RISKMETRICS

RiskGrades™ Technical Document

Jongwoo Kim and Jorge Mina
Contributors: Alan J. Laubsch and Alvin Lee

RiskMetrics Group
44 Wall Street
New York, NY 10005
www.riskmetrics.com

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Foreword

In 1994 J.P. Morgan, the global investment bank, launched RiskMetrics®, a transparent approach to measuring the risk of financial assets. RiskMetrics educated the world on the importance of understanding financial risk, and provided a data set to give institutions the ability to calculate their own risk exposures, as well as a technical document explaining all the mathematics behind the methodology. RiskMetrics was fully transparent and open, and free to all market participants and observers.

RiskMetrics quickly became the standard for institutions around the world to measure and manage their financial risks. Shortly after the launch of RiskMetrics, the regulators from the G-7 countries adopted a requirement that all banks report their market risk exposure. Over the last few years similar requirements have been extended to non-G-7-country banks, as well as non-financial institutions. Value-at-Risk, or VaR, the approach RiskMetrics made public, has now become the standard risk measure for over 5,000 institutions around the world.

The success of RiskMetrics was obvious. It filled a market need. Global markets were becoming more volatile and interrelated, more complex instruments such as derivatives were being traded, and firms were deriving an increasing amount of their profits from trading and investments in financial assets. Institutions were able to quantify the returns of these activities, but few were able to accurately measure their risks. They knew that some investments were riskier than others, but they didn't know by how much, or how to quantify the total risk of their portfolios of investments.

RiskMetrics was successful because senior managers, regulators, and shareholders recognized that *R*eturn Is Only Half the Equation. No decision should be made without understanding both the risk and the expected return of the outcome. RiskMetrics gave managers for the first time a transparent and consistent approach to quantify the risk of each of their financial investments and compare it to their expected returns, so that they could make better and more informed investment decisions.

Today there is a similar market need. Individuals around the world are taking on more responsibility for their financial futures. Investors as well as their financial advisors are looking at increasingly volatile markets and a wider array of increasingly complex investment options. And an individual's financial investments are more important to his future life than ever before. While individuals are given detailed return information, the best risk information they can get is "this stock or portfolio is aggressive," which they understand is riskier than “this stock or portfolio is moderate or conservative.” They know that stocks are riskier than bonds, which are riskier than cash, but they don’t know by how much, or how to quantify the total risk of their entire portfolio of investments.

To meet this need, the group that was responsible for RiskMetrics at J.P. Morgan is launching RiskGrades™ . RiskGrades is an open and transparent benchmark for individuals and their financial advisors to measure financial risk. RiskGrades include several components. First there is an online course, *Understanding Risk*, explaining the basic concepts of financial risk. Second, there is the RiskGrade™ data set allowing individuals the ability to measure the risk of stocks, bonds, funds, and other financial assets around the world. Third, there are the RiskGrades online analytics giving individuals the tools necessary to manage the risk of their own investment portfolios. Finally there are two documents — *Return is Only Half the Equation*™ , a practical risk management guide for individual investors, and the *RiskGrades*™ *Technical Document*, fully exposing all the calculations behind the RiskGrades approach. And all of the RiskGrades components are based on the same RiskMetrics research and technology used by thousands of leading institutions and regulators around the world.

The intent of RiskGrades is to help individuals make better investment decisions. RiskGrades do not by themselves provide advice, or make buy and sell recommendations. RiskGrades do not tell you what stock will do better in any one year, or tell you what investment strategy is right for you. Instead RiskGrades provide information about risk — information that should be considered, along with the return information you are already getting, to determine the best investments for you and your portfolio.

We have all grown accustomed to the infamous phrase “historical performance is no guarantee of future results,” and can be sure that no mathematical model can predict the future. Ultimately, successful investing can only be achieved by people — people who have good information, good discipline, and good judgement. RiskGrades provide some of that information, and some of that discipline. The judgement is up to you, and when appropriate, your professional financial advisor.

Ethan Berman
CEO
RiskMetrics

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Chapter 1

Introduction

The RiskGrade™ statistic is a new measure of volatility, recently devised by the RiskMetrics Group to help investors better understand their market risk. RiskGrade measurements are based on the exact same data and analysis as RiskMetrics® Value-at-Risk (VaR) estimates and, in fact, can be translated back into VaR estimates.

The RiskGrade measure, however, is scaled to be more intuitive and easier to use than VaR. RiskGrades™ are measured on a scale from 0 to 1000 or more, where 100 corresponds to the average RiskGrade value of major equity market indices during normal market conditions from 1995 to 1999. You would expect cash to have a RiskGrade value of zero, while a technology IPO may have a RiskGrade value exceeding 1000. [1]

1.1 Features of RiskGrades™

RiskGrades vary over time. The RiskGrade measure is dynamic and adjusts to current market conditions: during turbulent times, such as the Asian Flu or the Russian Crisis, the RiskGrades of major stock markets can easily escalate beyond 200, while in calmer markets, RiskGrades could fall below 50. RiskGrades can help investors dynamically monitor exposure to market risk.

RiskGrades allow comparison between investments. RiskGrade is a standardized measure of volatility, and therefore allows "apples-to-apples" comparison of investment risk across all asset classes and regions. Thus we can say that a Brazilian stock with a RiskGrade of 300 is six times as risky as an Asian bond fund with a RiskGrade of 50.

RiskGrades capture currency risk. Does a European investor in Yahoo! stock have the same risk as an American holding Yahoo!? Yahoo! is a riskier proposition for the European investor, because in addition to

[1] Most of this chapter was written by Alan J. Laubsch of RiskMetrics. He also created the web-based risk education course *Understanding Risk* at `http://www.riskgrades.com`.

the stock price fluctuation, she is exposed to the USD/EUR exchange rate fluctuations (e.g., she loses if USD depreciates relative to EUR). The RiskGrade statistic captures this currency risk component, and predicts the total price risk of your investment. Clearly, the value of a RiskGrade varies, depending on the home currency of the investor.

1.2 History of RiskMetrics® RiskGrades™

While the RiskGrade measure was launched only recently, the concepts behind RiskGrade have evolved from more than a decade of risk management research and practice. J.P. Morgan became an early pioneer of market risk measurement in the 1980s, when former CEO Dennis Weatherstone made a famous request: to have a one-page summary report of all the firm's risks on his desk by 4:15 PM. This 415 Report became a cornerstone of J.P. Morgan's day-to-day risk measurement, and provided management with timely information to manage enterprise-wide market risk.

By the early 1990s, J.P. Morgan had perfected VaR measurement and made a ground breaking decision to publish its proprietary risk methodology for free over the Internet. J.P. Morgan created the first global standard for risk measurement when it launched RiskMetrics in October 1994. The *RiskMetrics Technical Document* is still available for free download from RiskMetrics.

Since then, RiskMetrics methodology has been adopted as a universal standard for risk measurement by more than 5,000 institutions globally, including regulators, central banks, global financial institutions, money managers, and large corporations. RiskMetrics was spun off from J.P. Morgan in 1998 to expand its focus on delivering leading-edge risk management research, software, data and education.

In publishing RiskGrades, we are committed to giving individual investors access to the same risk analytics used by professional risk managers. Investors can now explore and measure their portfolio risk online, at `www.riskgrades.com`.

1.3 Comparison to Other Risk Measures

RiskGrade is not the only risk measure available for investors. Two other popular risk measures are standard deviation and beta. This section briefly introduces and describes these measures, and with a view of their uses and limitations.

1.3.1 Standard Deviation

Standard deviation is a general statistical measure of volatility. It can be used to measure the dispersion from the mean of any data series, such as a time series of returns. Standard deviation has been a classical portfolio risk measure since Markovitz used it in the 1950s to demonstrate the diversification effect of stocks. As a measure of volatility, standard deviation is similar to RiskGrade, although there are two main differences.

The first is that RiskGrade estimates are based on exponential weighting of historical data, which makes them more adaptive to current market conditions than plain standard deviations. When it released the RiskMetrics methodology, J.P. Morgan revealed a study which demonstrated that exponential weighting significantly improves forecasting accuracy and responsiveness in extreme market conditions.

The second difference is that RiskGrades have been calibrated to be easier to interpret for the general public, with a RiskGrade of 100 representing the typical risk of the global equity markets. Standard deviations, however, do not have such an intuitive reference point: we can easily tell that a standard deviation of 5% represents more risk than 2%, but it's not obvious how risky that is (e.g., how risky is that compared to the risk of a well-diversified equity portfolio?).

1.3.2 Beta

Beta measures how much an individual stock is likely to move with the general market. A beta of 1 means that a stock will tend to move lockstep with the general market, while a beta of 1.5 means that, on average, the stock will rise 1.5% for any 1% rise in the stock market, and fall 1.5% with any 1% fall in the stock market, compared to the risk-free interest rate. Beta can be used to compare the systemic risks of various stocks: the higher the beta, the more risk a particular stock is likely to contribute to a portfolio of stocks.

While elegant in its simplicity, beta has several limitations which are rooted in its parent theory of the Capital Asset Pricing Model (CAPM). First, it is only a relative risk measure: beta is only a measure of how a stock is likely to move relative to an overall stock index, and gives no indication of the stock's unique volatility (or the overall stock market's volatility). Beta can be misleading because two stocks with the same beta generally have a different unique risk. Second, it only measures incremental systemic risk for a perfectly diversified portfolio of stocks (i.e., a stock with a beta of 1 could easily contribute twice as much volatility as the broader stock market, if you have an undiversified portfolio). Third, CAPM focuses only on the risk premium of equities relative to risk-free assets, does not address fixed income and currency investments, and consequently, is difficult to apply across asset classes. In sum, we can say that RiskGrades account for both systematic and unique risk and thus show the whole picture of risk.

1.4 Beyond RiskGrades™

The RiskGrade measure is a representative and universal scaler of risk for all investment instruments of the individual investor. However, because investment purpose, time horizon, and attitude toward risk vary across investors, and the financial market occasionally jumps from quiet to turbulent, we provide even more specific measures of risk. Among these measures are XLoss™ (Loss in Extreme Markets) for short-horizon abnormal markets, Chance of Losing Money for short- or long-horizon normal markets, Worst Losing Streak, and Worst-Case Performance for long-horizon abnormal markets.

In addition to such risk measures, we have devised "warning lights" against potential market crashes. Warning lights indicate the degree of vulnerability to large market moves and identify portfolio components that can create the most serious portfolio losses. As warning lights, we provide two stress tests: the Historical Event Stress Test, based on actual historical events, and the User-Defined Event Stress Test, based on investors' assumptions.

Furthermore, because each investor has a preferred combination of risk and return, we introduce ReturnGrades, a portfolio optimization tool, and a sector analysis. ReturnGrades is a modified risk adjusted return for the individual investor to combine risk and return on one dimension. The risk-return optimization tool adjusts portfolio composition based on RiskGrades and expected return. It either maximizes the expected return given a predetermined maximum RiskGrade, or it minimizes the RiskGrade given a predetermined minimum expected return. The optimization tool handles both the overall optimization problem and the incremental optimization problem based on the current portfolio. The sector analysis provides a tool to identify concentration of investor's portfolio and decompose its risk through sectors.

Computing risk measures, performing stress tests, and optimizing portfolios all require years of historical data. Many recent IPO stocks, however, lack that much historical data, but cannot be excluded from a portfolio analysis. We therefore designed the Ghost Series Generator and based it on the one-factor model to construct reasonable proxy data for short-history assets (e.g., IPO stocks) by using their short historical data and industrial indices.

Chapter 2

RiskGrades™

2.1 What is a RiskGrade?™

The RiskGrade™ measure is a risk indicator that is based on the volatility of returns. The higher the volatility of returns, the higher the RiskGrade of an asset. RiskGrade values are available for a wide set of investments including individual equities, equity indices, mutual funds, bonds, and currencies. RiskGrades are comparable across asset classes in the sense that any asset with a RiskGrade of 800 is twice as volatile as any other asset with a RiskGrade of 400.

It is important to understand that since RiskGrade values are based on volatility estimates, they vary according to the base currency of each investor. For a British investor who views the world from a British pound perspective, having pounds under the mattress is a riskless investment, and hence has a RiskGrade of zero (from a market/price perspective, not an inflation perspective). For an American investor, having pounds under the mattress carries currency risk, and thus the RiskGrade of the pound from a U.S. dollar perspective is greater than zero. Similarly, an American investor who buys a German stock is exposed to two sources of risk: equity risk and currency risk. Therefore, the RiskGrade of a German stock from the point of view of an American investor is different from the RiskGrade of the same stock from a German investor's viewpoint.

RiskGrades are not constant through time. There are times when assets are more or less volatile, leading to RiskGrades that change through time. To account for rapidly changing market conditions, we recalculate RiskGrades on a daily basis.

2.2 Calculation of RiskGrades™

RiskGrades are scaled volatilities, where the scaling factor is chosen to simplify the interpretation of a RiskGrade. We set a RiskGrade of 100 to be equivalent to an annual volatility of 20%. While the scaling factor is somewhat arbitrary, it can be justified by noting that the market-cap weighted average volatility over the January 1995–December 1999 period of a group of international equity indices is approximately 20%. [1] The constituents of this group and their relative weights are shown in Table 2.1.

Table 2.1: **Volatility of Equity Indices**

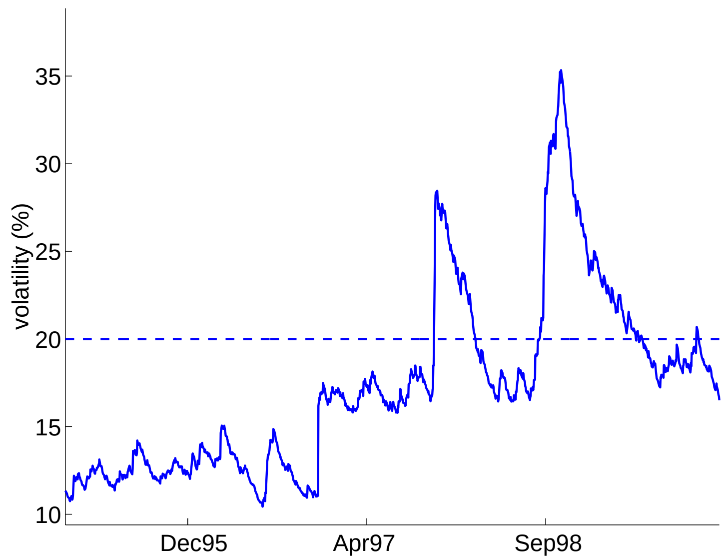
Index		Market-Cap Weights, (%)			Annualized
Ticker	Country	Europe	Asia/Pacific	Americas	Volatility, (%)
AEX	Netherlands	2.4			40.52
All Ordinaries	Australia		1.3		13.06
ASIN	Singapore		0.6		21.32
BEL20	Belgium	1.0			15.49
CAC40	France	3.9			19.80
DAX	Germany	4.3			21.10
FTSE100	UK	9.4			15.31
HangSeng	Hong Kong		1.4		30.96
HEX	Finland	0.6			26.48
IBEX	Spain	1.4			20.72
IPC	Mexico			0.4	30.08
KOSPI200	Korea		0.5		34.76
Merval	Argentina			0.5	37.10
MIB	Italy	2.2			23.65
Nasdaq	US			10.3	28.11
NIKKEI225	Japan		9.7		22.54
OMX	Sweden	1.1			21.09
NYSE	US			43.1	13.89
SMI	Switzerland	2.7			18.40
TSE100	Canada			2.2	14.80
TWII	Taiwan		1.0		23.99
Total		29.0	14.5	56.5	18.85

Table 2.1 also shows the volatility of each index and the market-cap weighted average volatility across all indices. The volatility is calculated from RiskMetrics methodology by using a 0.97 decay factor and 151 days of historical data. (The methodology is discussed in Section 2.3.) The market-cap weights are based on market capitalization of December 31, 1998 from London Stock Exchange Statistics.

[1]The market-cap weighted average volatility is the average of the volatilities of the equity indices with market-cap weights. It is *not* the volatility of the global equity portfolio, which is composed of market-cap weighted indices. The portfolio approach results in a very low volatility because of the large global diversification benefit. Since we cannot expect such global diversification for individual investors, it is unrealistic to implement the low volatility of the global portfolio as a reference volatility.

Figure 2.1 shows the weighted average volatility of the equity indices in the international group over the January 1995–December 1999 period. The weighted average volatility fluctuated between 10% and 20%, except for the two abnormal market conditions, when it hit 28% in the Asia Exchange Crisis of July 2, 1997, and 35% in the Russian Debt Crisis of August 21, 1998.

