Assignment - 7 Recommendation systems

Question 1

	M	N	P	Q	R	Mean
A	1	2	3	4	5	3
B	2	3	2	5	3	3
C	5	5	5	3	2	4

Normalising the rating by row

value - Mean

	M	N	P	Q	R
A	-2	-1	0	1	2
B	-1	0	-1	2	0
C	1	1	1	-1	-2

-0.66 0 0.66 0

Normalising the resultant matrix by column

	M	N	P	Q	R
A	-1.34	-1	0	0.34	2
B	-0.34	0	-1	1.34	0
C	1.66	1	1	-1.66	-2

Largest value is 2 for A, R

Entry of C, P is 1

Question 2

						Rating
A	1	0	1	0	1	2
B	1	1	0	0	1	6
C	0	1	0	1	0	2

$\alpha = 0, 0.5, 1, 2$

a) $\cos(A,B) = \frac{AB}{|A||B|} = \frac{1 + 1 + 2}{\sqrt{1^2+1^2+1^2+1}\sqrt{1^2+1^2+6^2}}$

$= \frac{2 + 12\alpha^2}{\sqrt{3+2\alpha^2}\sqrt{3+36\alpha^2}}$

For $\alpha = 0$

$\cos(A,B) = \frac{2 + 12(0)}{\sqrt{3+2(0)}\sqrt{3+36(0)}} = \frac{2}{\sqrt{3}\cdot\sqrt{3}} = \frac{2}{3} = 0.6$

For $\alpha = 0.5$

$\cos(A,B) = \frac{2 + 12(0.5)^2}{\sqrt{3+2(0.5)^2}\sqrt{3+36(0.5)^2}} = 0.72254$

For $\alpha = 1$

$\cos(A,B) = \frac{2 + 12(1)^2}{\sqrt{3+2(1)}\sqrt{3+36(1)^2}} = 0.84731$

For $\alpha = 2$

$\cos(A,B) = 0.9461$

b) $\cos(B,C) = \dfrac{B \cdot C}{|B|\,|C|} = \dfrac{1+12\alpha^2}{\sqrt{3+36\alpha^2}\,\sqrt{2+4\alpha^2}}$

For $\alpha = 0$, $\cos(B,C) = \dfrac{1+12(0)^2}{\sqrt{3+36(0)^2}\,\sqrt{2+4(0)^2}} = 0.4$

For $\alpha = 0.5$ $\cos(B,C) = 0.66$

For $\alpha = 1$ $\cos(B,C) = 0.8498$

For $\alpha = 2$ $\cos(B,C) = 0.9526$

c) $\cos(A,C) = \dfrac{A \cdot C}{|A|\,|C|} = \dfrac{4\alpha^2}{\sqrt{3+4\alpha^2}\,\sqrt{2+4\alpha^2}}$

for $\alpha = 0$, $\cos(A,C) = 0$

for $\alpha = 0.5$, $\cos(A,C) = 0.288$

for $\alpha = 1$ $\Rightarrow \cos(A,C) = 0.6172$

for $\alpha = 2$, $\cos(B,C) = 0.865$

Question 3 Utility Matrix

	a	b	c	d	e	f	g	h
A	4	5		5	1		3	2
B		3	4	3	1	2	1	
C	2		1	3		4	5	3

3, 4, 5 are treated as 1
1, 2 and blank are treated as 0

Resultant Matrix

	a	b	c	d	e	f	g	h
A	1	1	0	1	0	0	1	0
B	0	1	1	1	0	0	0	0
C	0	0	0	1	0	1	1	1

Jaccard distance = 1 − JS

$JS(a,b) = \frac{1}{2} \Rightarrow J(a,b) = 1 - 1/2 = 1/2$

$JS(a,c) = \frac{0}{2} = 0 \Rightarrow J(a,c) = 1 - 0 = 1$

$JS(a,d) = \frac{1}{3} \Rightarrow J(a,d) = 1 - 1/3 = 2/3$

$JS(a,e) = \frac{0}{1} = J(a,e) = 1 - 0 = 1$

$JS(a,f) = \frac{0}{2} \Rightarrow J(a,f) = 1 - 0 = 1$

$J(a,g) = 1/2$
$J(a,h) = 1$
$J(b,c) = 1/2$
$J(b,d) = 1/3$
$J(b,e) = 1$

$J(b,f) = 1$
$J(b,g) = 2/3$
$J(b,h) = 1$
$J(c,d) = 2/3$
$J(c,e) = 1$

$J(c,f) = 1$
$J(c,g) = 1$
$J(c,h) = 1$
$J(d,e) = 1$

$J(d,f) = 2/3$
$J(d,g) = 1/3$
$J(d,h) = 2/3$
$J(d,f) = 1$

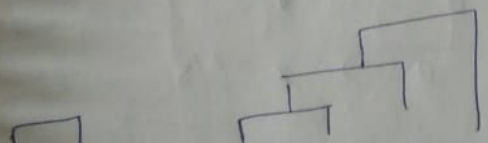
$J(e,g) = 1$
$J(e,h) = 1$
$J(f,g) = 1/2$
$J(f,h) = 0$
$J(g,h) = 1/2$

$J(f,h) = 0$ ⇒ This is the used.

Followed by $J(b,d) = 1/3$

$J(d,g) = 1/3$

So initially we merge (b,d) and then we merge the result with (g)



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C, C are separate clusters.

Question 1 UV decomposition

1	2	3
4	5	6
7	8	9

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad -(1)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 5 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 5x & 5x & 5x \\ 5y & 5y & 5y \\ 5z & 5z & 5z \end{bmatrix} \quad -(2)$$

RMSE between (1) & (2) is

$$[(5x-1)^2 + (5x-2)^2 + (5x-3)^2] + [(5y-4)^2 + (5y-5)^2 + (5y-6)^2] + [(5z-7)^2 + (5z-8)^2 + (5z-9)^2]$$

To minimize the RMSE, we should have derived each term of x, y, z should be equal to '0'.

$$150x - 60 = 0 \Rightarrow x = 2/5$$

$$150y - 150 = 0 \Rightarrow y = 1$$

$$150z - 240 = 0 \Rightarrow z = 2/5$$

$\therefore$ for RMSE to be minimum

$$x = 2/5, \quad y = 1, \quad z = 2/5$$