MACM 201 Homework 3 (Quiz Sep. 26)

Textbook problems:

Section	Question
11.2	16
12.1	4
12.1	16
12.2	2
12.2	4
12.2	10
12.2	12
12.2	20

Instructor question(s):

1. Let $G = (V, E)$ be a loopless graph with $|V| = n$. The **complement** $\overline{G}$ of G is the loopless graph $\overline{G} = (V, \overline{E})$ where $\overline{E} = \{\{x, y\} \mid x \neq y \text{ and } \{x, y\} \notin E\}$. In other words, the edge set of $\overline{G}$ is the set of all edges that are not in G. See Fig. 11.9 in your textbook for an illustration.

   (a) If $|E| = |\overline{E}|$ what is $|E|$ in terms of n?

   (b) If $|E| = |\overline{E}|$ show that either $n = 4k$ or $n = 4k + 1$ for some integer k.

   (c) Draw a graph on 5 vertices that is isomorphic to its complement.

   (d) Prove that if a graph G is not connected, then $\overline{G}$ is connected. (Hint: draw a small example with $n = 5, 6$ where G is not connected. Try to see why $\overline{G}$ is connected)

2. In this problem we consider spanning trees in the complete graph K_n

   (a) Draw all spanning trees of K_4

   (b) Let v be an arbitrary vertex of K_n. Find a formula for the number of spanning trees where v is a pendant vertex.