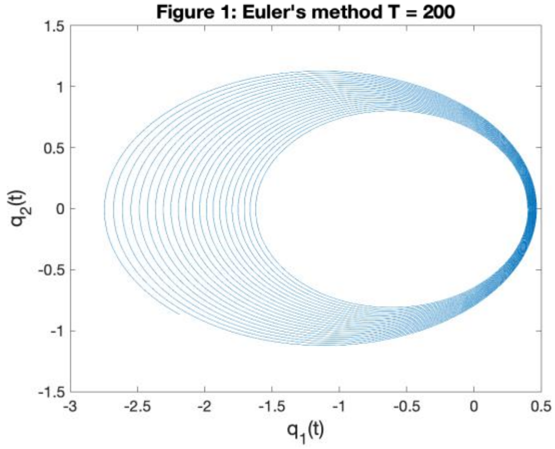

Figure 1: Euler's method T = 200

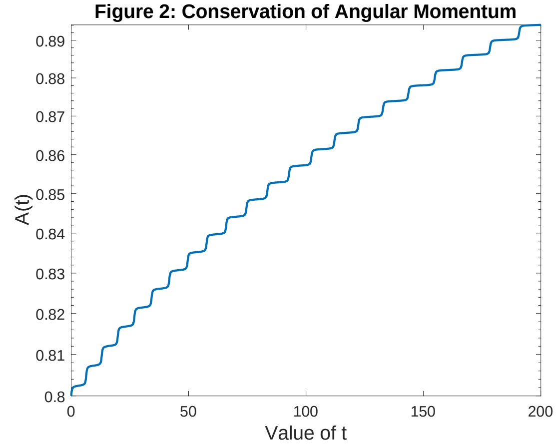

Figure 2: Conservation of Angular Momentum

(a) Using Euler's method along with the initial values and the ordinary differential equations (ODE) of q(t) and p(t), we can plot points in the $q_2(t)$ - $q_1(t)$ plane at time t_n where $q_2(t)$ and $q_1(t)$ are the position coordinates of the moving planet and where $n \in [0,N]$. N = T/h where h is the step size. As depicted in Figure 1, for T = 200, $q_1(t)$ converges to a value of -2.1846 whereas $q_2(t)$ converges to a value of -0.8685. We can also deduce from Figure 1 that most of the values of $q_1(t)$ are negative whereas the data for the value of $q_2(t)$ is split between negative and positive values.

