Bridge Design Report

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Introduction

The matboard bridge “The Bridge of Unyielding Passive Aggression” is designed to maximize the load it can withstand, with iterations focusing mostly on compression, type one and shear buckling. The design began with Design Zero’s geometry, with each iteration improving its design. Following each iteration, the new values were put through MATLAB Code to compare the factor of safety between the new and previous design. After running the MATLAB code for Design Zero with load case one, the design was unable to achieve a factor of safety over one for type one buckling, with a FOS value of 0.5965. Considering load case 2, Design Zero’s smallest FOS was 0.5312 for type one local buckling followed by compression with a factor of safety of 0.9225. To increase the maximum type one buckling factor of safety, we must increase the σ_{crit} of the bridge for type one buckling using the formula:

```
FOS_tens = 4.3621

FOS_comp = 1.0375

FOS_shear = 2.6670

FOS_glue = 9.3863

FOS_buck1 = 0.5974

FOS_buck2 = 4.0620

FOS_buck3 = 5.0889

FOS_buckV = 3.5048
```

Figure 1: Factor of Safety for Design Zero computed through MATLAB with loading being the first trial of load case one.

$$\sigma_{crit} = \frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$

Our design incorporated a bridge deck that was three layers or 3.81mm thick in the area between the webs which increased the t value and σ_{crit}. After this modification, the type one buckling FOS was 10.40. The second area of failure would be failure due to compression. This was partially addressed through increasing the thickness of the bridge. To further increase the bridge’s resistance to compression, the height of the bridge’s webs was increased which increased the bridge’s second moment of area given by the equation:

$$I_{total} = \sum_{i=1}^{n} A_i y_i^2 + \sum_{i=1}^{n} \frac{b_i h_i^3}{12}$$

The compression stress is given by the equation:

$$\sigma_{compression} = \frac{My}{I}$$

Since the area of the bridge’s cross section increased, the second moment of area increased. The distance from the centroid to the top of the bridge also decreased, which means that there will be a smaller *y* and a larger *I* which will increase the compression FOS. After these iterations, the lowest factor of safety was due to shear buckling given by the equation.

$$\tau_{crit} = \frac{5\pi^2 E}{12(1-\mu^2)}[(\frac{t}{a})^2 + (\frac{t}{h})^2]$$

The h term represents the distance between the diaphragms. To reduce this term, additional diaphragms were added to reduce the distance between the diaphragms on the bridge. The diaphragms increased to six since the remaining materials only allowed for the addition of six. This improved the shear buckling FOS but it is still the lowest. If the bridge is built exactly as designed, the bridge will fail at a load of 776N due to shear buckling.

Design Iterations

Multiple design iterations were implemented to design zero. Each iteration improves upon one or more properties of the bridge with the focus of reducing materials and increasing the strength of failure mechanisms with lower factors of safety through changing certain values on the bridge or the design of the cross-section. Throughout the design iterations, the main issue in the design that was addressed was the poor factor of safety against compression, type one and shear buckling, as the provided matboard had a very low compressive strength of 6 MPa compared to the tensile strength of 30 MPa.

```
FOS_tens = 3.8787

FOS_comp = 0.9225

FOS_shear = 2.2568

FOS_glue = 7.9427

FOS_buck1 = 0.5312

FOS_buck2 = 3.6119

FOS_buck3 = 4.5251

FOS_buckV = 2.9658
```

Figure 2: Factor of Safety for Design Zero computed through MATLAB with loading being the first trial of load case two.

Design Zero

The final design was built upon Design Zero. However, the bridge could not be built according to Design Zero since Design Zero had factors of safety under one for type one buckling and compression that would lead to the bridge failing. When Design Zero's values were calculated, the lowest factor of safety was due to type one local buckling with a FOS value of 0.531 for load case two as shown in Figure 2. In addition, Design Zero had a very low factor of safety against compression, where the FOS was 0.923 for loading case two. The future iterations try to address these issues.