Figure 2.1: **Weighted Average Volatility of Equity Indices**



The most important part of the RiskGrade calculation is estimating the return volatility. Once we have the volatility, we simply apply a fixed multiplier to obtain the RiskGrade. Equation 2.1 presents the formula used to calculate the RiskGrade of asset i.

$$\begin{aligned} \text{RiskGrade}\ (i) &= \frac{\sigma_i}{\sigma_{base}} \times 100 \\ &= \frac{\sigma_i \sqrt{252}}{0.2} \times 100, \end{aligned} \tag{2.1}$$

where σ_i and σ_{base} denote the RiskMetrics® volatility of asset i and the base volatility, respectively. Multiplying $\left(\frac{\sqrt{252}}{0.2} \times 100\right)$ by σ_i results in a RiskGrade of 100, equivalent to an annual volatility of 20%. Because RiskMetrics volatilities are a key input into the equations for RiskGrade, we discuss them in detail in the next section.

2.3 RiskMetrics Volatilities

RiskMetrics volatilities are calculated by using an exponentially weighted moving average, where the latest observations carry the highest weight in the volatility estimate. The exponential weighting scheme allows volatilities to react faster to shocks and therefore rapidly incorporates changes in the state of events. Following a turmoil, the volatility estimate decreases exponentially as the recent extreme observations are forgotten. By contrast, an equally weighted scheme delays the incorporation of extreme events into the volatility estimate, but once taken into account, their effects persist for long periods of time.

The exponentially weighted volatility for asset i at time t estimate can be written as

$$\sigma_{i,t} = \sqrt{(1-\lambda)\sum_{j=0}^{\infty} \lambda^j r_{t-j}^2}, \tag{2.2}$$

where λ is the decay factor. The return $r_{i,t}$ of asset i at time t is a one-day logarithmic return computed by $\ln(P_{i,t}/P_{i,t-1})$, where $P_{i,t}$ denotes the price of asset i at time t.

RiskMetrics assumes that the mean value of daily returns is zero. This assumption is unlikely to cause a large bias in the volatility estimate because the volatility of returns of a single asset dominates that asset's expected return in the short term (see the *RiskMetrics Technical Document* [7], pp. 91–92).

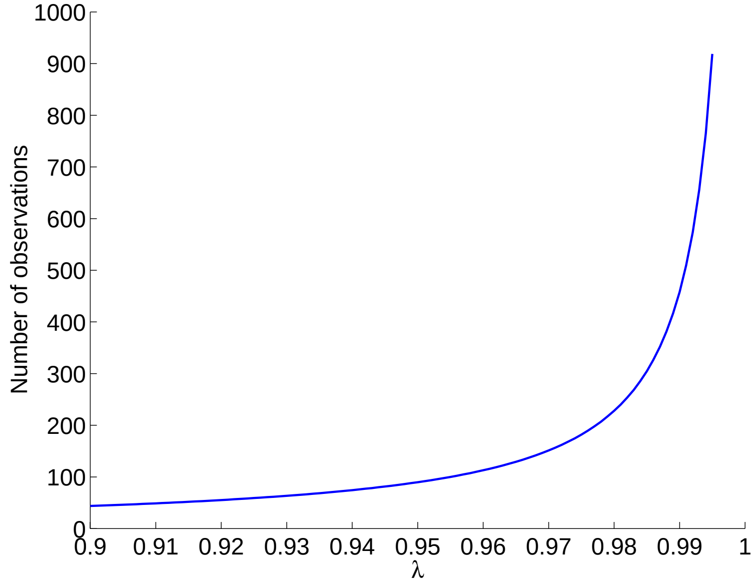
Given that we do not have an infinitely long history of returns, we need to define a cutoff point. Our criterion for defining the cutoff is to use as many returns as necessary to incorporate 99% of the information contained in an infinitely long history of returns. We can formalize this idea by observing that the total weight of an infinitely long history is equal to $1/(1-\lambda)$, whereas the weight of a finite series consisting of n returns is equal to $(1-\lambda^n)/(1-\lambda)$. Therefore, to incorporate 99% percent of the weight, we need to set $n = \ln(0.01)/\ln(\lambda)$. Note that the effective number of observations used in the volatility estimate depends on the decay factor λ — the higher the decay factor λ, the higher the number of observations. Figure 2.2 shows the effective number of observations as a function of decay factor in an exponentially weighted scheme.

Hence, the actual RiskMetrics volatility estimate is

$$\sigma_{i,t} = \sqrt{\frac{1-\lambda}{1-\lambda^n}\sum_{j=0}^{n} \lambda^j r_{i,t-j}^2}. \tag{2.3}$$

For calculating the volatility of a portfolio, we first need to estimate the covariance between all the portfolio components. The RiskMetrics covariance estimate between assets 1 and 2, based on an exponentially weighted average is given by Equation 2.4,

Figure 2.2: **Effective Number of Observations as a Function of Decay Factor**



$$\sigma_{12,t} = \sqrt{\frac{1-\lambda}{1-\lambda^n}\sum_{j=0}^{n}\lambda^j r_{1,t-j} r_{2,t-j}}. \tag{2.4}$$

Therefore, the RiskMetrics volatility of the portfolio is

$$\sigma_{p,t} = \sqrt{\omega \text{VCV} \omega'}, \tag{2.5}$$

where ω denotes the weights of the portfolio components, and VCV denotes the RiskMetrics variance-covariance matrix for the components. Note that $\sigma_{p,t}$ can be used in Equation 2.1 to calculate a portfolio RiskGrade™ .

2.4 Choosing a Decay Factor

Assuming an average daily return of zero, we can write $E[r_{i,t+1}^2] = \sigma_{i,t}^2$. Therefore, one means of obtaining λ is to minimize the average square error $\frac{1}{n}\sum_{t=1}^{n}(r_{i,t+1}^2 - \sigma_{i,t}^2)^2$, where the variance $\sigma_{i,t}^2$ is a function of λ.

From this methodology it has been found that the optimal decay factor λ is 0.94 for one-day volatility estimates, and 0.97 for one-month volatility estimates (see the *RiskMetrics Technical Document* [7], pp. 97–101).

The choice of decay factor is basically linked to the investor's time horizon. For individual investors, the relevant horizon is usually longer than one day and hence a somewhat stable volatility estimate is desirable. Volatility estimates constructed with a decay factor of 0.97 are more stable than those built with a decay factor of 0.94. This result is intuitive because the relative weight given to recent observations is lower when we use a higher decay factor. Since our objective is to provide an intuitive and stable measure of risk, we chose to use a decay factor of 0.97 in the calculation of RiskGrades. Therefore, the effective number of days used in a volatility estimate with a decay factor of 0.97 is 151.

2.5 Why Are RiskGrades™ Good Measures of Risk?

The strength of RiskGrades is derived from the ability of RiskMetrics volatilities to predict extreme events. RiskMetrics volatilities have been used and tested by professionals in financial institutions and regulatory bodies throughout the world, and have proven to be a consistent and reliable basis for calculating market risk. It has also been shown that RiskMetrics volatilities produce accurate VaR numbers across various markets and asset classes. [2] Figure 2.3 shows the 95% confidence limits for the Nasdaq index over the 1995–1999 period. You can check how robust the RiskMetrics volatilities are from Figure 2.3 The actual number of outliers (5.5%) from the 95% confidence interval is very close to the expected number of outliers (5%).

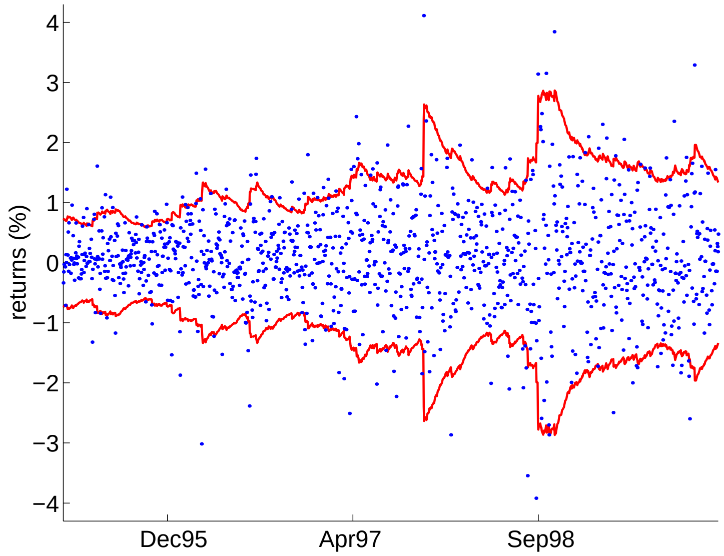
In addition, a recent study by Malz [6] shows that RiskMetrics volatilities contain predictive information regarding future large-magnitude returns. Furthermore, the predictive power of RiskMetrics volatilities is sometimes comparable to that of implied volatilities, as shown in Figure 2.4, in which the probabilities of a 5% devaluation of the Mexican peso over a one-month period are computed from both RiskMetrics and implied volatilities. Note how both probability estimates jump up at the same time and by about the same amount before an extreme event.

2.6 RiskImpact™

To determine the contribution of a single asset to the total portfolio volatility, we introduce the RiskImpact™ measure. We define RiskImpact as a Marginal RiskGrade. In other words, RiskImpact pertains to a specific asset and reflects how the RiskGrade of an investor's portfolio would change if the investor were to sell that asset and keep the cash proceeds. We can express the RiskImpact of asset i as in Equation 2.6,

[2] See the *RiskMetrics Technical Document* [7] for a general discussion on the accuracy of RiskMetrics volatilities, and Finger [1]'s discussion for a test of RiskMetrics volatility forecasts on emerging markets data.

Figure 2.3: **Confidence Limits for Nasdaq Returns**



$$\text{RiskImpact } (i) = \text{RiskGrade of the entire portfolio} - \text{RiskGrade of the portfolio without asset } (i). \quad (2.6)$$

We can also report the RiskImpact as a percentage of the portfolio RiskGrade:

$$\% \text{ RiskImpact } (i) = \frac{\text{RiskImpact } (i)}{\text{RiskGrade of the entire portfolio}} \times 100. \quad (2.7)$$

2.7 An Example of RiskGrades™

In this section we provide an example of how the RiskGrade and RiskImpact measures are interpreted and used to assess portfolio risk. We use this example throughout the document. A U.S.-based investor owns a portfolio worth USD 20,000 at the end of December 31, 1999. The portfolio consists of USD 10,000 of Coca-Cola shares (NYSE:KO) and USD 10,000 of Cisco Systems shares (Nasdaq: CSCO). Table 2.2 shows the RiskGrade and RiskImpact values of the two stocks and their portfolio.

Figure 2.4: **Probability of a 5% Devaluation in the Mexican Peso**

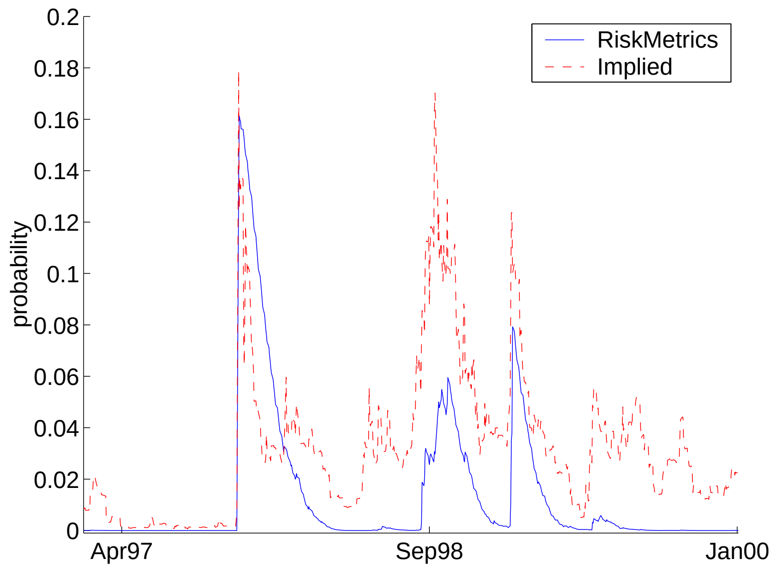


Table 2.2: **RiskGrade and RiskImpact Values on Dec. 31, 1999**

Asset	Price, (USD)	MV, (USD)	RiskGrade	RiskImpact, (%)
Coca-Cola	58 1/4	10,000	188	28
Cisco	107 1/8	10,000	179	25
Div. Benefit	...	...	59	...
Portfolio	...	20,000	125	...

Coca-Cola's RiskGrade is larger than Cisco's. On an absolute basis, how much riskier is the investment in Coca-Cola compared to the investment in Cisco, based on annual volatility? Since Coca-Cola's RiskGrade exceeds Cisco's by 9, and a RiskGrade of 100 is equivalent to an annual volatility of 20%, Coca-Cola is more volatile than Cisco by an annual volatility of 1.8% ($= (9 \times 20\%)/100$).

Table 2.2 also shows how the diversification benefit reduces the portfolio's RiskGrade to a value that is less than the sum of the component RiskGrades.

$$\text{RiskGrade Div. Benefit} = \sum_{i=1}^{N} \omega_i \text{RiskGrade of asset } (i) - \text{RiskGrade of the portfolio}, \tag{2.8}$$

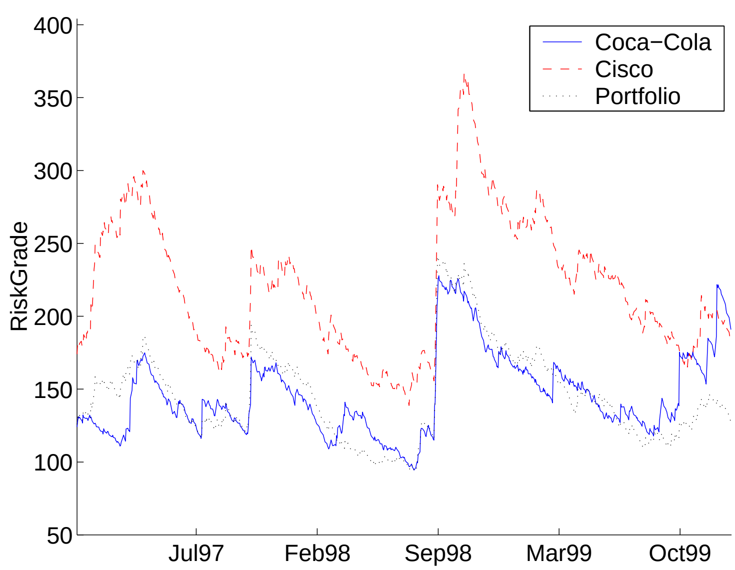
where N denotes total number of assets in the portfolio, and ω_i denotes the weight of asset i.

The RiskImpact values of Coca-Cola and Cisco are 28% and 25%, respectively, which means that if the investor closes her Coca-Cola position (sells all Coca-Cola stocks in her portfolio and keeps the cash proceeds), she can reduce her portfolio's risk by 28%. Why is the sum of the RiskImpact values of all portfolio components less than 100%? As long as a diversification benefit exists, it reduces portfolio risk such that the sum of RiskImpact values is less than 100%.

Figure 2.5 plots a three-year history of RiskGrades for Coca-Cola, Cisco, and the portfolio. It is worth noting that RiskGrade is a time-variant parameter which is updated daily by RiskMetrics to account for new information pertaining to an individual asset and to the financial market as a whole.

For example, on July 16, 1998, Cisco's RiskGrade fell to a minimum of 139, then jumped to a maximum of 367 on October 15, 1998. In three months, in the midst of the Russian Crisis of August 21, 1998, Cisco's RiskGrade jumped by almost a factor of three.

Figure 2.5: **Three-Year History of RiskGrades**



2.8 RiskGrades™ of Bonds and Options

In this section, we discuss issues related to the calculation of RiskGrades for bonds and options. Computing the volatility of equities is somewhat straightforward because we can observe consistent equity prices with only minor adjustments of stock splits and dividends. It is impossible, however, to observe daily bond and option prices under a constant maturity, strike price, and volatility. Thus, we need to construct an artificial historical return series for bonds and options by using a simple valuation model that is controlled by a fixed condition.

For example, if an investor holds a U.S. 10-year Treasury bond in her portfolio, she needs to know its historical daily return at a constant maturity of ten years. To obtain this information, we can construct an artificial return series by using the historical daily-yield series of the 10-year Treasury bond as follows:

1. Calculate the price of the bond from its discounted future coupons, and the maturity value of the bond from its yield at maturity:

$$P_t = \sum_{i=1}^{n} \frac{c}{(1+0.5y_t)^i} + \frac{M}{(1+0.5y_t)^n}, \tag{2.9}$$

   where P_t denotes the bond price at time t, c is the semiannual coupon interest, y_t is the yield to maturity at time t, n is the number of periods (number of years times two), and M is the bond value at maturity.