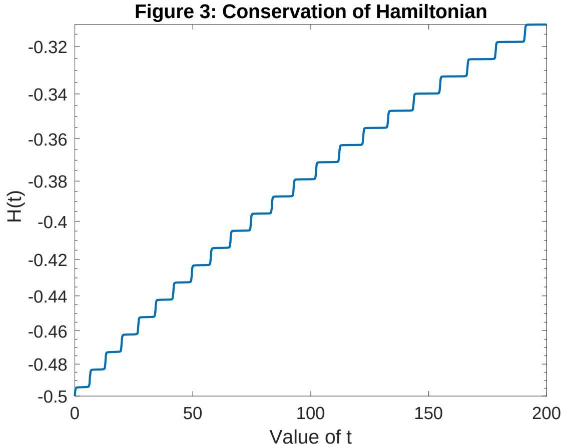

Figure 3: Conservation of Hamiltonian

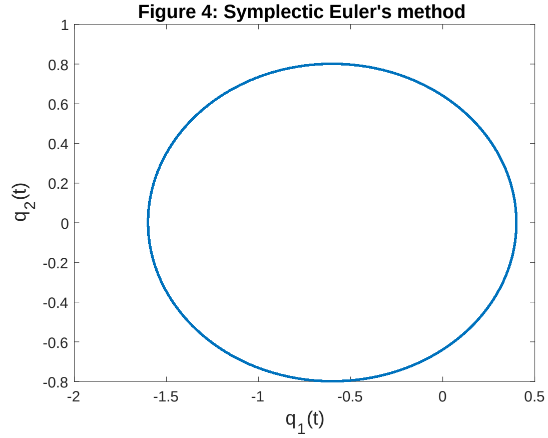

Figure 4: Symplectic Euler's method

(b) In my numerical solution, angular momentum is not conserved because it changes. As depicted in Figure 2, at time t = 0 in the time interval 0 <= t <= 200, the angular momentum starts off at a value of A(t) = 0.8 and increases to a value of A(t) = 0.8941. For larger T values, the magnitudes for the range of values will increase for $q_1(t)$ and $q_2(t)$, resulting in a larger semi-major axis and semi-minor axis in the ellipse when n = T/h. Hamiltonian is not conserved either because it also changes as time increases as depicted in Figure 3. The outwardly spiraling shape of the ellipse in Figure 1 illustrates the changing angular momentum and changing Hamiltonian.

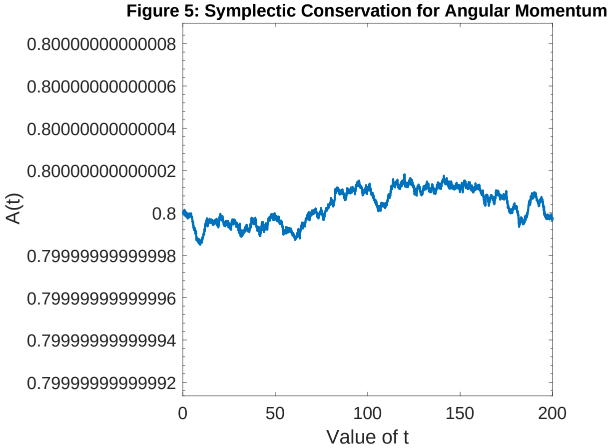

Figure 5: Symplectic Conservation for Angular Momentum

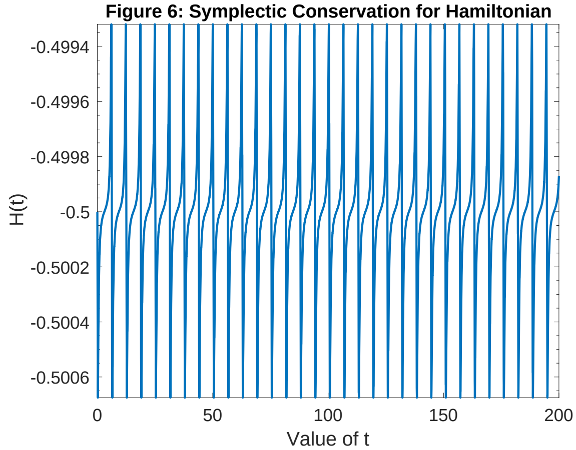

Figure 6: Symplectic Conservation for Hamiltonian

(c) When we use Symplectic Euler's Method, both Hamiltonian and Angular momentum are conserved. As depicted in Figure 4, the path of the moving planet is a circle instead of an outward spiral which verifies the conservation of these two quantities. As shown in Figures 5 and 6, the values for the angular momentum and Hamiltonian at time n = 0 and n = T/h are the same which further verifies conservation. Although the Angular Momentum and Hamiltonian quantities change, the change is very minor and the total quantities are always conserved in the entire time interval $n \in [0, T/h]$.

