Robust Asset Allocation Strategies

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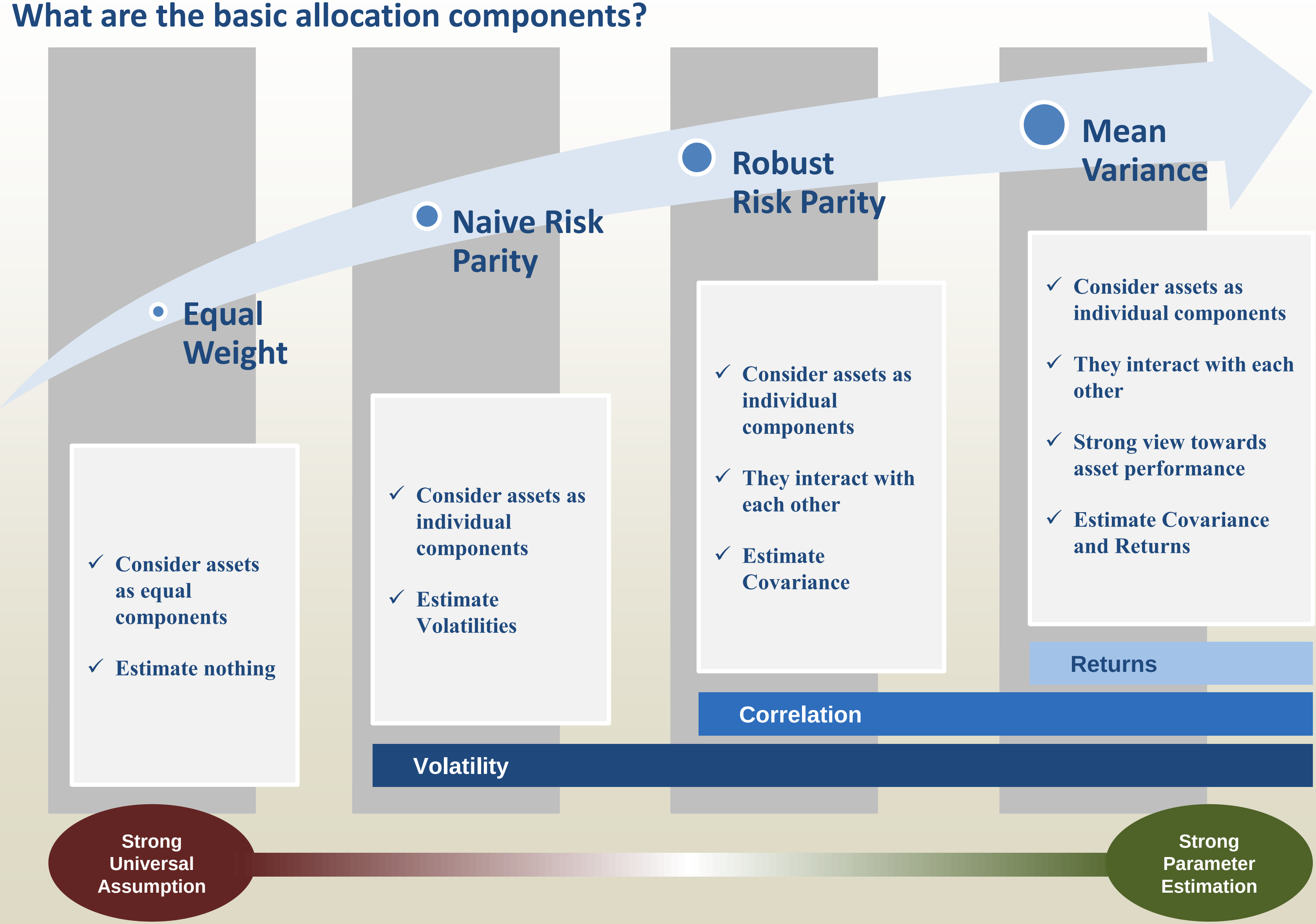
AGENDA

