Assignment - 8 Dimensionality Reduction

Question 1) Column follow orthonormal bias

$c_1 = [2/7, 3/7, 6/7]$

$c_2 = [6/7, 2/7, -3/7]$

$c_3 = [x, y, z]$

Given: $\boxed{x+y+z = 1}$

$c_1 : c_2 \Rightarrow \left[\frac{2}{7} \times \frac{6}{7}\right] + \left[\frac{3}{7} \times \frac{2}{7}\right] + \left[\frac{6}{7} \times \frac{-3}{7}\right] = 0$

$\frac{12}{49} + \frac{6}{49} \; \frac{-18}{49} = 0$

$c_2 : c_3 \Rightarrow \left[\frac{6}{7} \times x\right] + \left[\frac{2}{7} \times y\right] + \left[\frac{-3}{7} \times z\right] = 0$

$\boxed{6x + 2y - 3z = 0} \rightarrow (1)$

$c_1 : c_3 \Rightarrow \left[\frac{2}{7} x\right] + \left[\frac{3}{7} y\right] \left[\frac{6}{7} z\right] = 0$

$\boxed{2x + 3y + 6z = 0} \rightarrow (2)$

solve (1) & (2) $\rightarrow \boxed{y = -2x}$

· Put y in (2)

$6x + 9y + 18z = 0$

$6x + 2y - 3z = 0$

$- \quad - \quad (+)$

$7y + 21z = 0$

$\boxed{y = -3z}$

$x : y : z \Rightarrow -2 : 1 : -3$

Question 2 $A = \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix}$

Given: 1st component of eigen vector = 1

Find: One eigenvalue & one eigen vector

So eigen vector = $\begin{bmatrix} 1 \\ e \end{bmatrix}$

$Ax = \lambda x$

$\begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ e \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ e \end{bmatrix}$

$\boxed{2 + 3e = \lambda}$

$3 + 10e = \lambda e$

$(2 + 3e)e = 3 + 10e$

$3e^2 + 2e = 3 + 10e \Rightarrow 3e^2 - 8e - 3 = 0$

$(3e + 1)(e - 3) = 0 \Rightarrow e = 3, -1/3$

So eigen vectors are $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1/3 \end{bmatrix}$

The corresponding eigen values are

$\lambda = 2 + 3e = 2 + 3(3) = 11$

$\lambda = 2 + 3e = 2 + 3(-1/3) = \underline{1}$

Question 3 Unit Eigen vector

Eigen vector : [1, 3, 4, 5, 7]

$$P = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 5 \\ 7 \end{bmatrix}, \quad \text{Unit vector} = \frac{\text{Vector}}{\text{Magnitude}}$$

Unit vector => Divide each component by the sqrt of sum of squares

Magnitude = $\sqrt{1^2+3^2+4^2+5^2+7^2} = \sqrt{100} = 10$

$$\text{Unit eigen vector} = \begin{bmatrix} 1/10 \\ 3/10 \\ 4/10 \\ 5/10 \\ 7/10 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.7 \end{bmatrix}$$

Question 4

Points => (1,1), (2,2), (3,4)

Construct a matrix with rows as point & columns as dimension of space.

$$M = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{bmatrix}$$

$$M^T \cdot M = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 17 \\ 17 & 21 \end{bmatrix}$$

Question 5 Moore - Penrose Pseudoinverse

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Pseudo inverse

→ 1 divided by non-zero elements

$$M = \begin{bmatrix} 1/1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Question 6. Probability distribution for the rows

Matrix - $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$

Probability distribution = $\dfrac{\text{Sum of squares of elements in row}}{\text{Sum of squares of element in the matrix.}}$

$\cancel{P(R_1) = \dfrac{1^2+2^2+3^2}{1^2+2^2+3^2}}$

Sum = $1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2+11^2+12^2$

$P(R_1) = \dfrac{1^2+2^2+3^2}{\text{sum}} = 0.02$

$P(R_2) = \dfrac{4^2+5^2+6^2}{\text{sum}} = 0.12$

$P(R_3) = \dfrac{7^2+8^2+9^2}{\text{sum}}$

$= 0.298$

$P(R_4) = \dfrac{10^2+11^2+12^2}{\text{sum}}$

$= 0.56$ //