MACM 201 Assignment 3 Solutions September 23, 2018

Section 11.2

16. (a) $\binom{6}{3}(2^3) = \binom{6}{3}(2^{\binom{3}{2}})$ (b) $\binom{6}{4}(2^{\binom{4}{2}})$

(c) $\sum_{k=1}^{6} \binom{6}{k}(2^{\binom{k}{2}})$ (d) $\sum_{k=1}^{n} \binom{n}{k}(2^{\binom{k}{2}})$

Section 12.1

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4. $e = v - \kappa$

16. (1) This graph has $9 = 3 \cdot 4 - 3 = 3 + 3(4-2)$ vertices, so any spanning tree for it will have eight edges. There are $12 = 3 \cdot 4$ edges (in total) so we shall remove four edges. Two edges must be removed from one 4-cycle (a cycle on four vertices) and one edge from each of the other two 4-cycles. When two edges from a 4-cycle are removed one must be from the 3-cycle (induced by a, b, and c) – otherwise, we get a disconnected subgraph. There are three ways to select the 4-cycle for removing two edges and three ways to select the edge not on the 3-cycle. We then select one edge from each of the remaining 4-cycles in $4 \cdot 4$ ways. So the number of nonidentical spanning trees for this graph is $3(4-1)(4^2) = 144$.

(2) Here the graph has $8 = 4 \cdot 3 - 4 = 4 + 4(3-2)$ vertices and $12 = 4 \cdot 3$ edges. There are $4(3-1)(3^3) = 216$ nonidentical spanning trees.

(3) This graph has $16 = 4 \cdot 5 - 4 = 4 + 4(5-2)$ vertices and $20 = 4 \cdot 5$ edges. There are $4(5-1)(5^3) = 2000$ nonidentical spanning trees.

Section 12.2

2. (a)

Vertex	Level Number
p	35
s	36
t	36
v	37
w	38
x	38
y	39
z	39

(b) The vertex u has 37 ancestors.
(c) The vertex y has 39 ancestors.

4. (a) 5 (b) 2.1.3 (c) 4 (including the root)
(d) $2.1.3.x$, $1 \le x \le 5$; 2.1.3, 2.1.2, 2.1.1, 2.1, 2, 1.

10. (a) Here the maximum height is $n-1$.
(b) In this case n must be odd and the maximum height is $(n-1)/2$.

12. From Theorem 12.6 (c) we have
(a) $(\ell-1)/(m-1) = (n-1)/m \Longrightarrow (n-1)(m-1) = m(\ell-1) \Longrightarrow$
$n-1 = (m\ell - m)/(m-1) \Longrightarrow n = [(m\ell-m)/(m-1)]+1 =$
$[(m\ell-m)+(m-1)]/(m-1) = (m\ell-1)/(m-1)$.

(b) $(\ell-1)/(m-1) = (n-1)/m \Longrightarrow \ell - 1 = (m-1)(n-1)/m \Longrightarrow$
$\ell = [(m-1)(n-1)+m]/m = [(m-1)n+1]/m$.

20. The number of vertices at level $h-1$ is m^{h-1}. Among these we find $m^{h-1} - b_{h-1}$ of the l leaves of T. Each of the b_{h-1} branch nodes account for m leaves (at level h). Therefore, $l = m^{h-1} - b_{h-1} + mb_{h-1} = m^{h-1} + (m-1)b_{h-1}$.