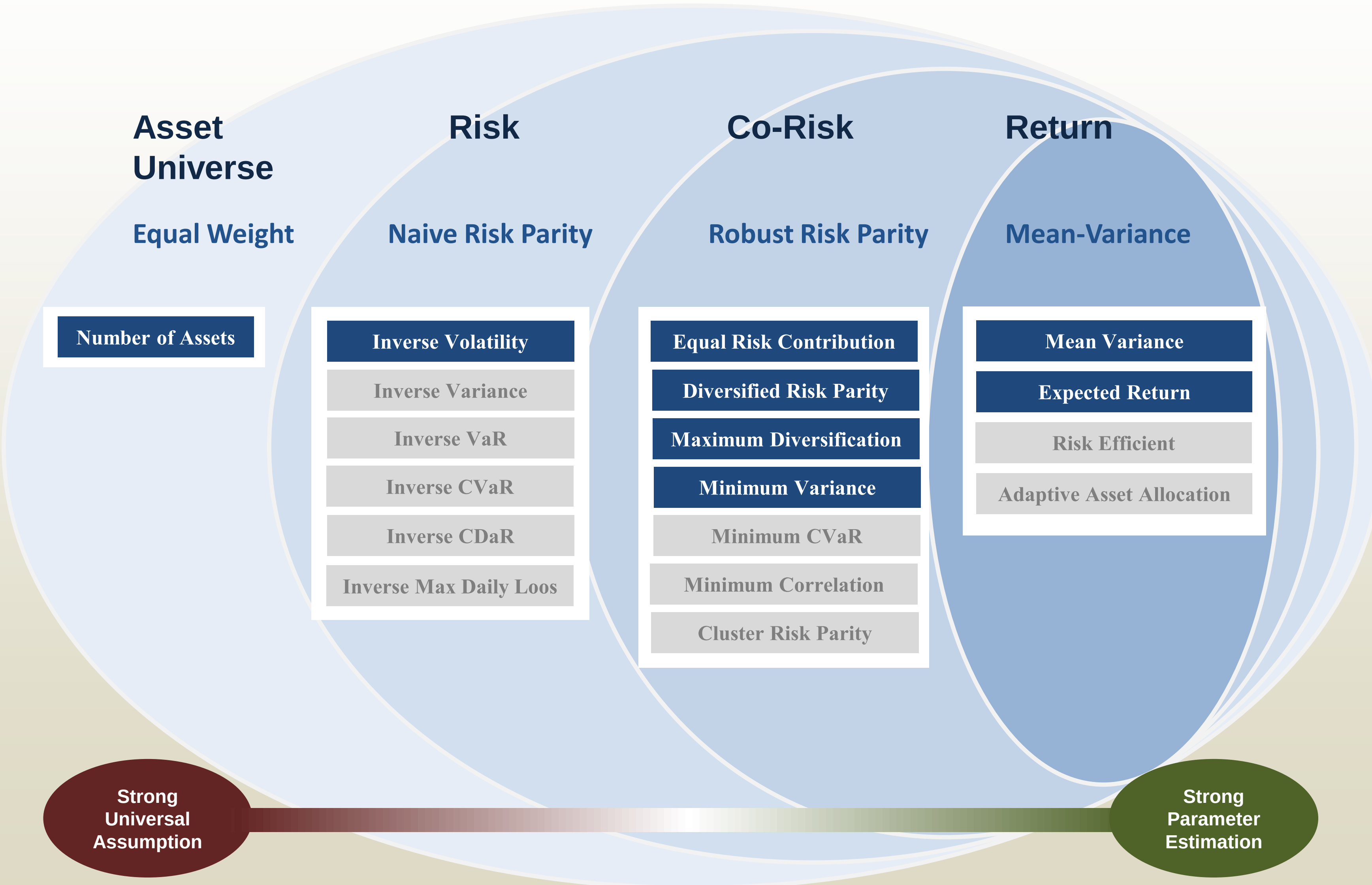
Thought Process	Methods	Robustness	Tactical Tools	Allocation 2015
• How to think about asset allocation? • Basic Components?	What are the approaches?	• Tests ○ Random Allocation ○ Leverage Invariance ○ Bootstrapping • Is their any approach that stands out?	Can trend help?	How should we allocate when interest rate increase?

Brainstorm → **Testing** → **Conclusion**

Understanding allocation framework:
What are the basic allocation components?



