Khemkaran Sevta(13349)
Dhondip Basumatary(13250)

MSE305A: Assignment 2(Bonus)

Department : MSE

23rd Feb 2016

Q.1

(a) Plot Temperature Vs Distance curve and Thickness Vs time in mold and solid at different time.

In this part we are using differential equation to plot the graph.

Code:

```
clear all
clc
%Aluminum
Ps = 2.7          %Density of Aluminum gm/cm3
H = 95            %Heat of Fusion cal/gm
Tm = 660          %Melting Temp (Degree celsius)
Ks = 0.5          %Thermal Conductivity (cal/sec)/(cm2 C/cm)
Cs = 0.215        %Specific Heat cal/gm C
alpha_s = Ks/(Ps*Cs);

%Sand Mold
Km = 0.00145       %Thermal Conductivity for mold(cal/sec)/(cm2 C/cm)
Pm = 1.5           %Density of Aluminum gm/cm3
Cm = 0.27          %Specific Heat cal/gm C
alpha_m = Km/(Pm*Cm);

Tm = 1000; % melting Temperature
T0 = 300;  % Ambient Temperature
tn = 1000;  % Total Time
dt = 1;   %Time Step
xn = 100;  %length of mold
alpha = 1;
dx = 2;
k_m = alpha_m*dt/(dx^2);% must be less then or equal to 0.5
k_m = 0.3;
k_s = alpha_s*dt/(dx^2);    % must be less then or equal to 0.5
k_s = 0.2;
T = zeros(xn+1,tn);
T1 = zeros(xn+1,tn);
k = 0.3;
a = 1:xn-1;

%temperature drop in solid
T(1,1) = T0;
for i = 2:tn
    T(1,i) = T(1,i-1)+2*100/tn;
end
for i = 2 : xn+1
    T(i,1) = Tm;
end

for t = 1:tn-1
 for x = 2:xn-1
          T(x,t+1) = T(x,t) + k_s*(T(x+1,t)-2*T(x,t)+T(x-1,t));
 end
  T(xn,t+1) = 1000;         %T(xn,t)+k_s*(T0 - 2*T(xn,t)+T(xn-1,t));
end
dtx = 2:tn-1
for t = 2:tn-1
```

```
 for x = 2:xn-1
    if(T(x,t) > 999)
        dtx(t) = (T(x-1,t)-T(x-2,t))/dt;
        break
    end
 end
end

xa = (-xn+1:1:xn)

%mold ----------------------
for i = 1:tn
    T1(1,i) = T(2,i);
end
for i = 2 : xn+1
    T1(i,1) = T0;
end

for t = 1:tn-1
 for x = 2:xn-1
         T1(x,t+1) = T1(x,t) + k_m*(T1(x+1,t)-2*T1(x,t)+T1(x-1,t));
 end
  T1(xn,t+1) = T1(xn,t)+k_m*(T0 - 2*T1(xn,t)+T1(xn-1,t));
end

 for j = 1 : dt : tn
    for i = 1:xn
            a(i) = T1(xn-i+1,j);
            if(i < xn)
                a(i+xn) = T(i+2,j);
            end
    end
    a(2*xn-1) = 1000;
    a(2*xn) = 1000;
    figure(1), clf
    plot(xa,a,'-r+','Linewidth',1,'Markersize',5);
    xlabel('Distance(cm)');
    ylabel('Temperature(degree Celsius)');
    title(['Temperature Vs Distance Curve in Mold at time : ',num2str(j)]);
    drawnow
 end
 l = 1:tn-1
 dst = 2:tn-1
 for i = 2:tn-1
     dst(i) = dtx(i)*Ks/(H*Ps);
 end
 s = 1:tn-1;
 s(1) = dst(1);
 for i = 2:tn-1
     s(i) = s(i-1)+dst(i);
 end
plot(l,s);
xlabel('Time(sec)');
ylabel('Thickness(cm)');
title('Thickness Vs Time');
```

Algorithm :