2. Compute the one-day capital gain from the change in bond price:

$$\text{Capital Gain}_t = ln(P_t) - ln(P_{t-1}). \tag{2.10}$$

3. Compute the one-day accrued interest return by converting the annual yield to a one-day yield:

$$\text{Accrued Interest}_t = \frac{y_t}{252}. \tag{2.11}$$

4. Sum the capital gain and the accrued interest return to calculate the total one-day return of the bond.

5. Construct the artificial return series for the bond, then calculate the RiskGrade as explained in Sections 2.2 and 2.3.

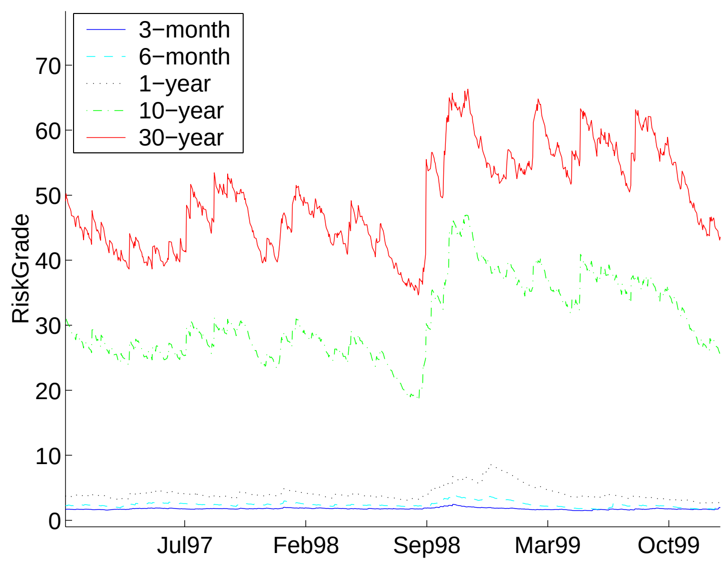
Table 2.3: **RiskGrades of U.S. Treasury Bonds on Dec. 31, 1999**

Maturity	3-month	6-month	1-year	10-year	30-year
RiskGrade	1	2	3	26	43

Table 2.3 shows the RiskGrades for U.S. Treasury bonds of various maturities. The 10- and 30-year Treasury bonds each have an 8% annual coupon. As we expect, the longer-maturity bonds have higher volatilities and thus, higher RiskGrades.

Figure 2.6 plots a three-year history for the RiskGrades of Treasury bonds of various maturities. Like the RiskGrades of equities, the RiskGrades of bonds are time-variant parameters. Although they move in the same direction because of the highly correlated yields, their swing becomes wilder as bond maturity increases.

Figure 2.6: **RiskGrades of U.S. Treasury Bonds**



In the case of options, constructing artificial return series is more complicated than for bonds because we need to consider many factors that affect an option's value. Some of these factors must be fixed at today's values, while other factors must be simulated according to their historical evolution. We use four basic steps to construct return series:

1. Calculate today's value of the option by using today's factors. The value of the option is a function

of the underlying asset price, strike price, time of expiration, volatility of the underlying asset price, risk-free interest rate, and dividend rate as follows:

$$C_t = F(S_t, X, T, \sigma_t, i_t, D), \tag{2.12}$$

where C_t denotes the price of the option at time t, S_t is the underlying asset price at time t, X is the strike price, T is the time of expiration, σ_t is the volatility of the asset price at time t, i_t is the risk-free interest rate at time t for an investment maturing at time T, and D is the dividend rate.

2. Compute the artificial price based on the historical price changes of the underlying asset and the risk-free interest rate. We fix the strike price, time of expiration, volatility of underlying asset price, and dividend rate at today's values:

$$C_{t-i} = F(S_t(1 + rs_{t-i}), X, T, \sigma_t, i_t(1 + ri_{t-i}), D), \tag{2.13}$$

where rs_{t-i} denotes the return of the underlying asset at time $t - i$, and ri_{t-1} denotes the change in the risk-free interest rate at time $t - i$.

3. Calculate the artificial historical return rc_{t-i} of the option by subtracting the option's value based on today's price from the option's value based on its historical price change:

$$rc_{t-i} = ln(C_{t-i}) - ln(C_t). \tag{2.14}$$

4. Construct the artificial return series for the option, then calculate the option's RiskGrade in the same manner as for equities, as explained in Sections 2.2 and 2.3.

Our method for handling options in a portfolio is a hybrid of historical simulation and parametric (delta-gamma) methods; i.e., the artificial return series of an option is calculated by valuing option prices based on the historical change in the values of the risk factors, but it is aggregated into the portfolio according to the parametric method. The hybrid method has the advantage of being able to incorporate the non-linear valuation of options while allowing the portfolio RiskGrade to be calculated from the parametric method of variance-covariance using the exponentially-weighted moving-average scheme.

Table 2.4 shows the RiskGrades of two portfolios composed of Cisco options. Each portfolio contains one unit of Cisco equity. The first portfolio, however, contains one unit of an at-the-money call option, while the other portfolio contains one unit of an at-the-money put option. As we expect, Cisco equity and the call option are highly correlated and the diversification benefit is relatively small. By contrast, the put option

Table 2.4: **RiskGrade and RiskImpact Values of Options on Dec. 31, 1999**

Portfolio	Asset	Price, (USD)	RiskGrade	RiskImpact, (%)
Call Option	Cisco	107 1/8	179	71
	Cisco Call Option	18 1/8	665	29
	Div. Benefit	...	22	...
	Portfolio	125 1/4	227	...
Put Option	Cisco	107 1/8	179	41
	Cisco Put Option	11 3/4	600	-58
	Div. Benefit	...	119	...
	Portfolio	118 7/8	102	...

efficiently hedges the investment. Its diversification benefit is large and its RiskImpact is negative, which means that if you close your put option position, your total portfolio risk will increase significantly.

Figure 2.7 plots a three-year history of the RiskGrades of various investments in Cisco: equity, a call option, a put option, and two option portfolios. The RiskGrades let you easily determine how the call option leverages your investment and results in higher risk, and how the put option hedges your investment and results in smaller risk.

2.9 RiskGrades™ of Leveraged Portfolios

Before concluding the discussion of RiskGrades, we add one brief section to explain how well RiskGrades measure the risk of portfolios leveraged by margin debt and short sale. [3]

How does leverage (that is, using borrowed money) affect the risk of your portfolio? Intuitively, we know that buying securities on margin debt (i.e., borrowing money to buy securities) or short sale (i.e., selling securities without holding them) increases the risk and return potential of our portfolios. Using RiskGrades, we can show how to quantify the effects of different leverage strategies.

The upper half of Table 2.5 shows two leveraged portfolios by margin debt. In this example, we compare portfolios that contain the same value of stock, but have different net values with respect to margin debt. Assume that the first portfolio (Portfolio 1) containing USD 10,000 of Coca-Cola stock has been purchased entirely in cash, whereas the second portfolio (Portfolio 2, upper center of table) has been purchased with a combination of 50% cash and 50% margin debt. The third portfolio (Portfolio 3, upper right of table) was purchased with a combination of 1% cash and 99% margin debt. [4]

[3] Part of this section was written by Alvin Lee of RiskMetrics.

Figure 2.7: **RiskGrade Values of Options**

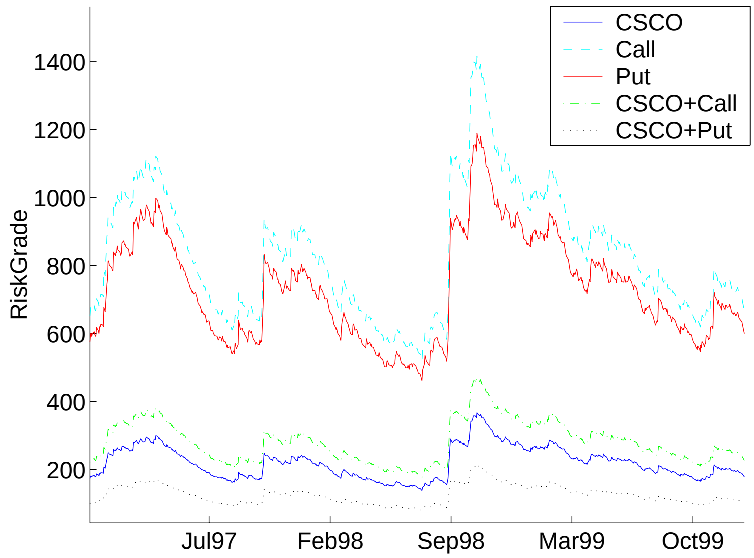


Table 2.5: **RiskGrades of Margin Debt and Short Sale on Dec. 31, 1999**

Asset	MV, (USD)	RG	RI, (%)	MV, (USD)	RG	RI, (%)	MV, (USD)	RG	RI, (%)
	Cash Purchase (Portfolio 1)			50% Margin Debt (Portfolio 2)			99% Margin Debt (Portfolio 3)		
Coca-Cola	10,000	188	100	10,000	188	100	10,000	188	100
Cash	0	0	...	-5,000	0	0	-9,900	0	0
Div. Benefit	0	...	...	...	0.00	...	...	0	...
Portfolio	10,000	188	...	5,000	376	...	100	18808	...
	Cash Purchase (Portfolio 1)			50% Short Sale (Portfolio 4)			99% Short Sale (Portfolio 5)		
Coca-Cola	10,000	188	100	10,000	188	58	10,000	188	33
Cisco	0	...	...	-5,000	179	12	-9,900	179	30
Div. Benefit	0			...	127	...	...	9759	...
Portfolio	10,000	188	...	5,000	428	...	100	26741	...

[4] In practice, 99% margin debt and 99% short sale are extreme cases since brokerage companies usually do not allow individual investors more than 65–75% margin debt or short sale without additional collateral.

Portfolio 1 contains USD 10,000 of Coca-Cola stock and zero cash. The portfolio's net market value is USD 10,000, and its RiskGrade of 188 is the same as Coca-Cola's RiskGrade. Portfolio 2 contains USD 10,000 of Coca-Cola shares and USD -5,000 of cash or margin debt, resulting in a net market value of USD 5,000 and a RiskGrade of 376. In Portfolio 3, USD 10,000 worth of Coca-Cola stock and USD -9,900 of cash or margin debt result in a net market value of USD 100 and a RiskGrade of 18808, or 3761% annual volatility.

Is the third portfolio as risky as its RiskGrade of 18808 indicates? Note that for leveraged portfolios, the RiskGrade measures the risk of the net value of the portfolio, not the value of the securities in the portfolio. While all three portfolios have the same amount of securities outflow, the net investment value of Portfolio 2 is only half that of the non-leveraged portfolio; for Portfolio 3, it is only one one-hundredth of the value of the non-leveraged portfolio. Therefore, a return calculated from the net investment value swings more in the case of leveraged portfolios.

In general, the RiskGrade of a leveraged portfolio is calculated as follows:

$$\begin{aligned}\text{Leveraged RiskGrade} &= \text{RiskGrade of Securities} \times \text{Leverage Ratio} \\ &= \text{RiskGrade of Securities} \times \left[\frac{\text{Securities Value}}{\text{(Securities Value - Margin Debt)}}\right].\end{aligned} \tag{2.15}$$

The lower half of Table 2.5 shows two leveraged portfolios by short sale. In this example, we compare portfolios that contain the same value of stock, but differ in net value because of the short sale. Assume Portfolio 1, (containing USD 10,000 of Coca-Cola stock) has been purchased entirely with cash, whereas Portfolio 4 (lower center of table) has been purchased with a combination of 50% cash and 50% short sale, and Portfolio 5 (lower right of table) with a combination of 1% cash and 99% short sale.

While the increase in RiskGrade from short sale is similar to the increase from margin debt, there is no simple equation, such as Equation 2.15, for calculating the RiskGrade of a leveraged portfolio by short sale. The reason is that the diversification benefit depends on the correlation between long and short securities.

This section shows how leverage increases the RiskGrade (and potential return) of a portfolio for a given investment of securities. As with most things in life, additional potential returns come with additional risk. Also worth noting is the fact that investors who make use of margin debt or short sales must contend not only with increased risk, but also with margin calls. Margin calls occur when the institution that extends the margin debt and short sale requires the investor to either deposit more cash into his account or sell securities to reduce the leverage in the portfolio. Investors may calculate the probability of a margin call by using another risk measure called "Chance of Losing Money," as explained in Section 3.2. As investors replace the initial investment value with the trigger value of a margin call, the Chance of Losing Money shows the probability of the current portfolio value shrinking under the trigger value of a margin call.

Chapter 3

Additional Risk Measures

The RiskGrade™ statistic is a good measure of risk. It is a representative and universal scaler of risk for all types of investment instruments. However, for specific situations, such as a relatively longer investment horizon or abnormal market conditions, we need other measurements in order to draw the whole picture of risk. For this purpose, we have devised several statistics to supplement RiskGrade: XLoss™ , Marginal XLoss, Chance of Losing Money, Worst-Case Performance, and Worst Losing Streak.

3.1 XLoss™

We define the XLoss value as the one-day expected loss that exceeds the loss at the fifth percentile (or a user-defined threshold) of the Profit and Loss (P&L) distribution. Intuitively, one can think of one-day XLoss as the average of the worst-case, one-day losses observed in each month (i.e., once among twenty days). We can also think of XLoss as an indicator of the average loss we would experience in an extreme scenario.

We can calculate XLoss as

$$\text{XLoss} = \text{E}[\text{P\&L}|\text{P\&L} < z] = \frac{\text{Portfolio Value}}{0.05} \int_{-\infty}^{z} x f(x) dx, \tag{3.1}$$

where

$$z : \int_{-\infty}^{z} f(x) dx = 0.05, \tag{3.2}$$

and $f(\cdot)$ is the density function of returns. The next step is to determine the density function either from the parametric method or from historical simulation.

3.1.1 Parametric Method versus Historical Simulation

The logic of when to use the parametric method and when to use historical simulation depends on the characteristics of the risk measurement.

The parametric method can be easily scaled up to longer horizons by use of the square root of time rule, because we assume the return process to be a random walk. We therefore require a small amount of data. The parametric method, however, cannot account for the fat tail of the return distribution, and consequently, should be used for short or long horizons, and for normal market conditions.

On the other hand, historical simulation can incorporate the fat tails of the return distribution, but cannot be scaled up to longer horizons because we do not assume the random walk process. Therefore, it has a heavy data burden for long-horizon calculations. For example, to calculate a quarterly return distribution, we need more than 100 historical quarterly returns (i.e., 25 years of returns). Furthermore, we cannot use overlapping data to reduce the data burden because the overlap leads to false autocorrelations. For example, if on one day the return suffers a big drop, all consecutive returns which include that day show an equally big drop.[1] Thus, historical simulation is better suited for short horizons and abnormal market conditions.

Since XLoss is a risk measure for the abnormal market condition, it concerns only the worst-case returns below a certain threshold (e.g., worst first or fifth percentile). Therefore, the computation of XLoss heavily depends on the left tail of the return distribution. Figure 3.1 plots the Coca-Cola's return distributions calculated from the parametric method and from historical simulation. Clearly, the return distribution based on the parametric method cannot account for the fat tail of historical returns. Consequently, the XLoss values calculated from the parametric method may underestimate actual risk.

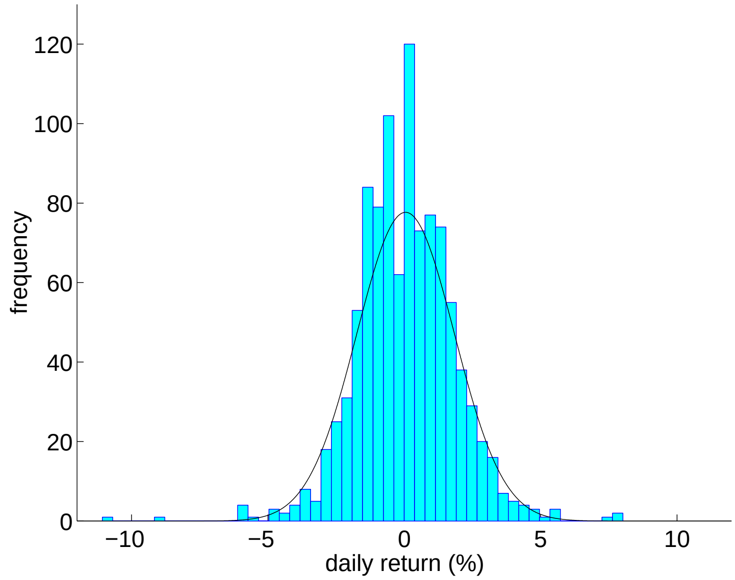
Using the previous example, let us compare the XLoss values obtained from the parametric method and from historical simulation. Table 3.1 shows the XLoss values at the first and fifth percentile thresholds.