Theory and methods: What are the methods we can employ in each category?



Methods Explained: Quantification and Intuition

Strong Universal Assumption

Method	Intuition	Optimization function ($\Sigma w = 1,\ 0 \le w \le 1$)	Basic Components	Simple Examples
Equal Weighted (EW)	Equal ***dollar amount*** in ***assets***	$w = \frac{1}{n}$	--	S: SP500 B: BONDS **wt(S) = 50% wt(B) = 50%**
Equal Volatility Allocation (EV)	Equal ***Volatility Contribution*** by different ***assets***	$w = \frac{1}{\sigma}$	Volatility	*Vol(S) = 10% Vol(B) = 5%* **wt(S) = 33% wt(B) = 66%**
Risk Parity (RP)	Equal ***Risk Contribution*** by different ***assets***	"equalize" $w_i \frac{\Sigma \cdot w}{w^T \cdot \Sigma \cdot w}$ $RC_i = w_i \frac{\partial \sigma_p}{\partial w_i}$	Volatility Correlation	*Vol(S) = Vol(N) = Vol(B)* *corr(S,N) = 1 corr(S, B) = 0 corr(N,B) = 0* **RC(S) = RC(N) = RC(B) = 33%** **wt(S) = 30% wt(N) = 30% wt(B) = 40%**
Diversified Risk Parity (DRP)	Equal ***Risk Contribution*** by different ***risk sources*** *(PCA based)*	"equalize" $W_{PC(i)} \frac{\tilde{\Sigma} \cdot W_{PC(i)}}{W_{PC}^T \cdot \tilde{\Sigma} \cdot W_{PC}}$ $w = E \cdot W_{PC}$	Volatility Correlation	*Similar assumptions as RP* PC1 = S + N, PC2 = B **RC(PC1) = 50% RC (PC2) = 50%** **wt(S) = 25% wt(N) = 25% wt(B) = 50%**
Most Diversified Portfolio (MDP)	Equal ***Risk Contribution*** by different ***risk sources*** *(Diversification Ratio)*	$\underset{w}{\mathrm{argmax}} \quad \frac{w\sigma}{\sqrt{w^T \cdot \Sigma \cdot w}}$	Volatility Correlation	*Similar assumptions as RP* **RC(S) = 25% RC(N) =25% RC(B) = 50%** **wt(S) = 25% wt(N) = 25% wt(B) = 50%**
Minimum Variance (MinV)	Minimize **Variance**	$\underset{w}{\mathrm{argmin}} \quad w^T \cdot \Sigma \cdot w$	Volatility Correlation	*Vol(S) = 20% Vol(B) = 2%* *Corr(S,B) = 0* **wt(S) = 0% wt(B) = 100%**
Mean Variance (MeanV)	Minimize **Variance** while Maximizing **returns**	$\underset{w}{\mathrm{argmin}} \quad w^T \cdot \Sigma \cdot w - q \cdot w \cdot R^T$	Volatility Correlation Return	*Vol(S) = 10% Vol(B) = 5%* *Corr(B,S) = 0* *Ret(S) = 20% Ret(B) = 10%* **wt(S) = 40% wt(B) = 60%**
Expected Return (ER)	Maximize **Returns**	$\underset{w}{\mathrm{argmax}} \quad w \cdot R^T$	Return	*Ret(S) : 20% Ret(B) : 10%* **wt(S) = 100% wt(B) = 0%**

Strong Parameter Estimation

n: Number of assets, σ: Volatility Vector, Σ: Covariance matrix, $\tilde{\Sigma}$: Eigenvalue matrix, R: Returns Vector, w: Weights vector, E: Eigenvector
S: SP500, N: NAS100, B:Bond; RC: Risk Contribution, PC: principle Components, T: Transpose q: risk tolerance

Data and Simulation

INPUTS

Use 500 days of in-sample returns (ISR) to compute Mean (Mu) and Covariance (S). Testing over the period of 2000 to 2014.

FUNCTION

Conduct Portfolio Optimization (F) using the aforementioned methods.

OUTPUTS

Compute weights for each asset (W) under each method.