Iteration One

As mentioned previously, the FOS against type one buckling in the top flange was very poor, with a value of 0.5312 when under load case two. To address this, the thickness of the top flange was increased by adding an additional layer to the deck. By doing this, this will increase the value of distance to the centroidal axis (y-bar) and the Second Moment of Area (I). If y-bar increases, this will also cause y-top to decrease as the distance between the top and y-bar will go down. According to Navier’s equation, with an increase of I and decrease in y-top, the compressive stress will decrease, thus yielding a better factor of safety. After applying the changes in the first iteration, all factor of safeties is greater than one shown in Figure 5.

```
ybar = 41.431

I = 4.1835e+0

y_top = 34.83
```

Figure 3: The ybar, second moment of area, and distance from centroidal axis to top of the bridge for Design Zero

```
ybar = 49.5402

I = 5.4165e+05

y_top = 27.9998
```

Figure 4: The ybar, second moment of area, and distance from centroidal axis to top of the bridge after increasing the thickness of the bridge’s deck

```
FOS_tension = 4.1943
FOS_compression = 1.4842
FOS_buck1 = 3.4182
FOS_buck2 = 5.8164
FOS_buck3 = 12.6559
FOS_shear = 2.2840
FOS_glue = 6.5834
FOS_shear_buck = 3.0015
```

Figure 5: The factor of safety for each mode of failure after the first design iteration

Iteration Two

Following iteration one, the new lowest factor of safety was compression. While it did increase between design zero and iteration one, improvements were still needed. As the compression stress is given by the formula:

$$\sigma_{compression} = \frac{My}{I}$$

We opted to once again add another layer to the top flange, now tripling the deck’s thickness to 3.81mm. Following this change the factor of safeties are shown in Figure 6.

```
FOS_tension = 4.3797
FOS_compression = 2.0074
FOS_buck1 = 10.4021
FOS_buck2 = 7.8668
FOS_buck3 = 27.3763
FOS_shear = 2.2865
FOS_glue = 6.1568
FOS_shear_buck = 3.0049
```

Figure 6: The factor of safety for each mode of failure after the second design iteration

Iteration Three

At this point, the lowest factor of safety was still compression, and the material constraints did not allow for another layer to the deck, therefore we opted to alter the height of the web. To further improve against compressive flexural stress, optimization code written in Python using *scipy’s* library was used to minimize the flexural stress function of the cross-section. By doing this, the code can give the set of optimal sides and parameters used for the bridge for a set of given dimension bounds of each of the cross-section's lengths. For a maximum bound placed at length of 1016mm (which is the longest side length of the Matboard), the code tries to maximize the web height, and top and bottom flange lengths, while minimizing the rest of the parameters (excluding the parameters of the glue tabs). In the parameters, the code prefers to maximize the web height by a factor of 5 compared to the other flange parameters, demonstrating that maximizing the web height is most significant in the design of this bridge. As a result, using this optimization method the bridge design was designed so that the height of the web is increased in this iteration which will further minimize compressive flexural stress.

```
Initial y_bar: 41.43109435192319
Initial I: 418352.2089994236
Optimal: [ 240.68413539    1.27          5.           5.           5.
 1016.          190.59137761    1.27      ]
```

Figure 7: Optimal parameters to minimize flexural stress in Python

As there was not enough material to further thicken the flange and the optimization code encouraged increasing web height, we opted to increase the web height from 73.73mm to 94.92mm (such that the total height is 100 mm) Note that unlike iterations one and two, the y-top value did not decrease as increasing the web height also increases the y-top value, however as the value of the second moment of area greatly increased, the factor of safety against compression still saw a net increase. After this iteration, the new factor of safeties are shown in Figure 8.

```
FOS_tension = 6.0906
FOS_compression = 2.6419
FOS_buck1 = 13.6900
FOS_buck2 = 10.3533
FOS_buck3 = 18.9665
FOS_shear = 2.8785
FOS_glue = 7.9373
FOS_shear_buck = 2.3317
```

Figure 8: The factor of safety for each mode of failure after the third design iteration.

Iteration Four

Following iteration four, the new lowest factor of safety is shear buckling. Note that this FOS went down between iterations three to four, which is because changing the geometry by increasing the height greatly affects the shear buckling formula:

$$\tau_{crit} = \frac{5\pi^2 E}{12(1-\mu^2)}[(\frac{t}{a})^2 + (\frac{t}{h})^2]$$

Increasing web height causes the maximum shear stress in buckling before failure to decrease as shown by the formula above. To counteract this change, the next easiest variable to change while not overly using matboard material is the *a* value, otherwise known as the distance between diaphragms. Design zero had four diaphragms and we

```
FOS_tension = 6.0906
FOS_compression = 2.6419
FOS_buck1 = 13.6900
FOS_buck2 = 10.3533
FOS_buck3 = 18.9665
FOS_shear = 2.8785
FOS_glue = 7.9373
FOS_shear_buck = 2.5527
```

Figure 9: The factor of safety for each mode of failure after the fourth design iteration.

