Figure 1: Recovered function where γ = 0.1

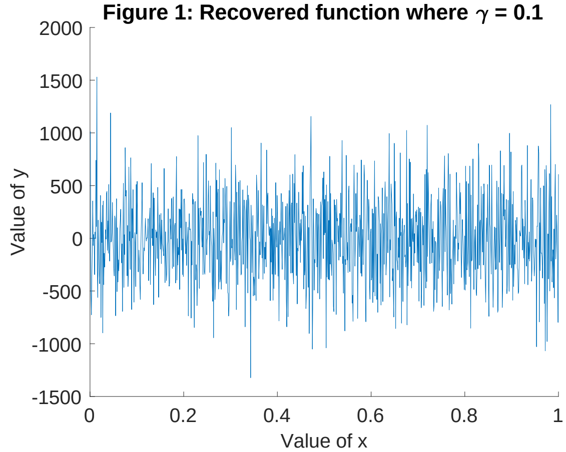
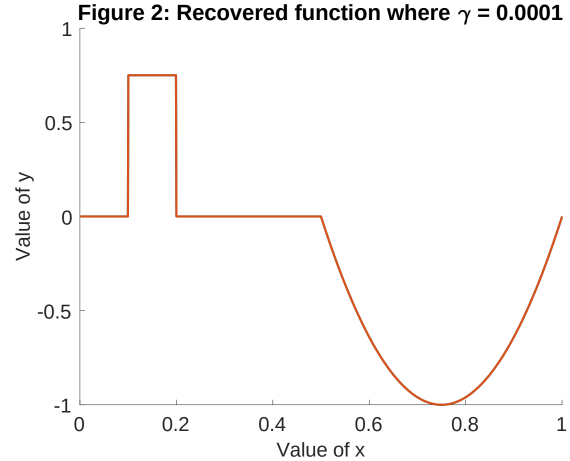


Figure 2: Recovered function where γ = 0.0001



(b) I computed the condition number of A using cond(A), with gamma values 0.0001, 0.001, 0.01, 0.1 and 0.5. For gamma = 0.0001, cond(A) = 1. For gamma = 0.001, cond(A) = 88.3509. For gamma = 0.01, cond(A) = 3.5608e+19. For gamma = 0.1, cond(A) = 1.0553e+21. For gamma = 0.5, cond(A) = 4.7041e+21. Because we get very large condition numbers (approximately 10^8) for gamma = 0.5, 0.1, 0.01 and 0.001, we can say that A is ill-conditioned for these specific gamma values. Gamma represents the amount of blurring and as gamma approaches 0, the condition number approaches 1. This means that A is becoming more well-conditioned. As depicted in Figure 2, the recovered function represents the true function f more closely when gamma is small, thus, a smaller value of gamma increases the accuracy of backslash on the linear system Af = g. This phenomenon is also depicted in Figure 1 where gamma is large, thus, we get a very large condition number for A. In this case, we get poor accuracy in solving the linear system Af = g due to A being ill-conditioned.

Figure 3: True f Vs reconstructions of f

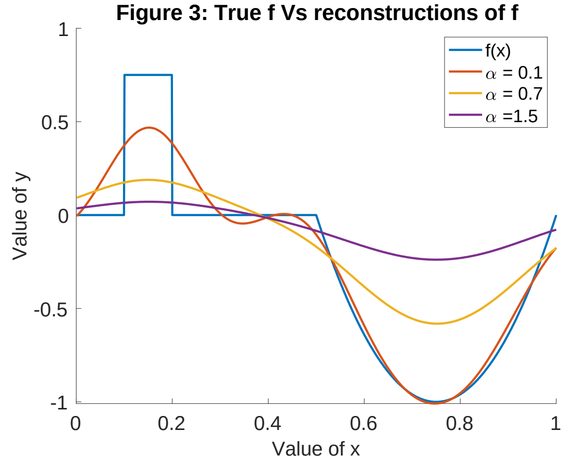
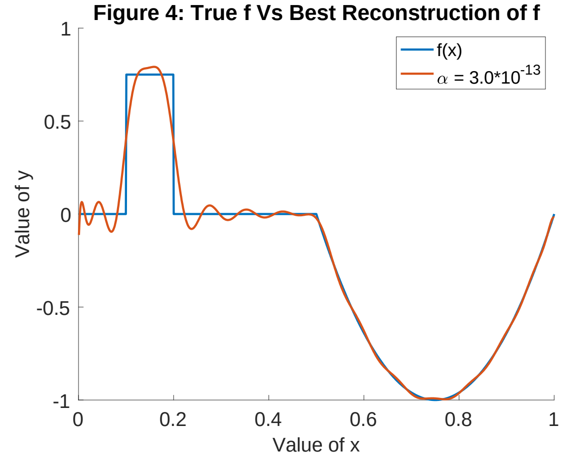