OUTSAMPLE

Apply weights to the next 60 days out-of-sample returns (OSR) to compute Sharpe Ratio (SR), Drawdown (DD) and Risk Contribution (RC).

Data Characteristics

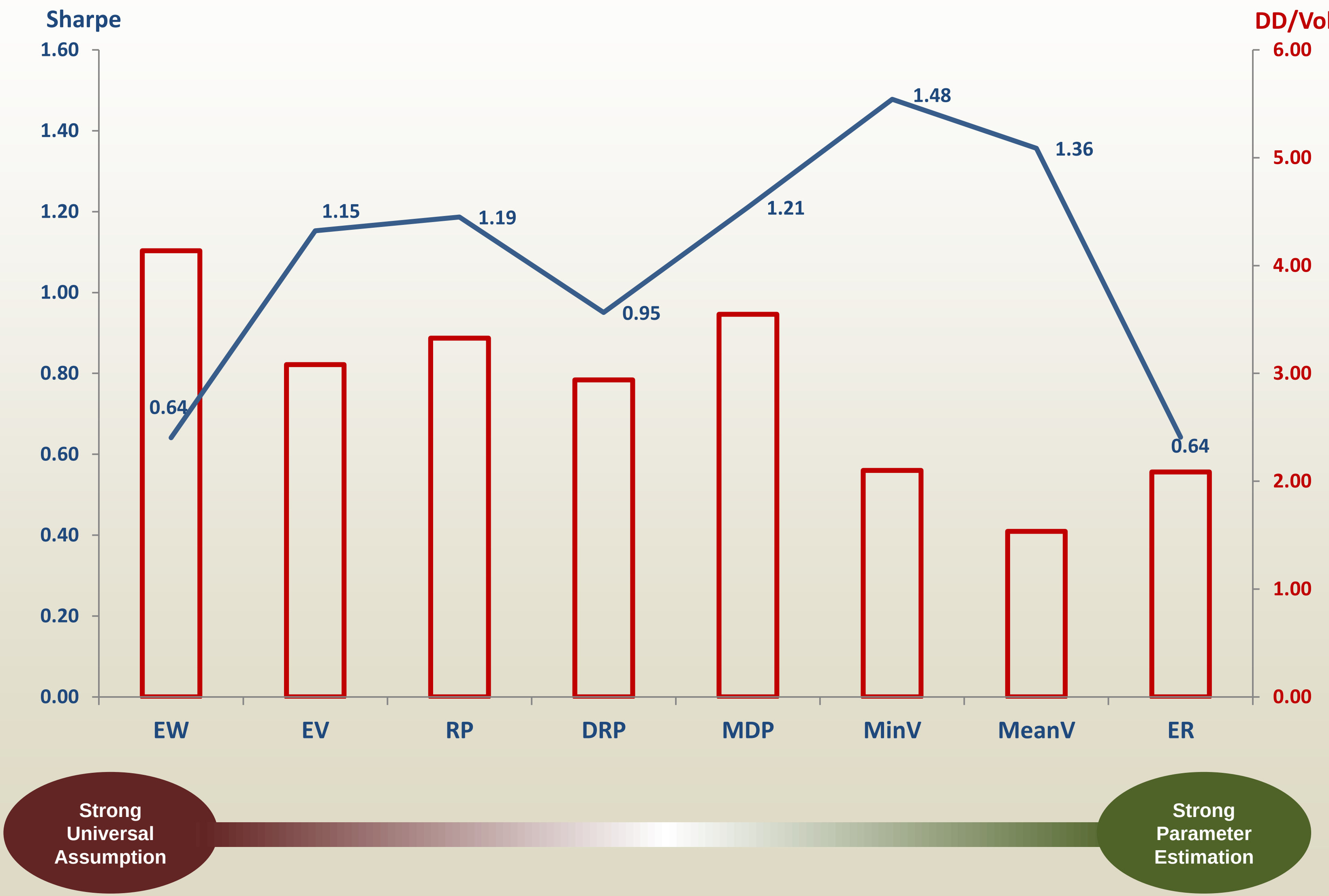
- Bonds generally performs better than other asset classes
- Commodity have overall poor performance
- Equity and Commodity are more volatile and they show significant Max Drawdown
- Bond and Equity shows negative correlation
- Bond and CTA shows positive correlation
- Equity and Commodity shows positive correlation
- CTA and Commodity shows positive correlation