At the fifth percentile threshold, the parametric XLoss underestimates loss by less than 10%, whereas at the first percentile threshold it underestimates loss by a significant amount. Therefore, our choice between the parametric method and historical simulation involves a trade-off. While the historical simulation for XLoss may account for the fat tails of the density function, it is unwieldy for longer-horizon measurements. The opposite is true for the parametric method. Since the accuracy of the XLoss measure depends heavily on the left tail of the return distribution and we have other good risk measures (Worst-Case Performance and Worst Losing Streak) for long-horizon abnormal markets, we use historical simulation to calculate XLoss and restrict the time horizon to span from one to five days.

We also need to determine the range of historical data and the method for calculating a portfolio's XLoss. The choice of the range of historical data depends on the XLoss threshold. If the threshold is the fifth percentile, one year of daily data (i.e., 252 observations) is enough to describe an abnormal market condition (i.e., 13 observations). If the threshold is the first percentile, one year of daily data is not enough and at least five years of daily data is required to describe an abnormal market condition.

[1] See the *LongRun Technical Document* [5] pp. 125–126 for more information about the problem of overlapping data.

Figure 3.1: **Return Distributions Based on the Parametric Method and Historical Simulation**



Simulation

Table 3.1: **XLoss Values Based on the Parametric Method and Historical Simulation**

Asset	Parametric (A, USD)	Historical (B, USD)	A/B, (%)
1st percentile:			
Coca-Cola	-472	-641	74
Cisco	-664	-918	72
Div. Benefit	-252	-416	...
Portfolio	-884	-1,144	77
5th percentile:			
Coca-Cola	-353	-382	92
Cisco	-532	-588	90
Div. Benefit	-182	-216	...
Portfolio	-704	-754	93

For the portfolio approach, we use the Portfolio Aggregation method to construct an artificial historical return series for the portfolio based on the current weights and historical returns of the portfolio components.

Zangari [8] summarizes Portfolio Aggregation in three basic steps:

1. Construct a time series of daily portfolio returns from a current set of portfolio positions and daily returns on individual securities.
2. Treat the portfolio return time series as a dynamic process.
3. Determine risk measures by fitting a statistical model directly to the time series of daily portfolio returns.

The critical problem of Portfolio Aggregation is the heavy computational resources that are required to calculate the daily P&L of nonlinear instruments in the portfolio. However, the problem is not serious for individual investors, as their portfolios generally contain small numbers of nonlinear instruments.

For more information about the Portfolio Aggregation method compared with the variance-covariance parametric approach and historical simulation, refer to Zangari [8].

The diversification benefit in Table 3.1 shows the reduction in the portfolio's XLoss compared to the sum of the XLoss values of the portfolio components.

$$\text{XLoss Div. Benefit} = \sum_{i=1}^{N} \omega_i \text{XLoss of asset } (i) - \text{XLoss of the portfolio}, \tag{3.3}$$

where N denotes the total number of assets in the portfolio and ω_i denotes the weight of asset i.

3.1.2 Marginal XLoss

Similar to RiskImpact™ indicating the contribution of an asset's RiskGrade to the RiskGrade of its portfolio, Marginal XLoss shows the contribution of each asset in the portfolio to the total XLoss of the portfolio. The Marginal XLoss for a specific asset reflects how the portfolio's XLoss would change if the investor were to sell that asset and keep the cash proceeds. We can also define Marginal XLoss as

$$\text{Marginal XLoss } (i) = \text{XLoss of the entire portfolio} - \text{XLoss of the portfolio without asset } (i). \tag{3.4}$$

We can also report the Marginal XLoss as a percentage of the XLoss of the entire portfolio:

$$\% \text{ Marginal XLoss } (i) = \frac{\text{Marginal XLoss } (i)}{\text{XLoss of the entire portfolio}} \times 100. \tag{3.5}$$

In the previous example, the Marginal XLoss values for the Coca-Cola and Cisco positions are 21.99% and 49.36%, respectively, which means that if the investor closes her Coca-Cola position, she can reduce her portfolio's XLoss by 21.99%. As long as the diversification benefit exists, it reduces the risk of the portfolio such that the sum of the Marginal XLoss values contributed by each of the portfolio components is always less than 100%.

3.2 Chance of Losing Money

The Chance of Losing Money shows the probability with which the value of an investment after a 1-, 3-, or 12-month horizon (or a user-defined investment horizon) can drop below its initial value (or a user-defined level). The method for computing the Chance of Losing Money runs opposite to the method for computing XLoss. While XLoss indicates the average shortfall that corresponds to a certain level of probability, the Chance of Losing Money provides the probability with which the P&L of an investment can fall below a certain level of shortfall.

We can calculate the Chance of Losing Money as

$$\text{Chance of Losing Money} = \text{Prob}[\text{P\&L}_{t,k} < z] = \int_{-\infty}^{z} f(x_{t,k})dx, \tag{3.6}$$

where $\text{P\&L}_{t,k} = x_{t,k}$ denotes the k-day P&L from time t and $f(\cdot)$ is the P&L density function. Parameter z is the threshold of the shortfall.

For short horizons, the Chance of Losing Money is a meaningless measure, given that the overnight Chance of Losing Money is always 50% when a zero expected return is assumed. Therefore, the Chance of Losing Money should be used as a risk measure for long-horizon investments. At long horizons, we avoid historical simulation, as it requires large amounts of data, and instead, use the parametric method to determine the density function of the returns.

Specifically, we assume that the k-day return follows a normal distribution, along with the k-day expected return and volatility, which are scaled up from the one-day expected return and volatility; i.e.,

$$r_{t,k} \sim N(k\mu_{t,1}, k\sigma^2_{t,1}), \tag{3.7}$$

where $r_{t,k}$ denotes the k-day return from time t. Parameters $\mu_{t,1}$ and $\sigma_{t,1}$ denote the one-day expected return and volatility, and $N(\cdot,\cdot)$ denotes a normal distribution.

Then, Equation 3.6 for the Chance of Losing Money expressed in terms of P&L can be rewritten in terms of returns as follows:

$$\text{Chance of Losing Money} = \text{Prob}[r_{t,k} < r_z] = \int_{-\infty}^{r_z} \Phi(r_{t,k})dr, \tag{3.8}$$

where r_z denotes the return that falls above the shortfall threshold (z), and ϕ denotes the normal probability density function (PDF) described in Equation 3.7.

The volatility parameter ($\sigma_{t,1}$) is computed from an asset's historical returns, but its calculation is somewhat different from that of RiskGrade. Since RiskGrade measures risk at relatively short horizons, it should be time-variant, following the recent behavior of the return. Thus, we put a higher weight on the more recent returns (i.e., the decay factor is less than one), which reduces our requirement for data (i.e., 151 days for a decay factor of 0.97). However, since the Chance of Losing Money is valid for the relatively long horizon, it should represent the long-term behavior of the return. Therefore, we assign the same weight to all historical returns and use a long history of returns — as long as five years.

While RiskGrade assumes the expected return parameter ($\mu_{t,1}$) to be zero because of the short horizon, the Chance of Losing Money no longer assumes a zero expected return. In addition, while RiskMetrics provides robust forecasts of volatility, no current methodology can provide robust forecasts of an expected return. To circumvent this problem, we take the expected return to be an average return that we compute from a long history of data. It is important to bear in mind that the average historical return is only a reference value, and the individual investor still needs to define her expected return.

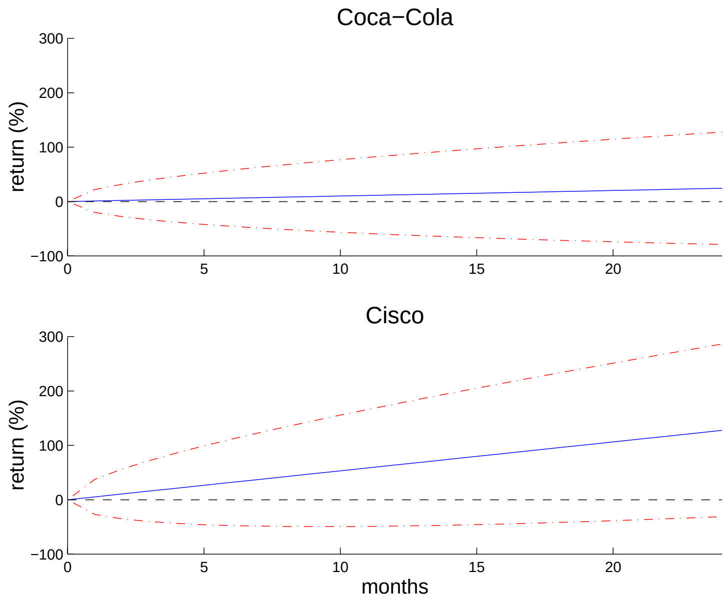
The Chance of Losing Money decreases as the expected return rises and the volatility drops. Using the Coca-Cola and Cisco example, Figure 3.2 shows how the expected return (or long-term trend) affects the Chance of Losing Money.

Each graph in Figure 3.2 shows three curves that plot the mean expected return and the 99% confidence intervals. The area below the zero-return line (horizontal dash line) indicates the Chance of Losing Money based on today's investment. At the one-year horizon, the Chance of Losing Money from the investment in Cisco is less than from the investment in Coca-Cola, i.e., 7.20% compared to 33.41%. While Cisco's annual volatility (43.65%) is greater than Coca-Cola's (28.42%), its annual expected return (63.78%) is much greater than Coca-Cola's (12.18%).

3.3 Worst-Case Performance

So far, we have established that XLoss is a risk measure for the short-horizon abnormal market, and the Chance of Losing Money is for the long-horizon normal market. How, then, do we measure risk for the

Figure 3.2: **Chance of Losing Money**



long-horizon abnormal market condition? The long-horizon abnormal market is the Achilles Heel of risk management methodology. Any kind of precise approach for determining the tail distribution of long-term returns requires tremendous amounts of data.

In this section and the next, we introduce two empirical types of risk measures for the long-horizon abnormal market. While empirical measures are not based on objective statistical inference, they still provide practical information about risk.

For example, if an investor is concerned that a 5% worst return during a particular quarter represents her losses, she can instead determine her potential losses from the Worst-Case Performance measure for any quarter during the last five years. Since there is a total of 20 quarters in the last five years, she can expect to experience the same Worst-Case Performance once in every 20 quarters.

Worst-Case Performance is the amount of loss (base currency or percent) that can occur if the worst 3-, 6-, or 12-month horizons (or a user-defined horizon) during a given historical period were to occur again. To obtain a Worst-Case Performance figure, we search for the worst k-period return in a given set of historical daily data by using a fixed-size, rolling, daily-updated window, and then apply the worst return to today's portfolio. We can calculate the k-period Worst-Case Performance as

$$\text{Worst-Case Performance} = \text{Min}\,[r_{t-i,k}, r_{t-(i-1),k}, \ldots, r_{t-(k+1),k}, r_{t-k,k}], \tag{3.9}$$

where, Min denotes the minimum operator, and i denotes the total number of daily historical data points.

The method for calculating the k-period return $r_{t,k}$ for the Worst-Case Performance measure differs from the methods used to calculate the k-period return $r_{t,k}$ for other risk measures. (The same is true for calculating the k-period return for the Worst Losing Streak measure, i.e., the amount of loss that would occur if the largest drop in asset price, from peak to trough, were to occur again. See Section 3.4 for a detailed discussion.) For the other risk measures, we use a logarithmic return, computed from $\ln(P_{t+k}/P_t)$, since it is additive and easily scaled up to longer terms. The logarithmic return is similar to the absolute return $(P_{t+k} - P_t)/P_t)$ for small changes in the return. For large changes, however, the two methods provide very different results. Specifically, if an asset price suffers a big drop, the logarithmic return overestimates the actual drop. For example, if a stock price drops from 100 to 50, the absolute return is -50%, while the logarithmic return is -69.31%. Since Worst-Case Performance (and Worst Losing Streak) are typically used to report large drops in asset price, we use the absolute return to avoid the overestimation that is inherent to the logarithmic return.

Using the Coca-Cola and Cisco assets from our example, we show in Table 3.2 their Worst-Case Performance figures for a single year (252 business days) in the last five years.

Table 3.2: **One-Year Worst-Case Performance**

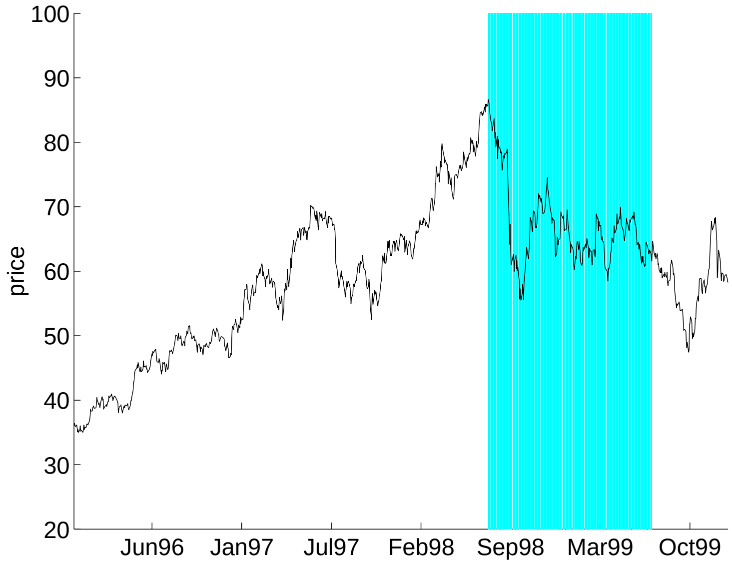
	Worst-Case	Starting		Ending	
Asset	Performance, (%)	date	price	date	price
Coca-Cola	-28.97	14-Jul-1998	86.65	14-Jul-1999	61.55
Cisco	-11.46	26-Apr-1996	11.64	25-Apr-1997	10.31
Portfolio	4.75	07-Oct-1997	40.32	07-Oct-1998	42.24

In Coca-Cola's case, the worst one year started on July 14, 1998 at USD 86.65 and ended on July 14, 1999 at USD 61.55, which is marked by the dark area in Figure 3.4. The worst one year was determined by using a fixed-size 252-day window, rolling and daily updated. The worst one-year return was calculated from the absolute return to be -28.97%. The Worst-Case Performance for the portfolio was computed from the Portfolio Aggregation method explained in Section 3.1.

3.4 Worst Losing Streak

The Worst Losing Streak is another empirical type of risk measure for the long-horizon abnormal market. We define it as the amount of loss (base currency or percent) that would occur if the largest drop in asset price, from peak to trough, during the last five years were to occur again. The definition is similar to that of Worst-Case Performance, except that no predetermined time horizon is required. Therefore, the worst period is selected not from fixed-period windows, but from flexible peak-to-trough windows.

Figure 3.3: **One-Year Worst-Case Performance, Coca-Cola**



We can express the Worst Losing Streak as

$$\text{Worst Losing Streak} = \text{Min}\,[\quad \begin{aligned} &\text{Min}\,[r_{t-i,1}, r_{t-i,2}, \ldots, r_{t-i,i-1}, r_{t-i,i}], \\ &\text{Min}\,[r_{t-(i-1),1}, r_{t-(i-1),2}, \ldots, r_{t-(i-1),i-2}, r_{t-(i-1),i-1}], \\ &\ldots, \\ &\text{Min}\,[r_{t-2,1}, r_{t-2,2}], \\ &\text{Min}\,[r_{t-1,1}]\ \ ], \end{aligned} \tag{3.10}$$

where Min denotes the minimum operator, and i denotes the total number of daily historical data points.

When we search for the largest drop from peak to trough, we ignore the local peaks and troughs. Figure 3.4 plots Coca-Cola's Worst Losing Streak, which starts on July 14, 1998 at USD 86.65 and ends on October 4, 1999 at USD 47.44. Within this period are several local peaks and troughs but they are ignored, as the difference in the returns on July 14, 1998 and October 4, 1999 is the largest among them.

Table 3.3 shows the Worst Losing Streak during the last five years for our Coca-Cola and Cisco example. We calculate returns by using the absolute method and apply the Portfolio Aggregation method to the portfolio. It is worth noting that for the same historical sample, the Worst Losing Streak is always worse than the Worst-Case Performance because it does not have a fixed investment horizon.

