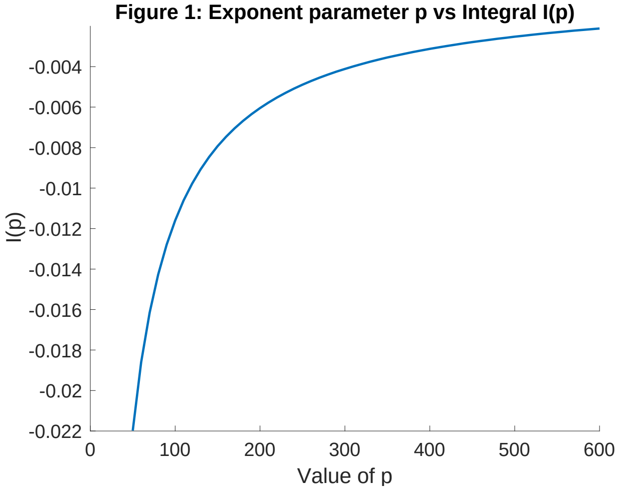
| Table 1: Qn vs Qhat approximation of I | | |
|---|---|---|
| | Qn | Qhat_n |
| N = 50 | -0.464289002017376 | -0.459361333772362 |
| N = 100 | -0.461917922130111 | -0.459360970845284 |
| N = 150 | -0.461094455803492 | -0.459360930686411 |
| N = 200 | -0.460674560911444 | -0.459360920518306 |
| N = 250 | -0.460419485047569 | -0.459360916813490 |
| N = 300 | -0.460247926419383 | -0.459360915150366 |
| N = 350 | -0.460124553884726 | -0.459360914294815 |
| N = 400 | -0.460031522253631 | -0.459360913810249 |
| N = 450 | -0.459958839353907 | -0.459360913515406 |
| N = 500 | -0.459900471625607 | -0.459360913325759 |

As depicted in Table 1, when we increase n from 50 to 500 in intervals of 50, the value of $Q_n$ will approach the value -0.459900471625607. As N grows to 450 and greater, the digits -0.4599 will not change in $Q_n$, thus, we can say that we can accurately compute 4 digits of I between values of N ranging from N = 50 to N = 500. Additionally, the digits -0.4 remain the same in each $Q_n$ approximation of I when N = 50 to N = 500. Therefore, we can say that I must be -0.4 when we take it to a precision of 1 digit after the decimal.

When we approximate I with Qhat_n using Aitken's method, the value of Qhat_n will approach -0.459360913325759 . As N increases to 400 and greater, the digits -0.459360913 will not change in Qhat_n, thus, we can accurately compute 9 digits of I between values of N ranging from N = 50 to N = 500. Additionally, the digits -0.45936 remain the same in each Qhat_n approximation of I for all trials of N. Therefore, we can say that I must contain the value -0.45936 when we take it to a precision of 5 digits after the decimal and we can accurately compute 5 digits of I with Aitken's method. I chose the 5 digits -0.45936 because they do not change for all trials of N. When we look at all 10 trials of N from N = 50 to N = 500, the value of Qhat_n will always contain -0.45936.

**Figure 1: Exponent parameter p vs Integral I(p)**




As depicted in Figure 1, as the values of p increase, the value of I(p) gets smaller. The value of I(p) appears to be converging to 0. This is most likely attributed to the fact that the function sin in the integral

$$I(p) = \int_0^1 \frac{\sin(x^{-p}\log(x))}{x}\,\mathrm{d}x,$$

has oscillations. As we increase p, the oscillations become very rapid. Eventually the period between each oscillation will become so small such that the integral will become very close to 0.

