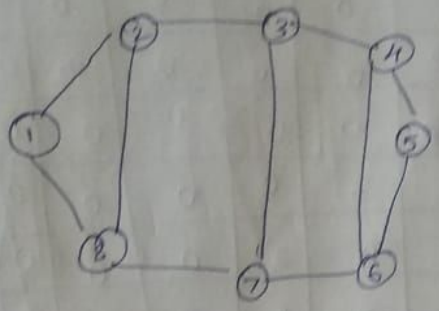
Assignment - 5

Question 1:




Find

Adjacency matrix, A

Degree matrix, D

Laplacian matrix, L

A =

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 2 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 3 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 6 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 7 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 8 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |

Sum of all entries = 22

No. of non zero entries A = 22

b) Degree Matrix

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |

Sum of all entries in D = 22

No. of non-zero entries in D = 8

c) Laplacian Matrix: $L = D - A$

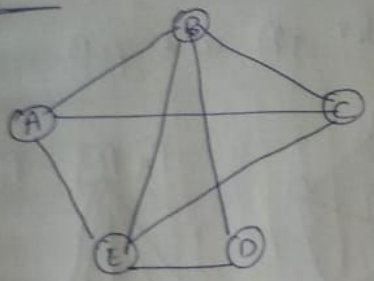
In laplacian matrix each row & column should be 0.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | -1 | 0 | 0 | 0 | 0 | 0 | -1 |
| 2 | -1 | 3 | -1 | 0 | 0 | 0 | 0 | -1 |
| 3 | 0 | -1 | 3 | -1 | 0 | 0 | -1 | 0 |
| 4 | 0 | 0 | -1 | 3 | -1 | -1 | 0 | 0 |
| 5 | 0 | 0 | 0 | -1 | 2 | -1 | 0 | 0 |
| 6 | 0 | 0 | 0 | -1 | -1 | 3 | -1 | 0 |
| 7 | 0 | 0 | -1 | 0 | 0 | -1 | 3 | -1 |
| 8 | -1 | -1 | 0 | 0 | 0 | 0 | -1 | 3 |

Sum of all entries = 0

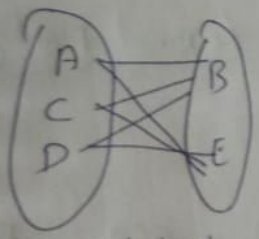
No of non zero entries of L = 30

Question - 2



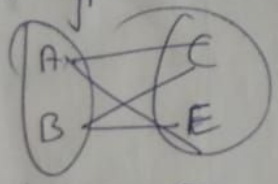

A → B, C, E
B → A, C, D, E
C → A, B, E
D → B, E
E → A, B, C, D

Bipartite subgraph of $K_{3,2}$ graph → {{A, C, D}, {B, E}}

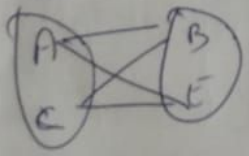



Bipartite subgraph of $K_{2,2}$ type are

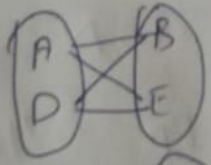
i) {{A, B}, {C, E}}



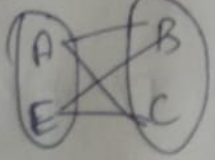

ii) {{A, C}, {B, E}}



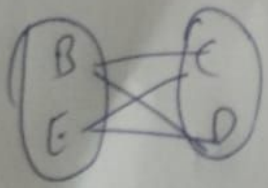

iii) {{A, D}, {B, E}}




iv) {{A, E}, {B, C}}

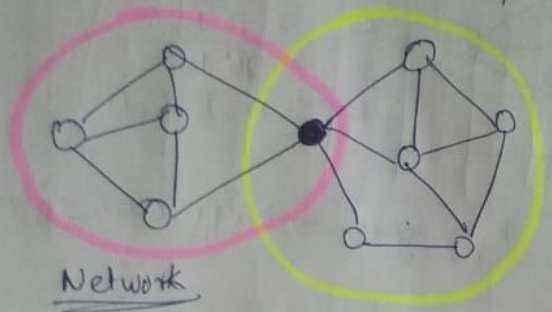



v) {{B, E}, {C, D}}

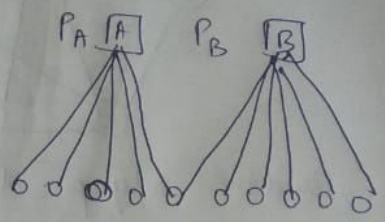


Question - 3

Find optimal values for $p_A$ and $p_B$



Network




Learned MSM parameters

For A, there are 5 nodes

Total no of possible edges for A = $5C_2$

$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2!\,(5-2)!}$

$= \frac{5 \times 4 \times 3 \times 2}{2 \times 3!} = \frac{5 \times 4 \times 3!\,/2}{2 \times 3!}$

$= 10$

No. of edges in A : 7

So, $P_A = 7/10 = 0.7$

Total no. of possible edges for B : $6C_2$

$= \frac{6 \times 5 \times 4 \times 3 \times 2}{2! \times 4!}$

$= 3\,\frac{6 \times 5 \times 4 \times 3 \times 2}{2 \times 4 \times 3 \times 2} = 15$

Edges in B is 9

So $p_B = 9/15 = 0.6$