Figure 3.4: **Worst Losing Streak, Coca-Cola**

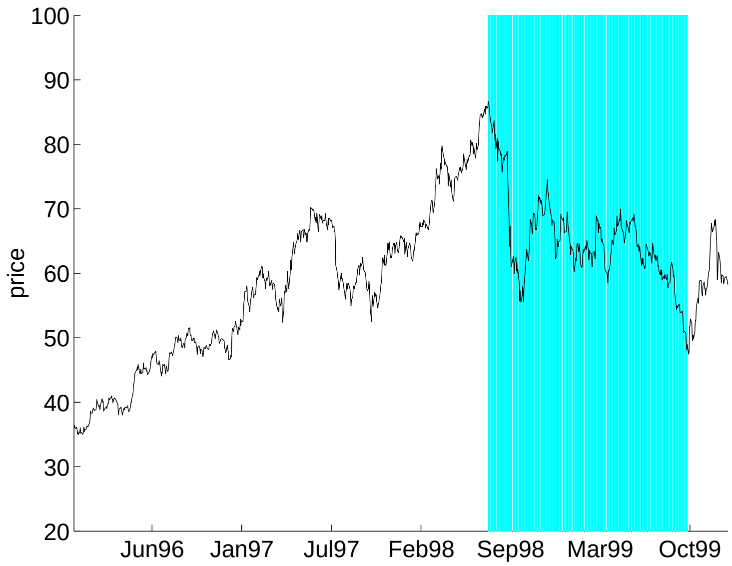


Table 3.3: **Worst Losing Streak**

Asset	Worst Losing Streak, (%)	Starting date	Starting price	Ending date	Ending price
Coca-Cola	-45.24	14-Jul-1998	86.65	04-Oct-1999	47.44
Cisco	-38.06	21-Jan-1997	16.64	25-Apr-1997	10.31
Portfolio	-28.88	15-Jul-1998	59.19	01-Oct-1998	42.09

Chapter 4

Stress Tests

RiskGrades™ and other measures introduced in Chapters 2 and 3 were designed to be intuitive and universal market risk indicators for the individual investor. Their underlying analytics have enabled professional risk managers to reliably measure market risk. As with all risk forecasts, however, RiskGrades and the additional risk measures have the limitation of being based only on historical market data. They are not a crystal ball that can forecast hidden risks, such as events. Observing historical return data gives no clues about when the next major earthquake, political scandal, or war could derail the markets. To capture such event risks, investors should complement statistical analysis with rigorous stress testing.

Stress tests are a common counterpart to the objective models used for day-to-day risk monitoring. Examples of the objective models are VaR and RiskGrades. The objective models typically forecast worst-case losses conditional on markets behaving generally as they have in the recent past. To make accurate forecasts, these models rely on a relatively short (one year at most) history of market factor returns. While certain models extrapolate from these returns and forecast losses greater than those observed in the historical period, the loss forecasts are always restricted by the historical returns. Stress tests are point estimates of portfolio losses based on market factor returns that have never occurred, or that occurred outside the relevant historical period for the model. Stress tests complement the objective model forecasts by providing a notion of losses deemed implausible by the model, but which certainly could occur.

Generically, stress tests involve specifying adverse market moves (scenarios) and revaluing the portfolio under these moves (Laubsch [4]). To specify scenarios, it is first necessary to select the market factors (the core assets) to be stressed, then define the amount by which to stress them and the time period over which the stress move will take place. For the remaining (peripheral) assets, there are a number of methods to specify the moves that would coincide with moves in the core assets.

The simplest specification for peripheral asset moves is to simply assume no change (call this the "zeroed-out" stress test). A second specification (the predictive stress test) utilizes current estimates of volatility and correlation to estimate the conditional expectation of a peripheral asset move, given the stress moves in the core assets (see Kupiec [3] for more details). A third specification (historical stress test) applies the moves in the peripheral assets that have coincided with large moves in the core assets historically. We summarize the three stress test methodologies in Table 4.1. [1] In this document, we discuss two types of stress tests: the User-Defined Event(or Hypothetical Event), which is based on the predictive stress test, and the Historical Event, which is based on the historical stress test.

Table 4.1: **Alternative Stress Tests**

Stress Test	Return of Peripheral Assets	Benefit	Drawback
Zeroed-out	Zero return	Implementation is quite easy	Ignoring co-movement is unrealistic
Predictive	Expected return based on correlation	Idiosyncratic errors are averaged out	Impossible to incorporate correlation breakdown
Historical	Actual return of the specific historical event	The stress condition is easily incorporated	Idiosyncratic errors of the historical event cannot be removed

4.1 User-Defined Events

The User-Defined Event Stress Test shows the amount of loss (base currency or percent) that could occur if the user-defined crisis were to happen again today. The User-Defined Event Stress Test is based on the predictive stress test, which computes the expected returns of peripheral assets in an event by using their historical correlations and volatilities.

Predictive stress tests use the conditional expectation of the returns of the peripheral assets, given that the return of the core asset (e.g., the USD S&P500) falls by a predetermined level (e.g., -30%). We rely on a linear relation between the returns $r_{y,t}$ of the peripheral assets and the returns $r_{x,t}$ of the core assets:

$$\left(\frac{r_{y,t}-\mu_y}{\sigma_y}\right) = \rho\left(\frac{r_{x,t}-\mu_x}{\sigma_x}\right) + \sqrt{(1-\rho^2)}\varepsilon_t, \tag{4.1}$$

where ε_t is a random error term with zero mean and unit variance. The conditional expectation of the peripheral asset's return, given the core asset's return, is then

[1] See Kim and Finger [2] for an in-depth comparison of the stress test methodologies.

$$E(r_{y,t}|r_{x,t}) = \mu_y - \left(\frac{\rho\sigma_y}{\sigma_x}\right)\mu_x + \left(\frac{\rho\sigma_y}{\sigma_x}\right)r_{x,t}. \tag{4.2}$$

Usually, we can assume that the user-defined event happens within a short period of time such that the expected returns of the core and peripheral assets (μ_x and μ_y) are close to zero. Then, Equation 4.2 is simply expressed as

$$E(r_{y,t}|r_{x,t}) = \beta_x r_{x,t}, \tag{4.3}$$

where β_x is $(\rho\sigma_y)/\sigma_x$.

Then, the individual asset price movement during the event is determined only by its β. The larger the β, the larger the drop. We apply the stress test to the previous example in which the S&P 500 drops 30%. Table 4.2 shows the estimated β based on one year of historical returns and the expected return in the event. Cisco's β is three times larger than Coca-Cola's, so that loss from investments in Cisco is three times greater than the loss from investments in Coca-Cola.

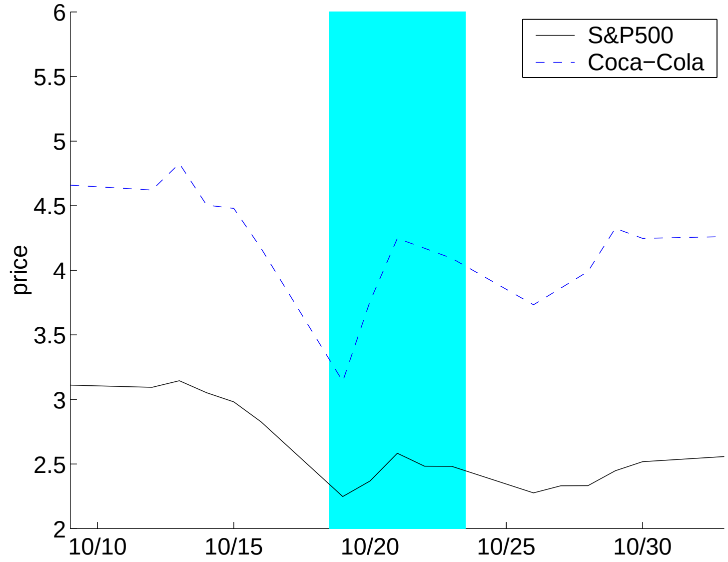
Table 4.2: **Stress Test for 30% Drop of S&P 500**

Asset	β	Expected Return, (%)
Coca-Cola	0.56	-16.92
Cisco	1.64	-49.19
Portfolio	1.10	-33.05

4.2 Historical Events

The Historical Event Stress Test shows the amount of loss (base currency or percent) that could occur if a specific historical crisis event were to occur again. Examples of historical events are the 1987 Stock Market Crash (October 19, 1987), Gulf War Crisis (January 16, 1991), Mexican Peso Crisis (December 14, 1994), Asian Crisis (July 2, 1997), Russian Crisis (August 21, 1998), and the Brazilian Crisis (January 13, 1999). The Historical Event Stress Test can eliminate many of the subjective assumptions by specifying all of the asset price movements based on the actual events. Each historical event, however, has many idiosyncratic properties which are not averaged out.

Figure 4.1: **Black Monday, 1987**



It is not an easy process to decide upon the calendar dates of an historical event. For example, the 1987 Stock Market Crash happened on October 19, 1987, the so-called Black Monday. However, the U.S. stock market started sliding on the previous Friday of October 16, 1987. As we can see in Figure 4.1, the S&P 500 dropped about 5% on that Friday. Therefore, there are some controversies about whether the crash started on Monday or on Friday. The end of the crash is just as unclear, because within one week the stock market recovered almost half of the drop it experienced on Black Monday. While the convention is five business days (one week) or one month after the event, the period of a historical crisis should be determined event by event.

Table 4.3 shows the result of an Historical Event Stress Test for the preceding example of the stock market

Table 4.3: **Stress Test for the 1987 Stock Market Crash**

Asset	Oct. 16–Oct. 19	Oct. 19	Oct. 19–Oct. 23	Oct. 19–Nov. 18
S&P 500	-28.20	-22.90	-13.01	-14.09
Coca-Cola	-35.51	-28.36	-1.87	-3.45
Cisco	-46.35	-37.65	-21.38	-23.16
Portfolio	-40.93	-33.00	-11.63	-13.31

crash. One problem is the lack of historical returns for that event. Cisco started trading in Nasdaq only on March 26, 1990, so that no historical returns exist for the stock market crash of 1987. In that case, we calculate the expected return by using the predictive stress test. For example, the S& P 500 dropped -13.01% during the five-day period from October 19 to October 23, 1987. Given that Cisco's β is 1.64, the expected loss from holding Cisco shares in the crash is -21.38% ($= 1.64 \times (-13.01)$).

Chapter 5

Risk-Return Profile

5.1 ReturnGrades

We receive numerous requests about forecasting returns. Generally, most want to know if we can forecast returns consistently better than a "random walk in the markets". Unfortunately, modern portfolio theory does not offer a robust methodology to forecast returns. Accordingly, we point to a natural benchmark or reference return - historic performance. Although a historical return is not necessarily a good forecast of future returns, it does provide an acceptable starting point. As a benchmark, we suggest using an annualized, longer-term historical return.

How much history should be included in computing a benchmark return? A trade-off exists between longer and shorter historic time series. Longer time series can remove abnormal price movements evidenced in the short run. Additionally, longer history can provide a better picture of larger scale trends. However, longer history has a higher chance of including structural changes. Structural changes can make past performance even less relevant to valuations moving forward. As such, arriving at an appropriate historic return is a case-by-case decision. A general rule of thumb is to match the length of historic time series to the future investment horizon. For short-term investments, we suggest a time series that covers more than two years but less than five years, which will reflect recent market dynamics. For long-term investments, the historic time series should cover more than one business cycle to include both a bull and bear market period.

With this in mind, we provide the following computation. The annualized n-day historical return of asset i is computed from

$$r_i = \left[1 + \left(\frac{P_{i,t} - P_{i,t-n}}{P_{i,t-n}}\right)\right]^{252/n} - 1, \tag{5.1}$$

where $P_{i,t}$ and $P_{i,t-n}$ denote today's asset price and the price n days ago.

It is worth noting that instead of the logarithmic return we use the absolute return, due to the fact that for periods exceeding a year price movements can be very large. Keep in mind that because our benchmark return is an ex-post observation, it sometimes shows extreme positive values; at other times, extreme negative values will be seen.

Investors should always consider both risk and return prospects before making any investment decision. A risk-return performance index that combines both dimensions of risk and return and translates into a single measure would prove extremely useful. However, building a risk-return performance index is an arbitrary matter, as it needs to make assumptions about each individual's appetite for risk. This is too subjective to provide any wide-scale utility.

Two popular approaches combine risk-return performance in a single coefficient - i) Sharpe ratio and ii) risk-adjusted return. Institutional and individual investors have used the former, the ratio of excess return to the underlying standard deviation, since the 1970's. The risk-adjusted return approach, on the other hand, has grown in popularity since the mass-scale application of VaR (Value at Risk) for capital requirements was introduced, and is now used widely by financial companies.

Our risk-return performance index, ReturnGrades, is founded on the principles of the latter, risk-adjusted returns. We do this for two reasons. A risk-adjusted return is typically presented as an annualized return, i.e. how much is earned after one year if one dollar is invested today. The Sharpe ratio, by contrast, is in ratio form and represents the return to standard deviation. For most investors, the concept of a return is more intuitive than a ratio. Furthermore, a risk-adjusted return can incorporate various degrees of risk aversion by simply adjusting the level of risk which is measured by VaR - a Sharpe ratio cannot.

Generally, the risk adjusted return for a financial company is defined as

$$\text{Risk Adjusted Return for asset i} = \frac{r_i - r_b + r_f \times VaR_i}{VaR_i}, \tag{5.2}$$

where r_b is a borrowing rate of the financial company and VaR_i denotes VaR of asset i. Both are average of last n days.

A financial company does not need to internally finance its investment. It can borrow one dollar at a cost of r_b and invest it to return r_i. Given this, the company earns $(r_i - r_b)$ from the risky investment. In addition to every initial dollar of risky investment, a financial company puts aside VaR_i of its own capital as a reserve or "buffer" against the risky investment. If the company applies a 99% level of confidence as buffer capital, it can cover losses up to the worst 1% case. In this vein, the level of VaR represents the degree of risk aversion of the company. A higher level of VaR denotes a higher degree of risk aversion. Generally, a company invests the buffer capital in risk free assets and obtains $r_f \times VaR_i$ on investments. In summary, the company invests VaR_i of its own capital and earns $(r_i - r_b + r_f \times VaR_i)$ after one year. Thus, the risk-adjusted return for the financial company is the return on its risky investments plus interest earned on reserves, which insures against losses from risky investments with a certain level of confidence. [1]

To make this concept of risk-adjusted return applicable to an individual investor, we have made two adjustments. It is impractical to think that individual investors have access to an unlimited pool of capital. Accordingly, the initial amount of investment is not VaR_i but $(1 + VaR_i)$ which includes one dollar of initial investment added to the risky asset. As such, an individual investor earns r_i from the risky investment, which ignores the borrowing cost of r_b. [2] The second thing to consider is that returns on risk free investments are limited for the individual. Moreover, the rate is usually negligible. A typical checking account does not offer significant interest; a cash account held at a brokerage company gives even less interest. Therefore, we ignore the risk free return for capital reserves, $(r_f \times VaR_i)$. [3]

After two adjustments are made to accommodate an individual investor, ReturnGrades, the modified risk-adjusted return for an individual investor, is defined as

$$\text{ReturnGrade for asset i} = \frac{r_i}{1 + VaR_i}. \tag{5.3}$$

Fortunately, the average $(1 - \alpha)$ level of VaR_i is easily calculated from our RiskGrades function.

$$VaR_i = z_\alpha \times 0.0020 \times \frac{1}{n} \sum_{j=0}^{n-1} RG_{t-j,i}, \tag{5.4}$$

where, z_α denotes the one tail entry of the standard normal distribution, i.e., $z_{0.001}$, $z_{0.01}$, and $z_{0.05}$ are 3.10, 2.33, and 1.64, respectively.

Next, we need to decide which level of VaR we will use as a default. As already mentioned, the level of VaR is representative of the investor's degree of risk aversion and is entirely a subjective decision. Table 5.1 illustrates how each level of VaR affects the minimum required return on the investment. For example, for a risky investment with a RiskGrade of 100, a 99.9% VaR requires a 9.7% return from the risky investment to put it on equal footing with a risk free investment. Using ReturnGrades with a 95.0% VaR level only requires an 8.0% return. This means that an investor who uses ReturnGrades with a 99.9% VaR level is more risk averse than an investor who uses ReturnGrades with a 95.0% VaR factor. The higher confidence level requires an additional 1.7% return for a comparable risky investment. Generally, a financial company will use a 99.0% VaR to compute its risk-adjusted return. [4] We make the assumption that an individual investor will likely have a higher degree of risk aversion than financial company and have set the ReturnGrade default level of VaR at 99.9%.

