| + |1-1| + |0-1| + |0-1| + |1-1| + |0-1| + |0-1| + |1-1| + |0-1| + |1-1| \\ &= 5 \end{aligned}$$



$$d(b,e) = |0-1| + |1-0| + |0-1| + |0-1| + |1-1| + |0-1| + |0-1| \\ + |1-1| + |0-1| + |1-1| \\ = 7$$

$$d(c,d) = |0-0| + |0-1| + |0-1| + |0-1| + |0-1| + |1-1| + \\ + |1-1| + |1-1| + |1-1| + |0-1| \\ = 5$$

$$d(a,d) = |1-0| + |1-1| + |1-1| + |1-1| + |0-1| + \\ |0-1| + |0-1| + |0-1| + |0-1| + |0-1| \\ = 7$$

$$d(c,e) = |0-1| + |0-0| + |0-1| + |0-1| + |0-1| + |1-1| + |1-1| \\ + |1-1| + |1-1| + |0-1| \\ = 5$$

$$d(d,e) = |0-1| + |1-0| + |1-1| + |1-1| + |1-1| + |1-1| + |1-1| \\ + |1-1| + |1-1| + |1-1| \\ = 7$$