

A)  $G$  is connected  
B)  $G$  is acyclic

} a tree

C)  $|V| = |E| + 1$

any two of A, B, C imply the other

$B + C \Rightarrow A$

Let  $G = (V, E)$  be an acyclic graph with  $|V| = |E| + 1$ .

Let  $G_1, G_2, \dots, G_k$  be the connected components of  $G$ . Each graph  $G_i$  must be a tree since it is connected and acyclic.

So (because we already proved  $A + B \Rightarrow C$ )

$$|V| = |V(G_1)| + |V(G_2)| + \dots + |V(G_k)|$$

$$= (|E(G_1)| + 1) + (|E(G_2)| + 1) + \dots + (|E(G_k)| + 1)$$

$$= |E(G)| + k. \quad \text{Thus } k=1 \text{ and A) holds.}$$