At any time t temp at solid-liquid interface temp is Tm(melting temp.).And at t = 0 for any x <0 value T is T0(temp. of mold).After that we just used the formula to get values at T(x,t+Δt) using values T(x,t), T(x +Δx , t), T(x,t), T(x-Δx,t).we used 1d array a to copy the elements of 2d arrayT at a particular time t and ploted it's graph with xa.where a 2d array T is used to store the value of temp as a function of temperature and time for Solid andT1 is used for mold. And then ploted graph for T Vs x and then calculate ds/dt for different value of t using this Eqn:

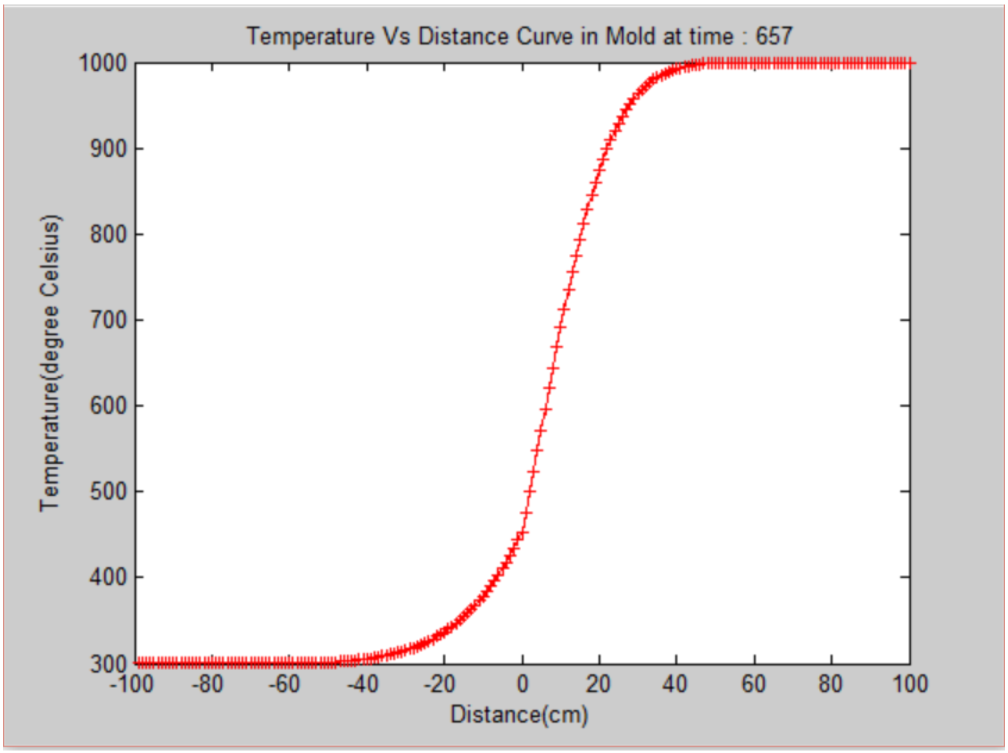
$$K_s \left(\frac{\partial T}{\partial x}\right)_{x=S} = H\rho_s \frac{\partial S}{\partial t}$$