Performance Measurement	Bonds	Equity	Commodity	CTA
Average Return	5.58	4.25	2.70	5.40
Average Vol	3.96	19.96	16.65	8.00
SR	1.41	0.21	0.16	0.67
Max DD	5.02	73.62	80.97	13.99
DD/Vol	1.27	3.69	4.86	1.75

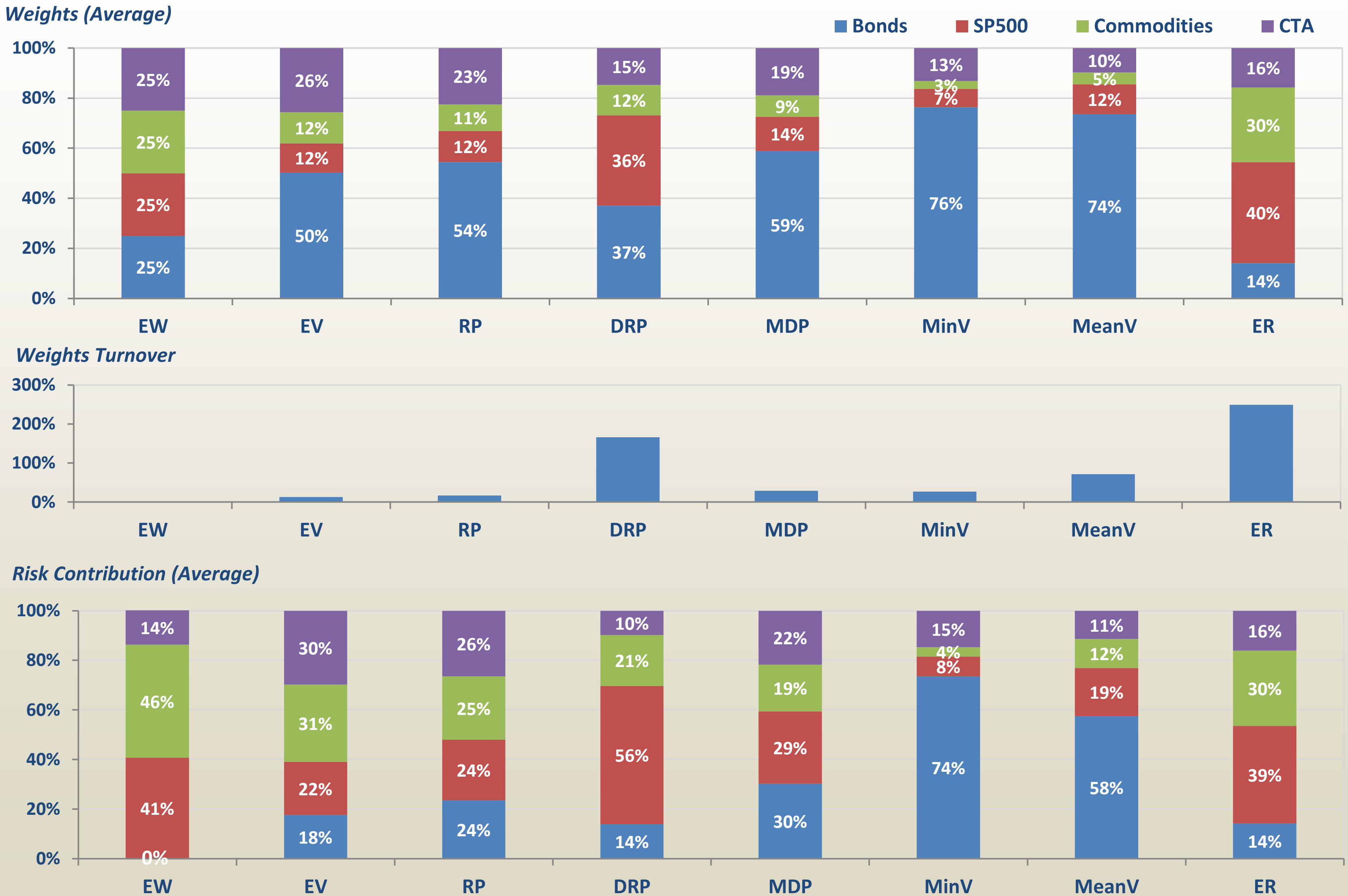
Correlation Matrix	Bonds	Equity	Commodity	CTA
Bonds	1			
Equity	-0.25	1		
Commodity	-0.08	0.24	1	
CTA	0.17	-0.11	0.17	1