[1] From a theoretical standpoint, it does not matter whether the company actually reserves against the risky investment.

Table 5.1: **ReturnGrades for Alternative Risk Aversion**

RetrunGrades	6.0	6.0	6.0	6.0	6.0	6.0
Risk Free Rate	6.0	6.0	6.0	6.0	6.0	6.0
RiskGrades	0	100	200	300	400	500
Return for 99.9% VaR	6.0	9.7	13.4	17.2	20.9	24.6
Return for 99.0% VaR	6.0	8.8	11.6	14.5	17.3	20.1
Return for 95.0% VaR	6.0	8.0	10.0	11.9	13.9	15.9

Figure 5.1: **Risk-Return Profile of UK Funds**

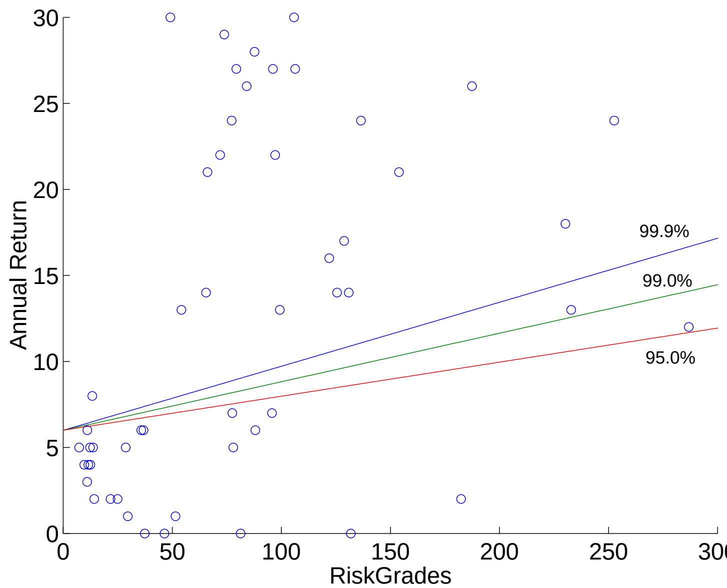


Figure 5.1 plots the actual risk-return profile of UK funds over one year. Three lines represent risk free equivalent ReturnGrades required for a risky asset at 99.9%, 99.5%, and 95.0% internal levels of VaR. The

[2]It is also a good idea to exclude the borrowing cost of r_b from the equation as the borrowing rate is different across individual investors.

[3]Excluding a risk free return is related to the degree of risk aversion. It penalizes riskier investments. If we believe that individuals have a higher degree of risk aversion than financial institutions, the exclusion of the risk free return is validated.

[4]Some suggest using a VaR level related to the bankruptcy probability of a company. If a company desires a BBB credit rating, it should use 98.5% VaR factor since the historical average bankruptcy probability of a BBB rating is 0.15%. Hence, investors should decide on the level of VaR based on individual risk tolerances.

funds that fall on the lines perform on par with risk-free assets. The funds that fall above the lines represent superior performance and the funds that fall below the lines suggest inferior performance over the given time horizon. Furthermore, we can directly rank each return-risk performance with RiskGrades.

Before we finish our discussion of ReturnGrades, it is necessary to point out some limitations. As we explained earlier, because we lack a reasonable expected return computation, almost by default we use a longer-term historical return as the benchmark. If an investor has found a better measure of expected returns, we strongly recommend using it. The return figure in Equation 5.3 is an expected return (ex-ante) and any expected return on a risky asset must be higher than the risk free rate. If the expected return of an asset is less than the risk free rate, an investor should not consider it as a worthy investment opportunity as long as it is not a hedge instrument which has strong negative correlation with other assets of her portfolio. However, because we use historical returns (ex-post), actual returns may be less than the risk free rate, which leads to some strange results – for assets of less than zero return, ReturnGrades are more favorable to higher risky assets. The basic reason for this is that a zero return from the buffer capital should no longer be viewed as a penalty and the zero return of the buffer dilutes the negative return on the underlying asset. Unfortunately, neither modern portfolio theory nor we have a perfect solution to this problem. Furthermore, assets with return prospects less than the risk free rate are not considered as worthy investments and have been excluded from our rankings. This problem is not serious as we can remove the risk free return on buffer capital from the numerator in Equation 5.2. Then, the ranking of assets with returns greater than zero and assets with returns less than zero is not inverted at any time.

5.2 Risk-Return Optimization

5.2.1 Overall Optimization

An investor opens her brokerage account today and deposits $10,000. The investor puts about 30 assets in her basket of candidate investments after considering various financial information and her favorite assets. Her next investment decision is how much of her money to allocate to the candidate assets in order to maximize her portfolio's return and minimize her portfolio's risk.

We can start the optimization problem from a simple setup that maximizes the return of the investor's portfolio, given a predetermined maximum level of the portfolio's RiskGrade™ .

$$\text{Maximize } (\omega) \;\; r_p = \sum_{i=1}^{N} \omega_i r_i, \tag{5.5}$$

Subject to

$$RG_p \leq RG_{max}, \tag{5.6}$$

$$\sum_{i=1}^{N} \omega_i = 1, \tag{5.7}$$

and

$$0 \leq \omega_i \leq \omega_{max} \text{ for } i = 1, 2, ..., N \in C, \tag{5.8}$$

where r_i and ω_i denote the return and weight of asset i, while r_p and RG_p denote the return and RiskGrade of the portfolio, respectively.

The objective function of Equation 5.5 is to maximize the portfolio return with respect to the weight of each asset in the portfolio. Three restrictions apply to the calculation: The first restriction, in Equation 5.6, requires that the portfolio RiskGrade be equal to or less than the maximum level (RG_{max}) predetermined by the investor. The second restriction, in Equation 5.7, requires that the sum of all weights be unity. That is, we do not include a margin debt decision in the optimization problem. The investor should decide on the margin debt amount before solving the optimization problem; to solve the optimization problem, she must multiply the total investment amount (including margin debt) by its weight. The third restriction, in Equation 5.8, requires that we do not allow either short sale (i.e., each ω should be non-negative) or extreme concentration (i.e., each ω must be less than the investor's predetermined maximum level, ω_{max}). Also, all assets must be selected by the investor from the predetermined candidate set (C).

The maximization problem can be easily converted to a minimization problem; i.e., the investor minimizes her portfolio's RiskGrade, given a predetermined minimum level of the portfolio's return. We use the following set of equations:

$$\text{Minimize } (\omega) \;\; RG_p, \tag{5.9}$$

Subject to

$$r_p = \sum_{i=1}^{N} \omega_i r_i \geq r_{p,min}, \tag{5.10}$$

$$\sum_{i=1}^{N} \omega_i = 1, \tag{5.11}$$

and

$$0 \leq \omega_i \leq \omega_{max} \text{ for } i = 1, 2, ..., N \in C. \tag{5.12}$$

In this case, the objective function of Equation 5.9 is to minimize the portfolio RiskGrade with respect to the weight of each asset in the portfolio. The first restriction, in Equation 5.10, shows that the return of the portfolio must be equal to or greater than the investor's predetermined minimum level ($r_{p,min}$). The second and third restrictions, in Equation 5.11 and Equation 5.12 are identical to the restrictions of the maximization problem.

5.2.2 Incremental Optimization

After opening her $10,000 account, the investor wants to increase her position by $1,000. Because of transaction costs, she decides to keep her current position ($10,000 current market value) and add only one other asset from her basket of candidate investments. To solve the incremental optimization problem, [5] we impose the following two additional restrictions on both the maximization and the minimization problems:

$$\omega_i = \omega_{i,cr} \text{ for } i = 1, 2, ..., N_{cr} \in C_{cr} \tag{5.13}$$

and

$$\omega_i = 0 \text{ or } 1 - \sum_{i=1}^{N_{cr}} \omega_i \text{ for } i = N_{cr} + 1, N_{cr} + 1, ..., N \in C_{ft}, \tag{5.14}$$

where C_{cr} denotes the assets of the current position, and C_{ft} denotes the candidate assets for future investment.

The first restriction maintains the positions of the current portfolio and assigns them to the new portfolio. The weight $\omega_{i,cr}$ of the current position is calculated by dividing the current value of the asset by the new

[5] Be aware that we cannot guarantee the solution of the incremental optimization to match the solution of the overall optimization problem, which starts from zero position for all assets.

total portfolio value. For example, if the investor holds USD 2,000 worth of Cisco shares, then Cisco's current weight is 0.20 ($= 2,000/10,000$). In the new portfolio, Cisco's weight $\omega_{i,cr}$ should be reduced to 0.18 ($= 2,000/11,000$). The second restriction stipulates that all further investment be placed in the one additional asset from the candidate basket. In the example, the weight of the additional asset should be 0.09 ($= 1,000/11,000$).

Now let us consider the opposite case of incremental optimization. After her initial investment, the investor wants to close $1,000 of positions from her brokerage account ($10,000 current market value). Because of transaction costs, the investor wants to adjust only a small number of current positions. Unfortunately, it is invalid to solve the problem by using the optimization framework explained in this section. Instead, we must use a pragmatic "what-if" approach. The investor closes some current positions, then calculates the return and the RiskGrade of the shrunken portfolio. The RiskImpact™ measure can also be a good guide for selecting which asset to close, because it shows the change in the RiskGrade of the total portfolio when one of the assets is closed. The investor can then choose the best strategy for a closing that will bring a higher return and a lower RiskGrade.

5.2.3 Optimization Inputs

Risk-return optimization requires two inputs: (1) the expected returns (r_i) of all the assets an investor considers for calculating the portfolio return. (2) the variance-covariance matrix of the candidate basket for calculating the portfolio RiskGrade. We have already explained in Section 2.3 how to calculate variance-covariance matrix and how robust it is as a forecast of future volatility. As an estimate for the expected return, we suggest using an annualized long historical return. Previously, in section 5.1, we demonstrated how to calculate historical return. Since our estimated return is an ex-post observation, it sometimes shows positive extreme values and at other times, negative extreme values. Under these conditions, the optimization provides a corner solution. It always concentrates on the asset that provides a positive extreme return, and it always excludes the asset that provides a negative extreme return. To solve the optimization problem in a reasonable manner, we fix a ceiling and a base for the expected return at 50% and 0%, respectively. [6] The historical return based approach is our suggestion for the estimation of the expected return. Yet, the investor may replace this benchmark return with her own forecast.

To illustrate the optimization procedure we use the following example. we choose nine stocks, place them in the candidate asset set and try to find the optimal weights. The nine stocks are CITIGROUP (NYSE:C), NEWS CORP LTD (NYSE:NWS), MOTOROLA INC (NYSE:MOT), AOL TIME WARNER (NYSE:AOL), APPLE COMP INC (Nasdaq:AAPL), FOUR SEASONS (NYSE:FS), SAKS INC (NYSE:SKS), DONNA KARAN (NYSE:DK), TRANS WORLD AIR (AMEX:TWA). Their historical returns and RiskGrades are shown in the second and forth column of Table 5.2 which are calculated by data from October 1, 1998 to December 14, 2000. Following our rule of setting expected return from historical return, we set expected

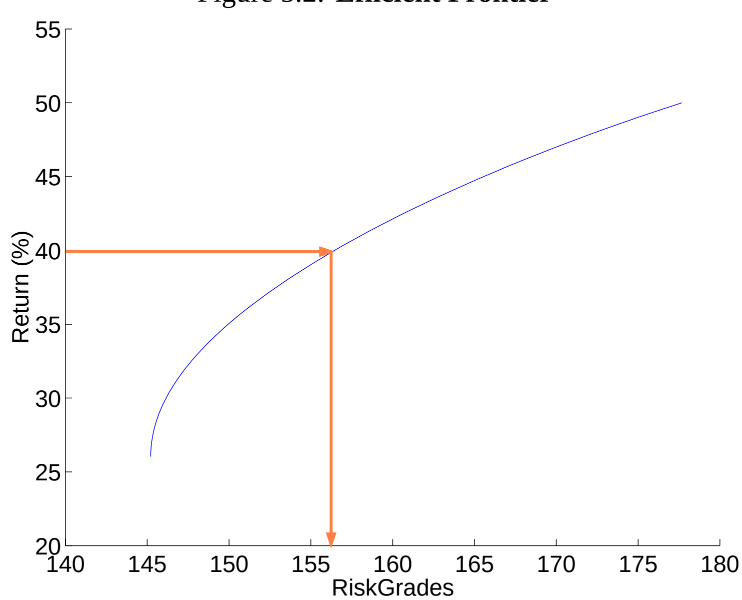
[6] If we include hedge instruments which have strong negative correlation with other assets in the candidate set and/or allow short sale for the asset of negative return, we should not apply the base of the return.

returns in the third column. With the restrictions of no short sale and no more than 50% weight for one asset, we obtain an efficient frontier as shown Figure 5.2. The efficient frontier shows the minimal RiskGrade that can be achieved subject to a required expected return. If an investor's required expected return is 40%, the minimal RiskGrades that can be obtained using this basket of assets is 156. The optimal portfolio weights are given in the last column of Table 5.2.

Table 5.2: **Optimal Weights**

Stock Ticker	Historical Return	Expected Return	RiskGrades	Optimal Weight
C	57.98	50.00	197	50.00
NWS	13.31	13.31	256	0.24
MOT	18.17	18.17	383	0.00
AOL	83.64	50.00	333	0.67
AAPL	-8.86	0.00	579	2.01
FS	54.17	50.00	230	28.44
SKS	-27.06	0.00	337	4.80
DK	-15.62	0.00	266	13.37
TWA	-41.88	0.00	593	0.45

Figure 5.2: **Efficient Frontier**



5.3 Sector Analysis

One key investment information about a fund is its sectors association. We introduce a simple method of sector analysis based on the variance decomposition technique of econometrics.

The traditional sector analysis calculates the weights of sectors as classifying actual holdings of a fund through sectors. Thus, it requires entire holding information and time consuming assignment process. However, the sector analysis based on the variance decomposition technique focuses on how much the value movement of a fund is explained through the value movement of each sector. Thus, it needs only net value information of the fund and index of sectors and quick multivariate regression. It is worth noting that our goal is not duplicate the actual holding weights of sectors but the relative sizes of impacts from sectors to return and risk of fund.

The technique consists of two steps - multivariate regression and variance decomposition. The multivariate regression fits return of fund to return of sector indices.

$$r_{i,t} = \alpha + \beta_1 s_{1,t} + \beta_2 s_{2,t} + ... + \beta_k s_{k,t} + \epsilon_{i,t}, \tag{5.15}$$

where $r_{i,t}$ denotes the return of fund i calculated from the net asset value of the fund. $s_{j,t}$ denotes the return of sector j calculated from the sector index. $\epsilon_{i,t}$ denotes the random error term. Thus, the correlation between the $s_{.,t}s$ and $\epsilon_{i,t}$ is zero.

To put heavier weight on recent returns, Equation 5.15 is estimated by weighted least squares. The weight is defined as RiskMetrics' exponentially weighted scheme with decay factor 0.97.

$$\omega_{t-h} = (1-\lambda)\lambda^h, h = 0, 1, ..., n, \tag{5.16}$$

where ω_{t-h} is weight for equation of h days before and λ is decay factor of 0.97. The effective number of observations, n, is defined as 151 days following decay factor 0.97 (h=1...151).

Then, the weighted least squares estimator of βs is

$$\hat{B} = (S'\Omega S)^{-1} S'\Omega R, \tag{5.17}$$

where $\hat{B} = [\hat{\alpha}, \hat{\beta}_1, ..., \hat{\beta}_k]'$, $S = [1 : s_1 : ... : s_k]$, Ω denotes n by n diagonal matrix of ω, and $R = [r_i]$.

Next, we do Concentration Identification and Risk Decomposition based on variance decomposition technique. We define Risk Decomposition as ratios of contributions of sectors to total variance of the fund. The variance of fund i is decomposed through the variances and the covariances of the sectors as

$$\mathrm{Var}(r_i) = \sum_{j=1}^{k} \hat{\beta}_j^{\,2} \mathrm{Var}(s_j) + 2 \sum_{j=1}^{k} \sum_{h \neq j} \hat{\beta}_j \hat{\beta}_h \mathrm{Cov}(s_j, s_h) + \mathrm{Var}(\epsilon_i), \tag{5.18}$$

where Var(.) and Cov(., .) denote variance and covariance operators with weights defined by Equation 5.16. Note that the s_js and ϵ_i are orthogonal by construction.