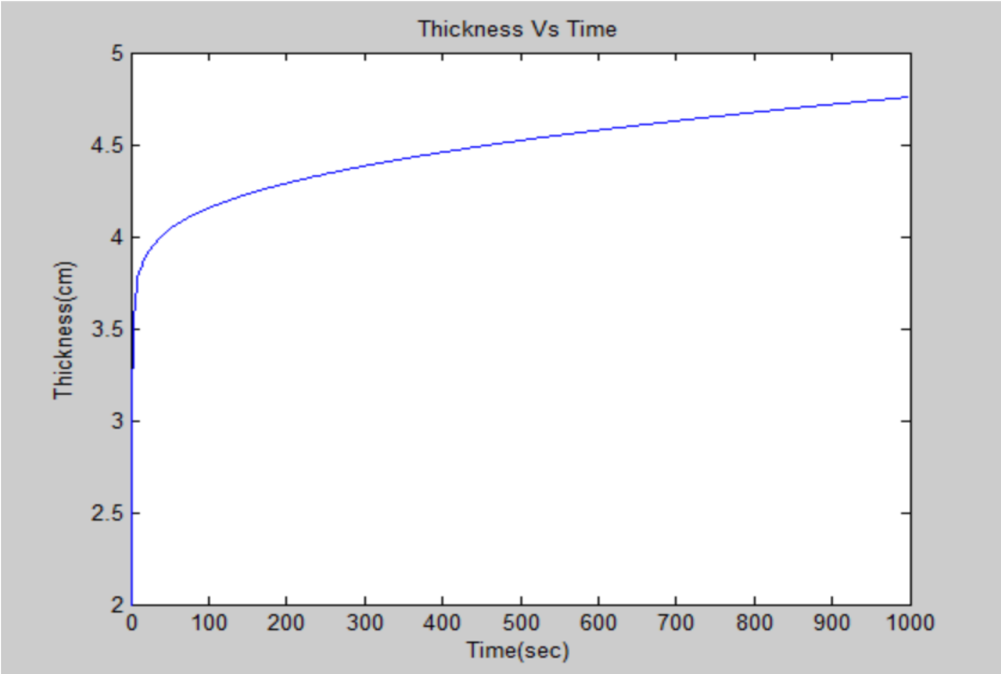
After that we plot a graph between S(thickness) vs t(time).which comes to be a parabola.

Formula Used :

- dT/dt = α (d^2)T/dx^2 , where alpha is heat diffusivity.
- (d^2)T/dx^2 = [T(x +Δx , t) – 2*T(x,t)+T(x-Δx,t)]/(Δx^2).
- dT/dt = [T(x,t+Δt)-T(x,t)]/Δt.
- T(x,t+Δt) = T(x,t)+k.(T(x +Δx , t) – 2*T(x,t)+T(x-Δx,t)).
- k = (Δt*α)/(Δx^2).
- The value of k should be less than or equal to 0.5.\

Graph :





(b) Solidification Thickness Vs Time curve using this eqn:

$$\gamma e^{\gamma^2}\left(\sqrt{\frac{K_s\rho_s C_s}{K_m\rho_m C_m}}+\operatorname{erf}\gamma\right)=\frac{C_s}{H\sqrt{\pi}}(T_M-T_0)$$

Code:

```
% Aluminum in Sand Mold
T0 = 25 %room Temp
%Aluminum
Ps = 2.7          %Density of Aluminum gm/cm3
H = 95            %Heat of Fusion cal/gm
Tm = 660          %Melting Temp (Degree celsius)
Ks = 0.5          %Thermal Conductivity (cal/sec)/(cm2 C/cm)
Cs = 0.215        %Specific Heat cal/gm C

%Sand Mold
Km = 0.00145       %Thermal Conductivity for mold(cal/sec)/(cm2 C/cm)
Pm = 1.5           %Density of Aluminum gm/cm3
Cm = 0.27          %Specific Heat cal/gm C

% Solving eqn for gamma(Y)
RHS = (Cs*(Tm-T0))/(H*sqrt(pi)); %RHS of eqn
b = (Ks*Ps*Cs)/(Km*Pm*Cm);
j = 1;
for i = 0 :0.0001: 1
    erf_Y = erf(i);                    %error function of gaama(Y)
    e_Y2 = exp(i^2);                   %exponential of Y^2
    LHS = (i*e_Y2*sqrt(b+erf_Y));      %LHS of eqn
    temp = LHS- RHS;                   %difference between LHS and RHS
    if(temp < 0)
        temp = temp*-1;
    end
    if(temp < 0.0009)                  %if difference between LHS and RHS is less
than 0.0009
```

```
        Y = i;                          %this will be the solution of equation
        break;
    end
    j = j+1;
end

s = 1 : 100                         %thickness of metal
t = 1 : 100                         %time
for k = 1 : 100
    s(k) = 2*Y*sqrt((Ks/(Ps*Cs))*k);
end
plot(t,s);
xlabel('Time(sec)');
ylabel('Distance');
title('Distance Vs Time ');
```

Graph: