Computational Physics

Class 1: First steps

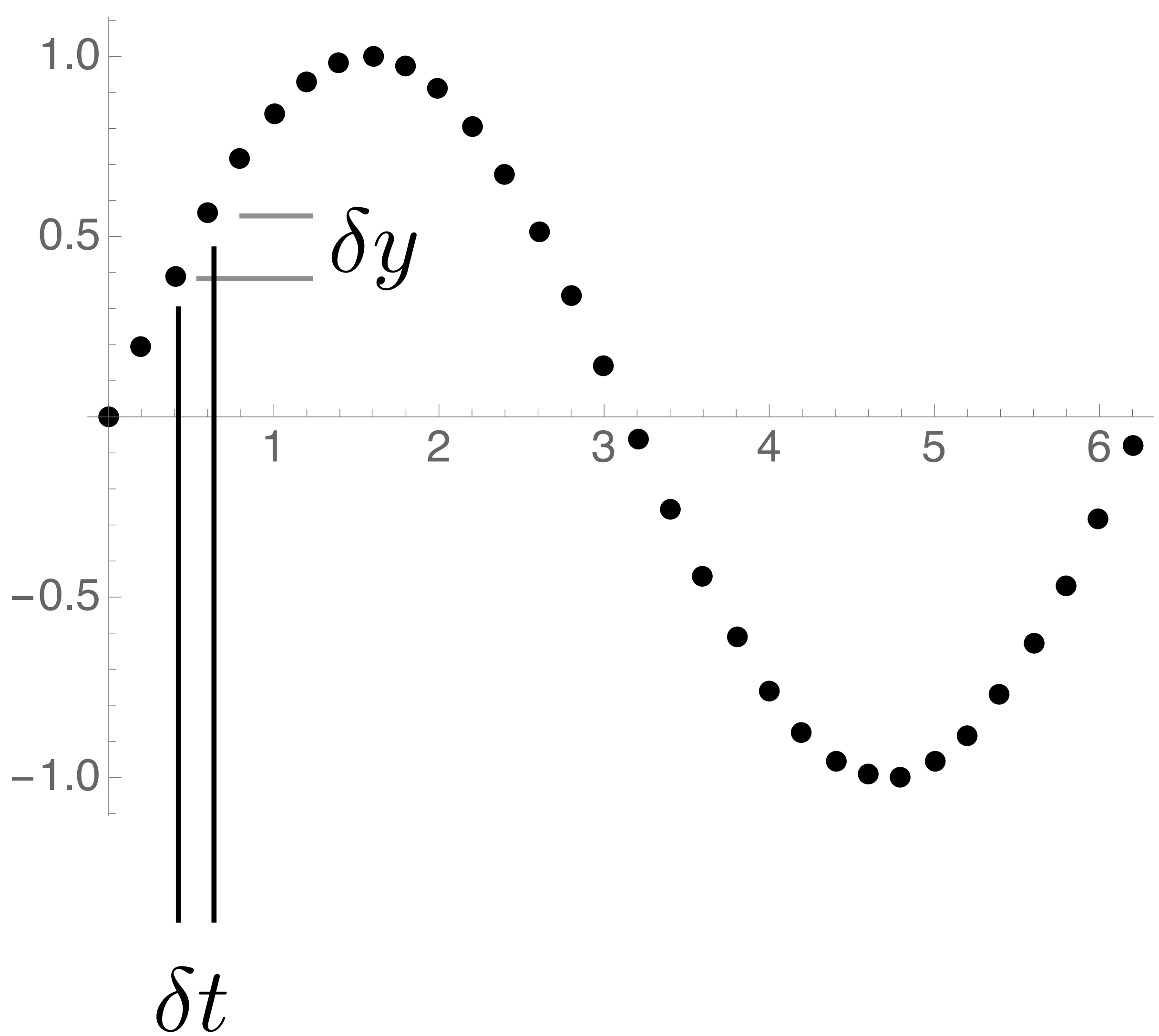
Discretization is the replacement of a continuous system with a discrete equivalent

$$q(t) \longrightarrow q_i$$

$$\nabla, \frac{d}{dt} \longrightarrow$$

$$\int dt \longrightarrow$$

To represent a function numerically, we can tabulate it at a sequence of points



Here, we have equally spaced points

$$y_i \in \{y_1, y_2, \ldots, y_N\}$$

Elementary calculus

$$\lim_{\delta t \to 0} \frac{y(t+\delta t) - y(t)}{\delta t}$$

suggests $\dfrac{dy}{dt} \approx \dfrac{y_{i+1} - y_i}{\delta t}$

Now for our ODE…

$$m\frac{d^2y}{dt^2} = F(y, y')$$ Newton's second law

$$= -mg$$ For just gravity

$$\frac{d^2y}{dt^2} = -g$$

1 second order ODE

$$\frac{d^2y}{dt^2} = -g$$

can be changed to 2 first order ODEs

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} = -g$$

Performing the discretization

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} = -g$$

$$\frac{dy}{dt} \rightarrow \frac{y_{i+1} - y_i}{\delta t}$$

Yields the discrete system

$$\frac{y_{i+1} - y_i}{\delta t} = v_i$$

$$\frac{v_{i+1} - v_i}{\delta t} = -g$$

and the iterative scheme

$$y_{i+1} = y_i + \delta t \ v_i$$

$$v_{i+1} = v_i - \delta t \ g$$

Euler method

Euler Algorithm for solving equation of motion

Initially

$$t_0 = 0$$

$$v_0 = ?$$

$$y_0 = ?$$

Loop over integer *i>0*

$$t_i = i \ \delta t$$

$$y_{i+1} = y_i + \delta t \ v_i$$

$$v_{i+1} = v_i - \delta t \ g$$