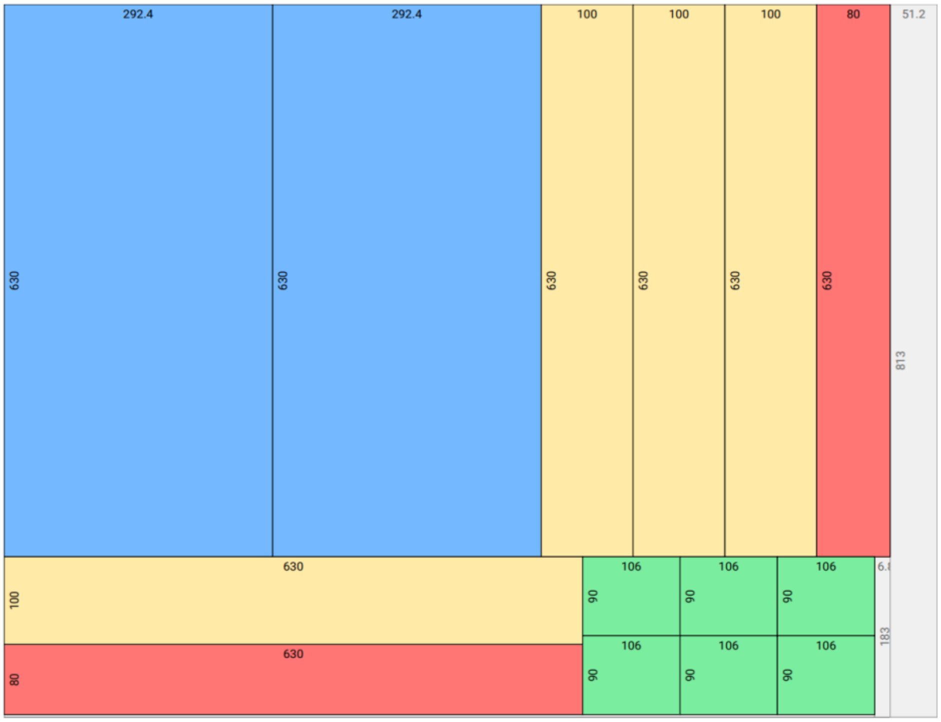
opted to increase this number to six with the remaining matboard left. The final factor of safeties are shown in Figure 9.

Iteration Five

The cut was made along the bridge since the matboards length was under the bridge's minimum length of 1250mm. The cut was made in the middle of the bridge. However, upon reflection, a slice closer to either end of the bridge would have been better since the bending moment would be less towards the end of the bridge. This difference would have resulted a bending moment at the slice of around 200Nmm less than the current bending moment which would not have a significant impact on the bridge's performance since the lowest FOS will still be due to shear buckling. When gluing the two halves of the bridge together, it was found that the corners of the bridge were unable to adhere properly to each other due to imperfections in the cutting of the matboard. So, it was decided for glue tabs to be added on the bottom and sides of the middle of the bridge where the two halves of the bridge connected to strengthen the connection at the center of the bridge as shown in Figure 10.

Figure 10: The glue tab connecting the two pieces of the bridge. Note: glue tabs are present on both sides and bottom of the bridge

Cutting of the Matboard



The dimensions of the matboard and the individual parts were determined and opticutter was used to determine how to optimize available matboard (OptiCutter).

Conclusion

This report outlines the major design iterations and decisions made to minimize materials and improve the factor of safety (FOS) against different types of buckling, tension, compression, and shear forces throughout the bridge. It outlines important considerations that were made and considered in the construction of the actual bridge, and how the bridge can be designed best to withstand as much applied load as possible. The design was carefully considered to meet requirements of passing base cases in Load Case one and two while also aiming to meet evaluation criteria such that it should withstand as much applied load as possible. Based on our lowest FOS of 2.55 the predicted failure load given our build will be 776N due to shear buckling if the bridge is built perfectly according to the design.

References

OptiCutter. (n.d.). *"Cutlist Optimizer" Opticutter.com, 2019, www.opticutter.com/cut-list-optimizer/print*