Bonds: **Barclay Capital Bond Index**
Equity: **SP500 Total Return Index**
Commodity: **DJUBS Commodity Index**
CTA: **NewEdge CTA Index**

Base Results:
Out-of-Sample Sharpe Ratio and Drawdown-to-Volatility



Base Results:
Weights, Out-of-Sample Risk Contribution (average over the entire period)



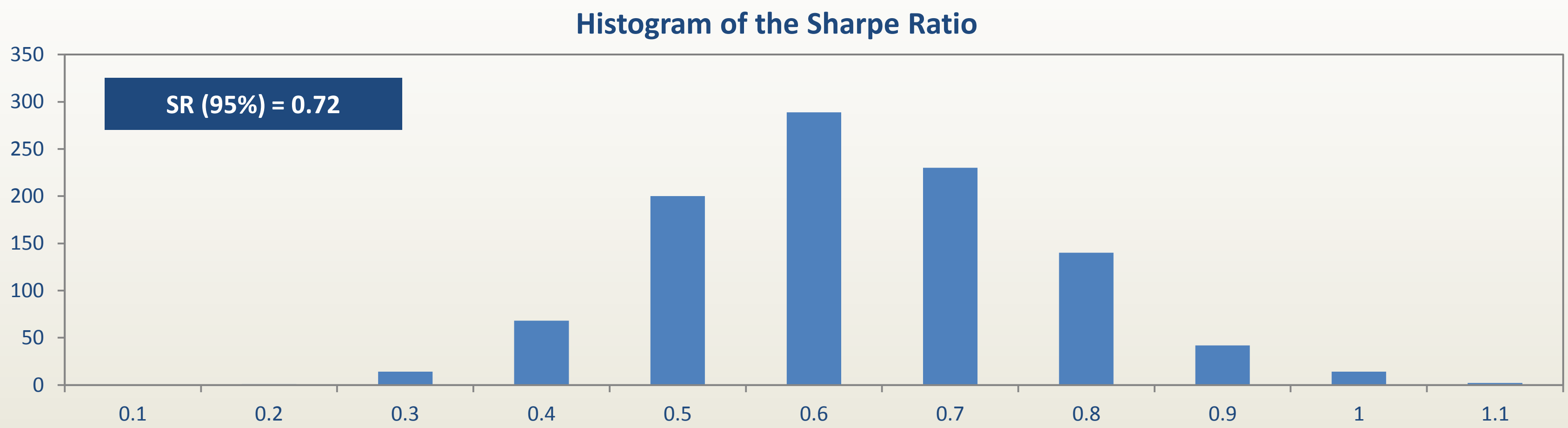
Robustness Testing

- **TEST 1: Random Allocation**
  - Does out-of-sample performance beat random allocation?
  - Method is expected to out-perform random allocation
- **TEST 2: Leverage Invariance**
  - Increase Bond volatility, Decrease Stocks volatility
  - Performance of consistent methods should be unchanged
- **TEST 3: Bootstrapping**
  - Null Hypothesis: Sharpe Ratio (base case) is true?
  - Method with higher *t*-statistic means it is likely to out-perform after accounting for sample bias

Robustness Testing:
TEST 1 – Random Allocation

Construction

- Randomly allocate across the 4 assets, $s.t. \Sigma w = 1$



	EW	EV	RP	DRP	MDP	MinV	MeanV	ER
Result	No (0.64 < 0.72)	Yes (1.15 > 0.72)	Yes (1.19 > 0.72)	Yes (0.95 > 0.72)	Yes (1.21 > 0.72)	Yes (1.48 > 0.72)	Yes (1.36 > 0.72)	No (0.64 < 0.72)