It follows that the contribution of sector j for the variance of fund i is defined as

$$risk_j = \frac{\hat{\beta}_j^{\,2} \mathrm{Var}(s_j) + \sum_{h \neq j} \hat{\beta}_j \hat{\beta}_h \mathrm{Cov}(s_j, s_h)}{\mathrm{Var}(r_i)} \times 100. \tag{5.19}$$

The residual portion is interpreted as the idiosyncratic risk of the fund. Thus, the contribution of idiosyncratic movement for variance of fund i is defined as

$$risk_\epsilon = \frac{\mathrm{Var}(\epsilon_i)}{\mathrm{Var}(r_i)} \times 100 = 100 - \sum_{j=1}^{k} risk_j. \tag{5.20}$$

We define Concentration Identification as ratios of absolute values of normalized coefficients of sectors to sum of them. The normalized coefficients mean the coefficients for the explanatory variables which are adjusted to have unit variance. Therefore, the normalized coefficient of sector j is computed by $\hat{\beta}_j \sigma_j$, where σ_j denotes the standard deviation of sector j, i.e., $\sqrt{\mathrm{Var}(s_j)}$.

$$concentration_j = \frac{|\hat{\beta}_j| \sigma_j}{\sum_{j=1}^{k} |\hat{\beta}_j| \sigma_j} \times 100. \tag{5.21}$$

Thus, the Concentration Identification does not concern about the direction of the value movement of fund but concerns only the relative size of the value movement of fund by the impact of value movement of sectors. Aware that the relative size of the value movement of fund is larger as the value movement of fund is more sensitive to unit value movement of a sector and/or the value movement of the sector is larger than the other sector. For example, if while the utility sector has 0.2 sensitivity to a fund (β_u) and 10% volatility (σ_u), the technology sector has 0.1 sensitivity (β_t) and 40% volatility (σ_t), then the total impact from the utility sector to the fund ($= 0.2 \times 10\%$) is half of that from the technology sector ($= 0.1 \times 40\%$).

As an example, Table 5.3 shows actual holdings and the sector analysis based on the variance decomposition technique as of August 11, 2000. We choose randomly 10 well known funds - DSPIX (Dreyfus Basic S&P

500 Stock Index), FSPTX (Fidelity Select Technology), VIVAX (Vanguard Value Index), JAWWX (Janus Worldwide), VFINX (Vanguard 500 Index), FSENX (Fidelity Select Energy), VGEQX (Vanguard Growth Equity), TVFQX (Firsthand Technology Value), AYEBX (AmSouth Value B), and NFBSX (Nvest Bullseye A).

The upper part of Table 5.3 shows the most recent holdings of the above funds from Yahoo's data. Some of the holding information have one-quarter lag (June 30, 2000) and others have two-quarter lag (March 31, 2000). The lower two parts of the table show the result of Concentration Identification and and Risk Decomposition based on the net asset value of the fund and the 11 sector indices of the S&P up to August 11, 2000. Aware that the sector classifications of Yahoo and S&P are a little different. It is worth noting that Risk Decomposition can have negative value because the covariances between one sector with the other sectors are large negative enough to dominate positive variance of the sector for Risk Decomposition.

Table 5.3: Actual Holdings and Sector Analysis

Actual Holdings	DSPIX	FSPTX	VIVAX	JAWWX	VFINX	FSENX	VGEQX	TVFQX	AYEBX	NFBSX
Industrial Cyc	10.81	2.03	11.10	3.09	10.42	3.23	4.15	7.63	11.21	23.87
Consumer Cyc	4.97	0.00	2.37	0.00	5.61	0.00	4.25	0.00	2.82	0.00
Consumer Non	1.88	0.00	3.86	5.76	1.52	0.00	0.54	1.46	3.84	0.00
Energy	5.90	0.00	12.93	1.97	5.86	95.13	3.16	0.00	14.04	10.06
Financials	13.02	0.00	27.09	2.77	12.73	0.00	0.00	0.00	16.21	0.00
Health	8.95	0.00	3.75	5.33	11.47	0.00	11.92	4.27	11.75	8.66
Retail	6.08	0.05	4.18	1.95	5.65	0.00	4.34	0.00	9.36	4.47
Services	13.63	1.44	20.21	41.13	12.62	0.30	11.29	2.62	18.68	8.95
Technology	32.95	96.48	10.52	37.98	32.16	0.00	59.48	84.01	7.05	34.08
Utilities	1.81	0.00	3.99	0.00	1.97	1.34	0.87	0.00	5.04	9.91
Concentration	DSPIX	FSPTX	VIVAX	JAWWX	VFINX	FSENX	VGEQX	TVFQX	AYEBX	NFBSX
Basic Materials	1.66	1.93	8.12	1.96	1.26	2.08	1.60	3.76	8.42	0.86
Capital Goods	6.01	0.01	0.68	14.91	5.21	2.94	0.06	4.08	3.72	15.47
Consumer Cyc	6.74	0.29	3.40	4.93	6.91	2.41	3.20	3.49	3.04	10.47
Consumer Non	6.38	9.13	3.73	10.86	6.17	0.32	0.46	10.09	4.13	0.97
Energy	4.20	6.01	10.34	6.18	4.27	77.53	5.78	5.13	15.23	1.18
Financial	13.35	5.92	32.85	4.41	14.49	3.36	6.05	0.01	26.23	5.74
Healthcare	11.59	0.04	5.42	2.38	11.54	4.17	9.71	1.39	4.68	10.18
Services	5.34	7.07	14.48	9.54	5.41	0.69	0.68	9.92	3.62	10.67
Technology	41.58	63.12	10.86	42.04	41.84	3.03	69.20	57.72	13.87	36.35
Transportation	0.40	6.12	2.39	0.16	0.05	0.52	2.74	2.71	5.42	6.61
Utilities	2.70	0.30	7.68	2.57	2.78	2.89	0.46	1.65	11.60	1.44
Risk Decomp	DSPIX	FSPTX	VIVAX	JAWWX	VFINX	FSENX	VGEQX	TVFQX	AYEBX	NFBSX
Basic Materials	0.55	-0.21	6.27	0.06	0.40	0.76	-0.01	-0.33	6.97	-0.04
Capital Goods	7.02	-0.01	-0.67	10.40	5.94	0.57	-0.05	2.73	-3.21	12.21
Consumer Cyc	5.82	-0.07	3.11	1.55	5.85	0.41	1.33	-0.62	2.68	-2.76
Consumer Non	5.24	-1.12	3.31	-1.32	4.99	-0.02	-0.16	-0.68	3.33	-0.34
Energy	0.38	1.76	4.48	1.56	0.37	85.38	1.23	1.56	10.51	0.16
Financial	13.90	2.73	48.17	1.95	15.07	-0.24	3.39	0.01	34.59	3.28
Healthcare	2.85	-0.02	1.39	0.61	2.69	-0.26	-0.63	-0.49	1.65	0.48
Services	3.99	5.45	10.48	6.25	4.00	-0.14	0.43	8.11	1.37	7.20
Technology	58.21	85.28	9.50	44.92	57.82	-0.19	89.10	77.65	9.97	38.80
Transportation	0.25	-1.30	2.02	0.04	0.04	0.05	-0.93	-0.64	4.49	3.02
Utilities	1.21	0.02	5.87	0.10	1.23	1.40	0.04	0.13	10.91	0.22
Idiosyncratic	0.59	7.49	6.05	33.86	1.61	12.27	6.27	12.58	16.73	37.76

Chapter 6

Ghost Series Generator

To calculate all the risk measures introduced in this document and to implement stress tests, we need up to five years of daily data. However, many Nasdaq stocks that recently went IPO do not have a five-year history. Therefore, we need to create proxy data for each instrument that has less than a year-year history. The proxy data should maintain the mean, volatility, and covariance structure of the instrument's existing data. In addition, the proxy data should keep the recent existing data. Recent, 151-day data is crucial for the calculation of RiskGrade™ values because we use the exponentially weighted moving average with a decay factor of 0.97. In this chapter, we introduce the Ghost Series Generator, which we base on the one-factor model to generate proxy data for market indices.[1]

6.1 Construction of the One-Factor Model

The one-factor model describes the returns $r_{i,t}$ of an individual equity as

$$\left(\frac{r_{i,t}-\mu_i}{\sigma_i}\right)=\rho_i\left(\frac{r_{m,t}-\mu_m}{\sigma_m}\right)+\sqrt{(1-\rho_i^2)}\varepsilon_{i,t}, \tag{6.1}$$

where $r_{i,t}$ denotes the return of equity i, and μ_i and σ_i denote its mean and standard deviation. The term $r_{m,t}$ denotes the return of the market portfolio, and μ_m and σ_m denote its mean and standard deviation. The term ρ_i denotes the correlation between the return of equity i and the market portfolio. The random error term $\varepsilon_{i,t}$ follows the standard normal distribution, and $\sqrt{(1-\rho_i^2)}\varepsilon_{i,t}$ denotes the idiosyncratic movement of equity i.

[1] We appreciate Alan Fang's helpful assistance with the research in this chapter.

We can rearrange Equation (6.1) with respect to $r_{i,t}$ as follows:

$$r_{i,t} = \left(\mu_i - \frac{\rho_i\sigma_i}{\sigma_m}\mu_m\right) + \frac{\rho_i\sigma_i}{\sigma_m}r_{m,t} + \left(\sqrt{(1-\rho_i^2)}\sigma_i\right)\varepsilon_{i,t} \tag{6.2}$$

How does the above one-factor model differ from the security market line of the CAPM and the univariate regression model?

The security market line is described as

$$\begin{aligned} r_{i,t} &= r_{f,t} + \beta_i(r_{m,t} - r_{f,t}) + \varepsilon_{i,t} \\ &= (1-\beta)r_{f,t} + \beta r_{m,t} + \varepsilon_{i,t}, \end{aligned} \tag{6.3}$$

where, $r_{f,t}$ denotes the risk-free asset return.

If you set β_i equal to $(\rho_i\sigma_i)/\sigma_m$, Equation (6.3) looks similar to Equation (6.2). The drift term, however, makes the security line and the one-factor model completely different from each other. The security line has only one unknown parameter β_i, which simultaneously determines both the slope and the drift. Therefore, the slope and the drift are closely related to maintain the CAPM theory that the equity with a large β_i provides a large expected return. However, it is doubtful that CAPM holds for current IPO stocks. Usually, recent IPO stocks show relatively small βs (small systematic, but large idiosyncratic movement) but provide a relatively large expected return. (For more information about IPO stocks, see Section 6.5.) In the one-factor model, we have another unknown parameter, $\alpha_i = \left(\mu_i - \frac{\rho_i\sigma_i}{\sigma_m}\mu_m\right)$, which may be determined independently of β_i. Thus, we need not restrict the relationship between the slope and the drift, as explained by CAPM. We can determine them from the individual time series.

The one-factor model is similar to the univariate regression model except for the coefficient $\left(\sqrt{(1-\rho_i^2)}\sigma_i\right)$ of the error term $\varepsilon_{i,t}$. The coefficient explicitly makes the total variance terms in the right-hand side of Equation (6.2) equal to the variance of the return of equity i:

$$\begin{aligned} \mathrm{VAR}(r_{i,t}) &= \left(\frac{\rho_i\sigma_i}{\sigma_m}\right)^2 \mathrm{VAR}(r_{m,t}) + \left(\sqrt{(1-\rho_i^2)}\sigma_i\right)^2 \mathrm{VAR}(\varepsilon_{i,t}) && (6.4) \\ &= \rho_i^2\sigma_i^2 + (1-\rho_i^2)\sigma_i^2 && (6.5) \\ &= \sigma_i^2, && (6.6) \end{aligned}$$

where VAR denotes the variance operator and $\mathrm{VAR}(\varepsilon_{i,t})$ equals unity because $\varepsilon_{i,t}$ follows the standard normal distribution. This is an important feature of the one-factor model, since it guarantees to maintain the volatility of equity i when we generate proxy data from the right-hand side terms of Equation (6.2).

6.2 Generation of Proxy Series

Let us assume that equity i has only one year of real historical data ($r_{i,T-1}, ..., r_{i,T-252}$). Then, we need to create proxy historical data ($\widehat{r}_{i,T-253}, ..., \widehat{r}_{i,T-1260}$) for the previous four years and combine it with the real historical data. The process of generating a proxy series is summarized in the following six steps:

1. Find the market portfolio return data (e.g., S&P Industrial Index) of the last five years ($t = T - 1, ..., T - 1260$). To find the market portfolio index of equity i, we can use the Standard Industrial Code (SIC) of equity i.

2. Estimate all parameters ($\mu_i, \sigma_i, \mu_m, \sigma_m$, and ρ_i) in Equation (6.2) by using real historical data.

3. Construct 1,008 daily returns from Equation (6.7) for the previous four years ($t = T - 253, ..., T - 1260$).

$$\widehat{r_{i,t}} = \left(\widehat{\mu_i} - \frac{\widehat{\rho_i}\widehat{\sigma_i}}{\widehat{\sigma_m}}\widehat{\mu_m}\right) + \frac{\widehat{\rho_i}\widehat{\sigma_i}}{\widehat{\sigma_m}} r_{m,t} + \left(\sqrt{(1 - \widehat{\rho_i}^2)}\widehat{\sigma_i}\right)\varepsilon_{i,t}, \tag{6.7}$$

   where $\widehat{\ }$ denotes the estimated parameters and returns.

   The random error term $\varepsilon_{i,t}$ is generated from the standard normal distribution. Then, the unique unknown variable is the proxy return series for equity i ($\widehat{r_{i,t}}$).

4. Compute the proxy return series of equity i.

5. Construct the five-year proxy return series $[r_{i,T-1}, ..., r_{i,T-252}, \widehat{r}_{i,T-253}, ..., \widehat{r}_{i,T-1260}]$ for equity i by combining real historical data and the generated proxy historical data.

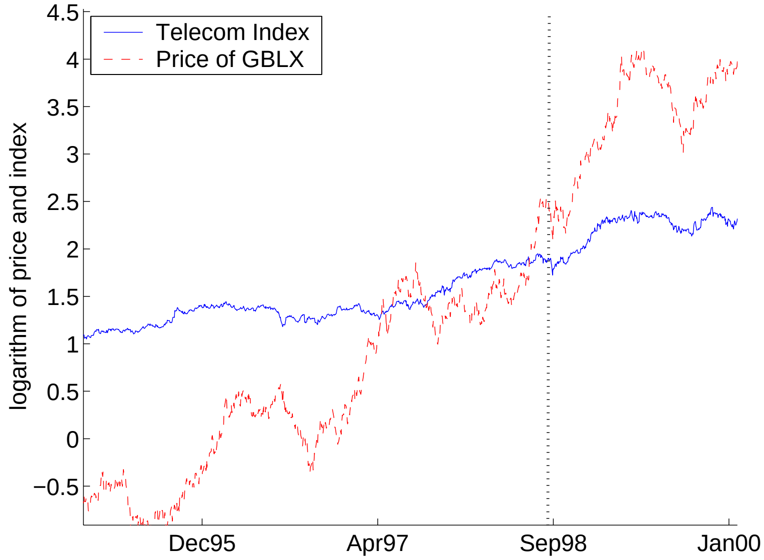
6. Construct the five-year proxy price series $[p_{i,T-1}, ..., p_{i,T-252}, \widehat{p}_{i,T-253}, ..., \widehat{p}_{i,T-1260}]$ for equity i by using the proxy return and current price of equity i.

6.3 Example of Global Crossing

Global Crossing (Nasdaq:GBLX) has been traded in Nasdaq since August 14, 1998. As of December 31, 1999, we have 374 days of real historical data and need to generate another 886 days of proxy data to fill in the history from January 1, 1995 through August 13, 1998.

Using real historical data, we estimate all parameters. We estimate $\mu_i, \sigma_i, \mu_m, \sigma_m$, and ρ_i to be 0.0038, 0.0514, 0.0012, 0.0220, and 0.3519, respectively. The mean and volatility of GBLX are more than twice

Figure 6.1: **Example of Proxy Data**



as large as those of the telecommunications market index. The correlation coefficient between GBLX and telecommunications is relatively high.

