Assignment - 1

Problem - 1

(P, i) → prime divisor, → the dividend

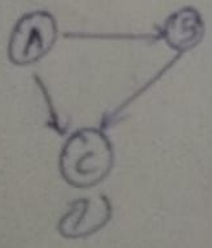
Input set : 15, 21, 24, 30, 49

map(15) : [(3,15), (5,15)]
map(21) : [(3,21), (7,21)]
map(24) : [(2,24), (3,24)]
map(30) : [(2,30), (3,30), (5,30)]
map(49) : [(7,49)]

reduce : (P, i_1, i_2, ...) :

reduce : (2, 24+30) = (2,54)
reduce : (3, 15+21+24+30) : (3,90)
reduce : (5, 15+30) = (5,45)
reduce : (7, 49+21) = (7,70)

Problem 2:



β : 0.7
nodes : 3

The sum of page ranks of A, B, C should be 3

Calculate transition matrix

$$M = \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 1 \end{bmatrix}$$

$A = \beta M + \left(\frac{1-\beta}{N}\right) \times e$

e → Matrix where is filled by 1s
shape of transition Matrix

$$= 0.7\begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 1 \end{bmatrix} + \frac{1-0.7}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.7/2 & 0 & 0 \\ 0.7/2 & 0.7 & 0.7 \end{bmatrix} + 0.1\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.45 & 0.1 & 0.1 \\ 0.45 & 0.8 & 0.8 \end{bmatrix}$$

Ranks $= \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$, we can calculate like this and finally multiply by 3

(or) we can use rank $= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Sum of ranks should be 3.

$$r = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

We need to iterate till we get repeated values.

Iteration 1:

$$r = Ar \Rightarrow \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.45 & 0.1 & 0.1 \\ 0.45 & 0.8 & 0.8 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$r = \begin{bmatrix} 0.099 \\ 0.2145 \\ 0.6765 \end{bmatrix}$$

Iteration 2:

$$r = Ar \Rightarrow \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.45 & 0.1 & 0.1 \\ 0.45 & 0.8 & 0.8 \end{bmatrix} \begin{bmatrix} 0.099 \\ 0.2145 \\ 0.6765 \end{bmatrix}$$

$$= \begin{bmatrix} 0.099 \\ 0.13365 \\ 0.7573 \end{bmatrix}$$

Iteration 3

$$r = Ar = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.45 & 0.1 & 0.1 \\ 0.45 & 0.8 & 0.8 \end{bmatrix} \begin{bmatrix} 0.099 \\ 0.13365 \\ 0.7573 \end{bmatrix}$$

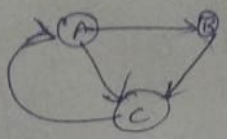
$$= \begin{bmatrix} 0.1 \\ 0.135 \\ 0.76 \end{bmatrix}$$

Multiply by 3 $= \begin{bmatrix} 0.3 \\ 0.405 \\ 2.295 \end{bmatrix}$

Page rank $\Rightarrow$ A = 0.3, B = 0.405, C = 2.29

Problem 3:

$\beta = 0.85$



Write equations for a, b, c

Transition matrix $M = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix}$ $N = 3$

$$A = \beta M + (1-\beta)\left(\frac{1}{N} e\right)$$

$$= 0.85 \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix} + \frac{(1-0.85)}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.05 & 0.05 & 0.9 \\ 0.475 & 0.05 & 0.05 \\ 0.475 & 0.9 & 0.05 \end{bmatrix}$$

$$a = 0.05a + 0.05b + 0.9c$$

$$a - 0.05a = 0.05b + 0.9c$$

$$0.95a = 0.05b + 0.9c$$

$$b = 0.475a + 0.05b + 0.05c$$

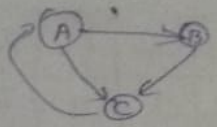
$$b - 0.05b = 0.475a + 0.05c$$

$$0.95b = 0.475a + 0.05c$$

$$c = 0.475a + 0.9b + 0.05c$$

$$0.95c = 0.475a + 0.9b$$

Problem 4:



Limit ⇒ when multiplying the distribution by M another time does not change the distribution.

Transition matrix: $M \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix}$

Iteration 1:

$r = Mr \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 1.5 \end{bmatrix}$

Iteration 2:

$r = Mr \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.5 \\ 1 \end{bmatrix}$

Iteration 3:

$r = Mr \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1.5 \\ 0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.75 \\ 1.25 \end{bmatrix}$

Iteration 4:

$r = Mr \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0.75 \\ 1.25 \end{bmatrix} = \begin{bmatrix} 1.25 \\ 0.5 \\ 1.25 \end{bmatrix}$

Iteration 5:

$r = Mr \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1.25 \\ 0.5 \\ 1.25 \end{bmatrix} = \begin{bmatrix} 1.25 \\ 0.625 \\ 1.125 \end{bmatrix}$

The limit is reached in Iteration ⇒ 5th

and the pageranks are $a = 1.2$, $b = 0.6$, $c = 1.2$ //