

$$\begin{matrix} 1 & 2 & 3 & 4 \\ \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} & \begin{matrix} \sigma(1)=3 \\ \sigma(2)=1 \\ \sigma(3)=4 \\ \sigma(4)=2 \end{matrix} \end{matrix}$$

$$\text{sgn } \sigma = (-1)^k, \quad k - \text{число пар } (i, j):$$

$$i < j, \sigma(i) > \sigma(j)$$

$$k: \begin{matrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{matrix} \quad k=3$$

$$(1, 2), (1, 4), (3, 4)$$

$$\text{sgn } \sigma = (-1)^3 = -1; \quad -\sigma_{13} \cdot \sigma_{21} \cdot \sigma_{34} \cdot \sigma_{42}$$

№1.

$$A = \begin{pmatrix} 1 & 2 & 6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 6 & 1 & 0 & 0 \\ -1 & 0 & 3 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\text{II} + \text{I}} \left(\begin{array}{ccc|ccc} 1 & 2 & 6 & 1 & 0 & 0 \\ 0 & 2 & 9 & 1 & 1 & 0 \\ 0 & -1 & -5 & -1 & 0 & 1 \end{array} \right) \rightarrow$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 6 & 1 & 0 & 0 \\ 0 & -1 & -5 & -1 & 0 & 1 \\ 0 & 2 & 9 & 1 & 1 & 0 \end{array} \right) \rightarrow$$

$$\xrightarrow{\text{III} + 2\text{II}} \left(\begin{array}{ccc|ccc} 1 & 2 & 6 & 1 & 0 & 0 \\ 0 & -1 & -5 & -1 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 2 \end{array} \right) \rightarrow$$

$$\xrightarrow{\begin{matrix} \text{I} + 6\text{III} \\ \text{II} - 5\text{III} \end{matrix}} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -5 & 6 & 12 \\ 0 & -1 & 0 & 4 & -5 & -9 \\ 0 & 0 & -1 & -1 & 1 & 2 \end{array} \right) \xrightarrow{\text{I} + 2\text{II}}$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -4 & -6 \\ 0 & -1 & 0 & 4 & -5 & -9 \\ 0 & 0 & -1 & -1 & 1 & 2 \end{array} \right) \xrightarrow{\begin{matrix} \text{II} \cdot (-1) \\ \text{III} \cdot (-1) \end{matrix}}$$

$$\rightarrow \left(\begin{array}{ccc|ccc} & & & 3 & -4 & -6 \\ \text{E} & & & -4 & 5 & 9 \\ & & & 1 & -1 & -2 \end{array} \right)$$

$$\begin{pmatrix} 3 & -4 & -6 \\ -4 & 5 & 9 \\ 1 & -1 & -2 \end{pmatrix} = A^{-1} \quad A = \begin{pmatrix} 1 & 2 & 6 \\ -1 & 0 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\delta) \quad AX = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad | \cdot A^{-1} \text{ слева}$$

$$X = A^{-1} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 3 & -4 & -6 \\ -4 & 5 & 9 \\ 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -3 & -10 & 0 \\ 5 & 13 & 0 \\ -1 & -3 & 0 \end{pmatrix}$$

№5.

$$a) \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = A \quad \text{Spec } A$$

$$\text{Хар. уравнение: } \det(\lambda E - A) =$$

$$= \det \left(\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right) = \det \begin{pmatrix} \lambda & 1 \\ -1 & \lambda \end{pmatrix} =$$

$$= \lambda \cdot \lambda - (-1) \cdot 1 = \lambda^2 + 1$$

$$\lambda^2 + 1 = 0 \quad \lambda^2 = -1, \quad \lambda = i, -i$$

$$\delta) \quad \begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix} = A$$

$$\det(\lambda E - A) = \det \left(\begin{array}{ccc|c} \lambda & -1 & 0 & 0 \\ 4 & \lambda - 4 & 0 & 0 \\ 2 & -1 & \lambda - 2 & 0 \end{array} \right) =$$

$$= \begin{vmatrix} \lambda & -1 \\ 4 & \lambda - 4 \end{vmatrix} \cdot (\lambda - 2) = (\lambda - 2)^3$$

$$\lambda(\lambda - 4) + 4 = \lambda^2 - 4\lambda + 4 = (\lambda - 2)^2$$

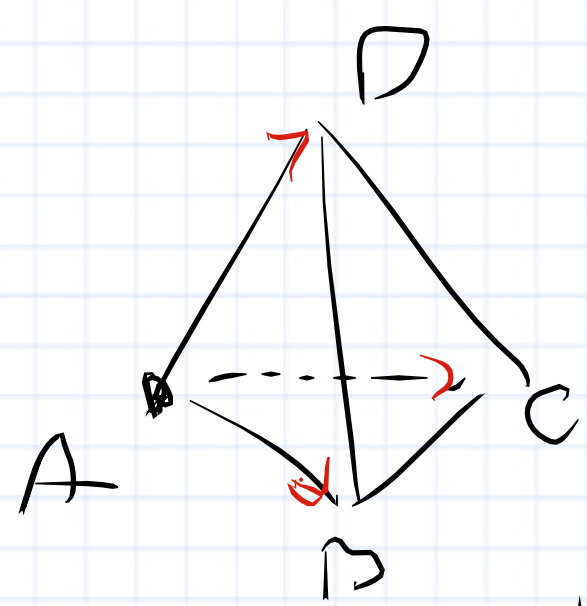
$$(\lambda - 2)^3 = 0 \Leftrightarrow \lambda = 2$$

Ответ: 2

$A - 2E$ - необратима

$A - 7E$ - обратима

№2.



$$V_{\text{тетр.}} = \frac{1}{3} \cdot S_{ABC} \cdot h$$

$$V_{\text{тетр.}} = h \cdot 2S_{ABC}$$

$$V_{\text{тетр.}} = \left| \frac{1}{6} \begin{vmatrix} 1 & 0 & -1 \\ 2 & 1 & 4 \\ -2 & -2 & -5 \end{vmatrix} \right|$$

№4.

$$\begin{vmatrix} x & y & 0 & 0 \\ 0 & x & y & 0 \\ 0 & 0 & x & y \\ 0 & 0 & 0 & x \end{vmatrix} = x \cdot (-1)^{1+1} \cdot x^3 + y \cdot (-1)^{4+1} \cdot y^3 = x^4 - y^4$$