MATLAB CODE

```
clc
close all;

T= 200;
h = 0.0005;
N = ceil(T/h);
t = 0:h:T;
tvals = zeros(N+1,1);
tvals(1)= 0;

Q_arr1 = zeros(N+1,1);
Q_arr2 = zeros(N+1,1);
Q_arr3 = zeros(N+1,1);
Q_arr4 = zeros(N+1,1);

P_arr1 = zeros(N+1,1);
P_arr2 = zeros(N+1,1);
P_arr3 = zeros(N+1,1);
P_arr4 = zeros(N+1,1);


Q_arr1(1) = 0.4;
Q_arr2(1) = 0;
Q_arr3(1) = 0.4;
Q_arr4(1) = 0;

P_arr1(1) = 0;
P_arr2(1) = sqrt(1.6/0.4);
P_arr3(1) = 0;
P_arr4(1) = sqrt(1.6/0.4);

A = zeros(N+1,1);
H = zeros(N+1,1);
A2 = zeros(N+1,1);
H2 = zeros(N+1,1);


for i = 1:(N+1)

 %Part A

 P_arr1(i+1) = P_arr1(i) + h*(-1/((Q_arr1(i).^2) + (Q_arr2(i)^2)).^(3/2))*Q_arr1(i);
 P_arr2(i+1) = P_arr2(i) + h*(-1/((Q_arr1(i).^2) + (Q_arr2(i)^2)).^(3/2))*Q_arr2(i);


 Q_arr1(i+1) =Q_arr1(i)+ h*P_arr1(i);
 Q_arr2(i+1) =Q_arr2(i) + h*P_arr2(i);


 %Part B

 A(i) = Q_arr1(i)*P_arr2(i) - Q_arr2(i)*P_arr1(i);
 H(i) = (0.5)*(P_arr1(i).^2 + P_arr2(i).^2) - 1/(sqrt(Q_arr1(i).^2 + Q_arr2(i).^2));

 %Part C
```

```
 Q_arr3(i+1) = Q_arr3(i) + h*P_arr3(i);
 Q_arr4(i+1) = Q_arr4(i) + h*P_arr4(i);

 P_arr3(i+1) = P_arr3(i) - (h/(Q_arr3(i+1).^2+Q_arr4(i+1)^2)^(3/2))*Q_arr3(i+1);
 P_arr4(i+1) = P_arr4(i) - (h/(Q_arr3(i+1).^2+Q_arr4(i+1)^2)^(3/2))*Q_arr4(i+1);

 A2(i) = Q_arr3(i)*P_arr4(i) - Q_arr4(i)*P_arr3(i);
 H2(i) = 1/2*(P_arr3(i).^2 + P_arr4(i).^2) - (1/(sqrt(Q_arr3(i).^2 + Q_arr4(i).^2)));


end


figure(1);
plot(Q_arr1, Q_arr2);
set(gca,'FontSize',14)
title('Figure 1: Euler''s method T = 1000','fontsize',18)
xlabel('q_1(t)','fontsize',18)
ylabel('q_2(t)','fontsize',18)

figure(2);
semilogy(t, A, 'linewidth', 2);
set(gca,'FontSize',14)
title('Figure 2: Conservation of Angular Momentum','fontsize',18)
xlabel('Value of t','fontsize',18)
ylabel('A(t)','fontsize',18)

figure(3);
semilogy(t, H, 'linewidth', 2);
set(gca,'FontSize',16)
title('Figure 3: Conservation of Hamiltonian','fontsize',18)
xlabel('Value of t','fontsize',18)
ylabel(' H(t)','fontsize',18)

figure(4);
plot(Q_arr3, Q_arr4, 'linewidth', 2);
set(gca,'FontSize',14)
title('Figure 4: Symplectic Euler''s method ','fontsize',18)
xlabel('q_1(t)','fontsize',18)
ylabel('q_2(t)','fontsize',18)

figure(5);
semilogy(t, A2,'linewidth', 2);
set(gca,'FontSize',16)
title('Figure 5: Symplectic Conservation for Angular Momentum','fontsize',16)
xlabel('Value of t','fontsize',18)
ylabel('A(t)','fontsize',18)


figure(6);
semilogy(t, H2,'linewidth', 2);
set(gca,'FontSize',16)
title('Figure 6: Symplectic Conservation for Hamiltonian','fontsize',17)
xlabel('Value of t','fontsize',18)
ylabel('H(t)','fontsize',18)
```