MATLAB Code

```
%Part 1

n = 50;
Q_arr = zeros(n,1);

I = @(x) ((sin((x.^(-1)).*log(x)))./x);

for n = 50:50:500

    %Calculate Qn
    I_arr = zeros(n,1);

    a = zeros(n+1,1);

    for i = 1:n+1
        b = fzero(@(x) x*exp(x)-i*pi,0);
        a(i) = exp(-b);
    end

    format long;
    I_arr(1) = integral(I,a(1),1);

    for i = 1:n
        I_arr(i+1) = integral(I,a(i+1),a(i));
    end

    format long;
    Q_n = sum(I_arr);

    %Calculate Qn+1
    I_arr1 = zeros(n+2,1);

    a1 = zeros(n+2,1);

    for i = 1:n+2
        b = fzero(@(x) x*exp(x)-i*pi,0);
        a1(i) = exp(-b);
    end

    format long;
    I_arr1(1) = integral(I,a1(1),1);

    for i = 1:n+1
        I_arr1(i+1) = integral(I,a1(i+1),a1(i));
    end

    format long;
    Q_n1 = sum(I_arr1);

    %Calculate Qn+2
    I_arr2 = zeros(n+3,1);
```

```
    a2 = zeros(n+3,1);

    for i = 1:n+3
        b = fzero(@(x) x*exp(x)-i*pi,0);
        a2(i) = exp(-b);
    end

    format long;
    I_arr2(1) = integral(I,a2(1),1);

    for i = 1:n+2
        I_arr2(i+1) = integral(I,a2(i+1),a2(i));
    end

    Q_n2 = sum(I_arr2);

    format long;
    Qhat_n = Q_n - (((Q_n1 - Q_n).^2)/(Q_n2-2*Q_n1 + Q_n));

    disp(n);
    disp(Q_n);
    disp(Qhat_n);

end

%Part 2
N_arr = zeros(0,30);
Qhat_arr = zeros(0,30);
pVal_arr = zeros(0,30);

k = 0;
for n = 50:10:600
    k = k+1;
    pVal_arr(k) = n;

    I = @(x) (sin(power(x,-n).*log(x)))./x;
    a = zeros(0,n);

    for i = 1:n+1
        b = fzero(@(x) x*exp(x*n)-i*pi,0);
        a(i) = exp(-b);
    end

    I_arr = zeros(0, n);
    format long

    I_arr(1) = integral(I, a(1), 1);
    for i = 1:n
        I_arr(i+1) = integral(I, a(i+1), a(i));
    end

    format long;
    Q_n = sum(I_arr);
```

```
    disp(Q_n);

    N_arr(i) = Q_n;

    a2 = zeros(0,n+1);

    for i = 1:n+2
        b = fzero(@(x) x*exp(x*n)-i*pi,0);
        a2(i) = exp(-b);
    end

    I_arr1 = zeros(0, n+1);

    format long
    I_arr1(1) = integral(I, a2(1), 1);
    for i = 1:n+1
        I_arr1(i+1) = integral(I, a2(i+1), a2(i));
    end

    format long;
    Q_n1 = sum(I_arr1);

    a3 = zeros(0,n+2);

    for i = 1:n+3
        b = fzero(@(x) x*exp(x*n)-i*pi,0);
        a3(i) = exp(-b);
    end

    I_arr2 = zeros(0, n+2);


    format long
    I_arr2(1) = integral(I, a3(1), 1);

    for i = 1:n+2
        I_arr2(i+1) = integral(I, a3(i+1), a3(i));
    end

    format long;
    Q_n2 = sum(I_arr2);

    format long;
    Qhat_n = Q_n - (((Q_n1 - Q_n).^2)/(Q_n2-2*Q_n1 + Q_n));

    Qhat_arr(k) = Qhat_n;

end

hold on;
plot(pVal_arr,Qhat_arr,'linewidth',2.0);
hold off;
set(gca, 'FontSize', 16);
```

```
xlabel('Value of p', 'fontsize',18);
ylabel('I(p)', 'fontsize', 18);
title('Figure 1: Exponent parameter p vs Integral I(p)');
grid on;
```