Figure 4: True f Vs Best Reconstruction of f



(c) As depicted in Figure 3, the reconstructions of f look more like the true f as alpha decreases. Decreasing alpha produces a better fit of the data by finding a value that agrees with the true f. Also, the second term in the least-squares equation [g ; zero_vector] affects the sum by producing a flatter function for increasing values of alpha. We can see this when we compare the reconstruction of f when alpha = 1.5 and alpha = 0.1 in Figure 3. When alpha = 1.5, the reconstruction of f more so resembles a flat line compared to when alpha = 0.1. This flattening effect as alpha increases negatively affects the deblurring process. Choosing small values of alpha makes it so that we get a reconstruction of f closer to the true f when we solve the least-squares equation with backslash.

(d) As shown in Figure 4, the value of alpha which I found to achieve the best deblurring was alpha = 3 * 10^{-13} = 0.00000000000003 where gamma = 0.1. Any value of alpha smaller than this would produce large errors in the deblurred function when compared to the true f. Any value of alpha larger would negatively affect the deblurring process as mentioned in part c. I believe the reason for the large errors when alpha < 3*10^{-13} is because if alpha is too small, the values of alpha*I appended to A in the least-squares equation will become very small which produces round-off errors and robustness issues when we solve the least-squares equation with backslash.

MATLAB Code

```
n = 1024;
A = zeros(n,n);
gamma = 0.1;


%Create matrix A (Part a (i))
for i = 1:n
    for j = 1:n
        A(i,j) = ((1./n)*(1./(gamma*sqrt(2*pi))))*exp(-(power((i-
j),2))/((2*(power(n,2))*(power(gamma,2)))));
    end
end

%Create true vector f (Part a (ii))
f = zeros(1,n);
x_grid = zeros(1,n);
for i = 1:n

    x_grid(i) = i./n;

    if x_grid(i) >= 0.1 && x_grid(i) <= 0.2
        f(i) = 0.75;
    elseif x_grid(i) >= 0.5 && x_grid(i) <= 1
        f(i) = 16*(x_grid(i) - 0.75)^2 - 1;
    else
        f(i) = 0;
    end

end

%Generate right hand side g = Af (Part a (iii))
g = A*transpose(f);

%Compute numerical solution f_tilda (Part a (iv))
f_tilda = A\g;

figure(1)
hold on;
plot(x_grid,f_tilda,'linewidth',2.0);
hold off;
set(gca,'FontSize',18);
xlabel('Value of x','fontsize',18);
ylabel('Value of y','fontsize',18);
title('Figure 1: Recovered function where \gamma = 0.1');

%Part b
condA = cond(A);
disp(condA);

%Part c Tikhonov regularization least-squares problem
a1 = 0.1;
```

```
a2 = 0.7;
a3 = 1.5;

%Define identity matrix,zero vector and g with zero vector
I = eye(n);
zero_vector = zeros(n,1);
g_zero = [g ; zero_vector];

%Reconstruct f with Tikhonov regularization for a1,a2,a3
I_a1 = I.*a1;
A_a1 = [A ; I_a1];
f_a1= A_a1\g_zero;

I_a2 = I.*a2;
A_a2 = [A ; I_a2];
f_a2= A_a2\g_zero;

I_a3 = I.*a3;
A_a3 = [A ; I_a3];
f_a3= A_a3\g_zero;

%Plot f and its reconstructions
figure(2);
hold on;
plot(x_grid,f,'linewidth',2.0);
plot(x_grid,f_a1,'linewidth',2.0);
plot(x_grid,f_a2,'linewidth',2.0);
plot(x_grid,f_a3,'linewidth',2.0);
hold off;
set(gca,'FontSize',18);
xlabel('Value of x','fontsize',18);
ylabel('Value of y','fontsize',18);
title('Figure 3: True f Vs reconstructions of f');
legend('f(x)','\alpha = 0.1', '\alpha = 0.7','\alpha =1.5');

%Part d Adjust alpha so that reconstruction matches f as closely as possible
a4 = 0.00000000000003;
I_a4 = I.*a4;
A_a4 = [A ; I_a4];
f_a4= A_a4\g_zero;

figure(3);
hold on;
plot(x_grid,f,'linewidth',2.0);
plot(x_grid,f_a4,'linewidth',2.0);
hold off
set(gca,'FontSize',18);
xlabel('Value of x','fontsize',18);
ylabel('Value of y','fontsize',18);
title('Figure 4: True f Vs Best Reconstruction of f');
legend('f(x)','\alpha = 3.0*10^{-13}');
```