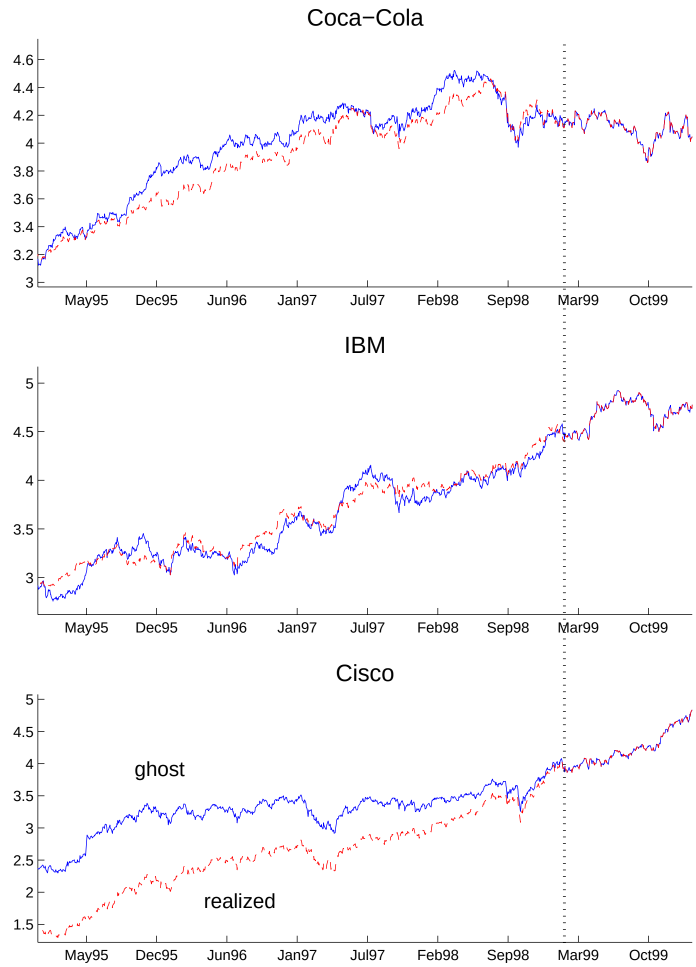
Next, based on the estimated parameters, we generate a proxy series. Figure 6.1 shows the real and proxy histories of GBLX. We can easily see that the proxy history of GBLX (left-hand side of dotted line) correlates with the real history of the market index (right-hand side of dotted line) and maintains its mean and volatility.

6.4 Backtesting Proxy Data

Let us discuss how well proxy data built by the Ghost Series Generator represents a period of missing data. To answer this question, we apply backtesting to several examples. First we assume that the series has only one year of historical data (in-sample period of December 31, 1999–January 1, 1999). We then estimate all the parameters of the one-factor model by using only in-sample data. Next, we generate a proxy series for the previous four years (out-of-sample period of December 31, 1998–January 1, 1995) by using estimated parameters from the in-sample data. Finally, we compare the proxy series and the realized series obtained from the out-of-sample data.

We applied the backtesting to Coca-Cola, IBM (NYSE:IBM), and Cisco and plotted the proxy and realized series in Figure 6.2. The dotted vertical line demarcates the in-sample period (left-hand side) and the out-

Figure 6.2: **Backtesting Examples of Proxy Data**



of-sample period (right-hand side). Since the random numbers that were generated for the error term are different in each trial, the proxy series are also different. However, basic statistics such as expected return, volatility, and correlation with market index are maintained in each construction of the proxy series regardless of the different error terms. Figure 6.2 shows that the volatility of the proxy series is similar to the volatility of the realized series, and the correlation between the proxy and realized series is high.

Table 6.1 shows the basic statistics of the proxy and realized series. As we expected, the volatility of the proxy series and the correlation of the proxy series with the market index are very close to those of the realized series. This consistent agreement of volatilities and correlations validates the use of the proxy series

Table 6.1: **Backtesting Examples of Proxy Data**

Basic Statistics	In Sample Dec. 31, 1999–Jan. 1, 1999 Realized	Out of Sample Dec. 31, 1998–Jan. 1, 1995 Realized	Ghost
Coca-Cola			
Expected Return	-0.0003	0.0009	0.0008
Volatility	0.0211	0.0160	0.0181
Corr. w/ Market Index	0.9333	0.9470	0.9055
Corr. w/ Realized and Ghost	...	0.8529	
IBM			
Expected Return	0.0016	0.0014	0.0020
Volatility	0.0264	0.0198	0.0231
Corr. w/ Market Index	0.8030	0.8138	0.7524
Corr. w/ Realized and Ghost	...	0.5922	
Cisco			
Expected Return	0.0038	0.0024	0.0014
Volatility	0.0262	0.0277	0.0284
Corr. w/ Market Index	0.9949	0.9238	0.9943
Corr. w/ Realized and Ghost	...	0.9298	

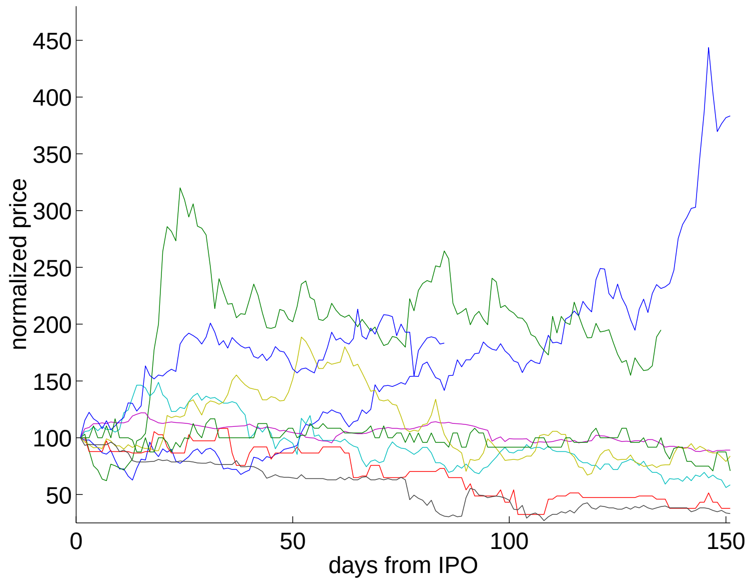
for volatility- and correlation-based risk measures, such as RiskGrades and XLoss™ . On the other hand, the expected returns behave differently. For Coca-Cola the expected return of the proxy series is close to the return of the realized series, while for IBM and Cisco it is not. This finding means that we must be cautious about using an expected return from a proxy series. Therefore, when we use a proxy series to obtain both the volatility- and the expected return-based risk measures such as Chance of Losing Money, we should be aware of its limitations. Overall, the high correlations between proxy and realized series show that a proxy series can be a reasonable replacement for a period of missing data.

6.5 Recent IPO Stock

If equity i has a very short history of real historical data (less than 151 days), the parameters estimated from the real history are very unstable. To avoid instability, in Step 2 of Section 6.2 we use the average parameters of the IPO stocks in the industry of equity i.

When we estimate the average parameters of IPO stocks, we must use only the data that immediately follows the IPO date of each equity. Since the IPO date is different across IPO stocks, we use the technique of event study. The process of estimating the average parameters of IPO stocks is summarized in the following five steps using the example of the telecommunication industry:

Figure 6.3: **IPO Event in the Telecommunication Industry**



1. Find all IPO stocks between 1995 and 1999. There are nine IPO stocks in the telecommunication industry.

2. Set the IPO date of an individual stock to "Date 1" and extract up to 151 days of data after IPO.

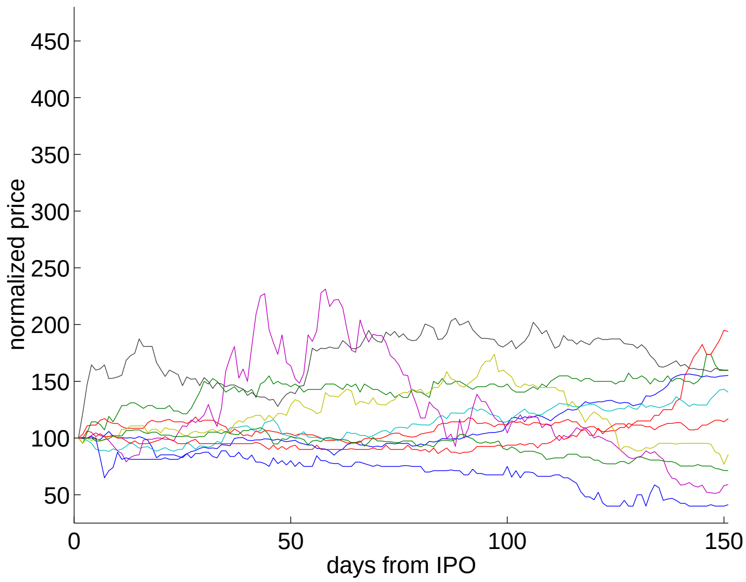
3. Normalize the closing price on the IPO date to 100, and overlap all IPO stocks following not the calendar date, but the IPO dates in the period Date 1–Date 151. Figure 6.3 plots the normalized prices of nine IPO stocks in the telecommunication industry.

4. Estimate parameters μ_i, σ_i, ρ_i, μ_m, and σ_m for individual IPO stocks by using real historical data from Date 1 to Date 151.

5. Average the estimated parameters for each industry.

Table 6.2 shows the average parameters of the 146 IPO stocks that appeared from 1995 through 1999 across twenty S&P industrial classifications. It is worth noting that the average volatility of IPO stocks (annualized volatility, 76.04%) is very high and the average correlation with the market index (0.1470) is relatively small.

Figure 6.4: **IPO Event in the Chemical Industry**



This means that IPO stocks have a very high idiosyncratic risk but a small systematic risk. This fact validates our method of estimating parameters from the data of only IPO stocks right after the IPO date.

Table 6.2 also shows that the average parameters vary widely across industry. Specifically, the volatility of the so-called High-Tech industries (technology, telecommunication, computer networking, computer hardware, pharmacy and semi-conductors) is very high. As an illustration, compare Figure 6.3 of the telecom IPOs and Figure 6.4 of the chemical IPOs. You can easily see that the telecom industry is twice as volatile as the chemical industry. This fact validates our method of applying each industry's characteristic parameters to the new IPO stock.

Table 6.2: **Estimated Parameters for IPO Stocks**

Industry	No. of IPOs	μ_i	σ_i	ρ_i	μ_m	σ_m
AIRLINE	4	0.0013	0.0374	0.0685	0.0018	0.0172
AUTO	5	0.0009	0.0330	0.0807	0.0010	0.0177
BEVERAGES NON-AL	4	-0.0001	0.0434	0.1393	0.0003	0.0150
RETAIL	8	0.0017	0.0385	0.1134	0.0009	0.0241
CHEM	10	0.0004	0.0353	-0.0101	0.0005	0.0140
COMP HARD	6	0.0028	0.0528	0.1639	0.0009	0.0199
TELE CELLULAR	6	0.0020	0.0459	0.0456	0.0029	0.0238
ELECT SEMI-CONDUCTORS	6	0.0009	0.0582	0.1830	0.0009	0.0251
FINANCE	10	0.0006	0.0388	0.3102	0.0003	0.0161
FOOD	6	-0.0009	0.0297	0.1283	-0.0005	0.0112
COMP NETWORKING	10	0.0033	0.0615	0.2147	0.0042	0.0256
BROADCASTING	9	0.0015	0.0395	0.1311	0.0019	0.0180
METALS	5	-0.0015	0.0304	0.0956	-0.0017	0.0169
OIL & GAS	3	0.0015	0.0247	0.2461	-0.0002	0.0133
PHARM	9	0.0011	0.0548	0.0898	0.0007	0.0144
TRANSPORTATION	5	0.0009	0.0251	0.1853	0.0008	0.0090
TELECOM	9	-0.0003	0.0620	0.1816	0.0008	0.0179
TECHNOLOGY	19	0.0045	0.0741	0.2530	0.0021	0.0186
BIOTECH	6	0.0019	0.0468	0.0662	0.0012	0.0195
UTILITIES	6	0.0006	0.0463	0.0143	0.0004	0.0109
Weighted Average	146	0.0013	0.0479	0.1470	0.0012	0.0178

Glossary

beta. The measure of a fund's or stock's risk in relation to the market, or an alternative benchmark. A beta of 1.5 means that a stock's excess return is expected to move 1.5 times the market excess returns; e.g., if the market excess return is 10%, then we expect, on average, the stock return to be 15%. Beta is referred to as an index of the systematic risk due to general market conditions that cannot be diversified away.

CAPM. Capital Asset Pricing Model. A model that relates the expected return on an asset to the expected return on the market portfolio.

Chance of Losing Money. Shows the probability with which the value of an investment after a 1-, 3-, or 12-month horizon (or a user-defined investment horizon) can drop below its initial value (or a user-defined level).

decay factor. Lambda (λ). The weight applied in the exponential moving average. The decay factor takes a value between zero and one. RiskMetrics uses a decay factor of 0.94 in the calculation of volatilities and correlations for the one-day horizon, and 0.97 for the one-month horizon.

diversification benefit. Measures risk reduction that arises from holding a collection of assets that are not perfectly correlated. The diversification benefit for your portfolio RiskGrade is the difference between the computed portfolio RiskGrade and the market-value weighted average of the individual asset RiskGrades. The portfolio diversification benefit for XLoss is the difference between the computed portfolio XLoss and the sum of the individual asset XLoss values.

exponential weighting. A method of applying weights to a set of data points (returns), with the weights declining exponentially over time. In a time series context, this results in weighting recent data more than data of the distant past.

Ghost Series Generator. An algorithm that, based on the one-factor model, generates proxy data for market indices with short histories of less than five years. The proxy data created by the Ghost Series Generator maintains the mean, volatility, and covariance structure of the instrument's existing data.

historical event. An actual historical event that is used in the Historical Event Stress Test to determine the amount of loss (base currency or percent) that could occur if the event were to happen again. Examples of historical events are the 1987 Stock Market Crash (Oct. 19, 1987), Gulf War Crisis (Jan. 16, 1991), Mexican

Peso Crisis (Dec. 14, 1994), Asian Crisis (Jul. 2, 1997), Russian Crisis (Aug. 21, 1998), and the Brazilian Crisis (Jan. 13, 1999).

historical simulation. A nonparametric method of using past data to make inferences about the future. One application of this technique is to take today's portfolio and revalue it by using past historical price and rates data.

lambda (λ). *See* **decay factor**.

Marginal XLoss. Shows the contribution of an asset in the portfolio to the total XLoss of the portfolio. The Marginal XLoss for a specific asset reflects how the portfolio's XLoss would change if the investor were to sell that asset and keep the cash proceeds.

market risk. Risk that arises from the fluctuating prices of investments as they are traded in the global markets. Market risk is highest for securities with above-average price volatility and lowest for stable securities such as Treasury bills.

parametric. When a functional form for the distribution of a set of data points is assumed. For example, when the normal distribution is used to characterize a set of returns.

RiskGrade. The RiskGrade for a single position or portfolio is a ranking which measures the potential volatility of the position or portfolio relative to the volatility of a standard benchmark. The benchmark used is the average daily volatility of the market-capitalization weighted average of international equity indices during the period 1995–1999, which is defined to have a RiskGrade of 100. For example, if a position or portfolio has a RiskGrade of 200, the position or portfolio is twice as volatile as the benchmark.

RiskImpact. The RiskImpact for a single position is the percentage amount that the portfolio's RiskGrade will decrease upon removal of that position.

stress testing. The process of determining how much the value of a portfolio can fall under abnormal market conditions. Stress testing consists of generating worst-case stress scenarios (for example, a stock market crash) and revaluing a portfolio under those stress scenarios.

systematic risk. *See* **systemic risk**.

Systemic risk. Also, systematic risk. The risk of a portfolio after all unique risk has been diversified away. Systemic risks may arise from common driving factors (for example, market and economic factors, natural disasters, or war) and can influence the whole market's well being.

unique risk. Exposure to a particular company; sometimes referred to as *firm-specific risk*.

user-defined event. A crisis, such as a drop in the S&P 500, that is defined by an individual for a User-Defined Stress Test in order to determine the amount of loss (base currency or percent) that could occur if the crisis were to happen again.

Value-at-Risk. A measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a preset horizon.

volatility. Means risk, as measured by the standard deviation of a security's price.

Worst-Case Performance. The amount of loss (base currency or percent) that can occur if the worst 3-, 6-, or 12-month horizon (or a user-defined horizon) during a given historical period were to occur again.

Worst Losing Streak. An empirical type of risk measure for the long-horizon abnormal market. RiskMetrics defines Worst Losing streak as the amount of loss (base currency or percent) that would occur if the largest drop in asset price, from peak to trough, during the last five years were to repeat. The definition is similar to that of Worst-Case Performance, except that no predetermined time horizon is required. Therefore, the worst period is selected not from fixed-period windows, but from flexible peak-to-trough windows.

XLoss. XLoss stands for "Loss in Extreme Markets". The XLoss for a single position or portfolio is the dollar value by which the position's or portfolio's value could fall during periods of high market volatility. High market volatility periods are defined as months in which market movements are in the 95th percentile or higher in terms of magnitude. The XLoss is calculated by using the expected value of the market moves only for these high market-volatility periods.

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