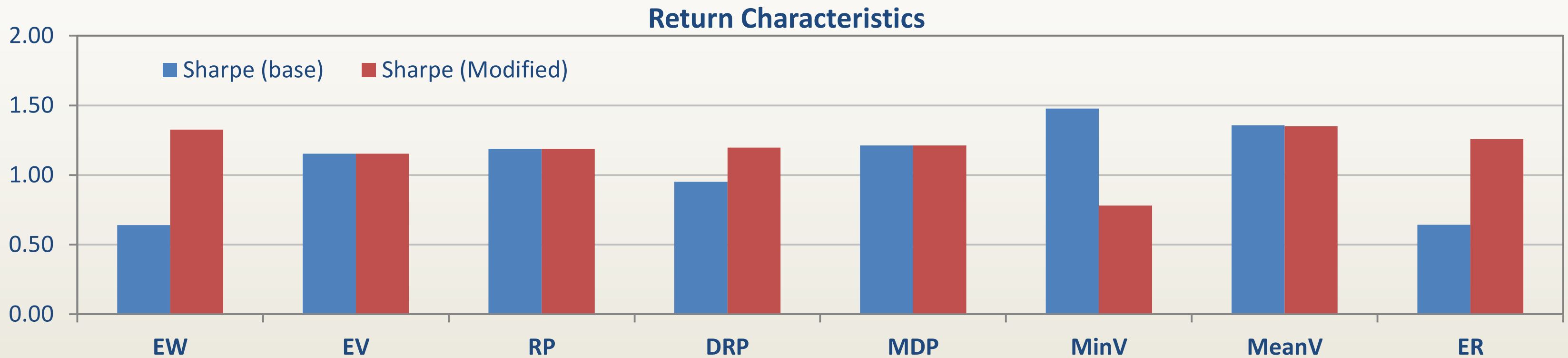
Takeaways

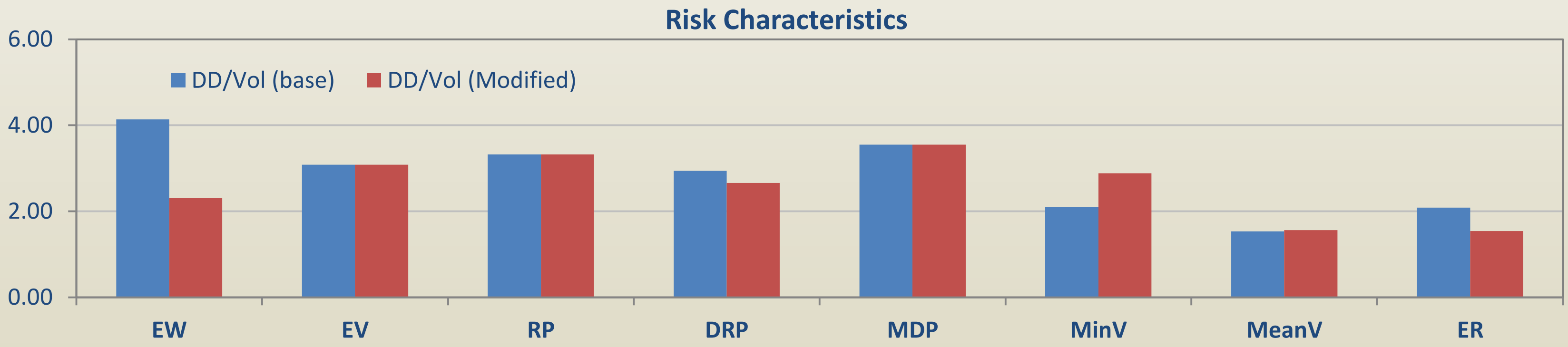
- Risk based allocation performs significantly better
- Random Allocation on an average yields a result similar to equal weighted (EW)
- ER also performs poorly: can be a result of mean reversion

Robustness Testing: TEST 2 – Leverage Invariance

Construction

- Increase the volatility of bonds from 4% to 30% & decreasing the volatility of stocks from 20% to 5%
- Keep correlation and risk adjusted return characteristics intact





	EW	EV	RP	DRP	MDP	MinV	MeanV	ER
Result	No	Yes	Yes	No	Yes	No	Yes	No

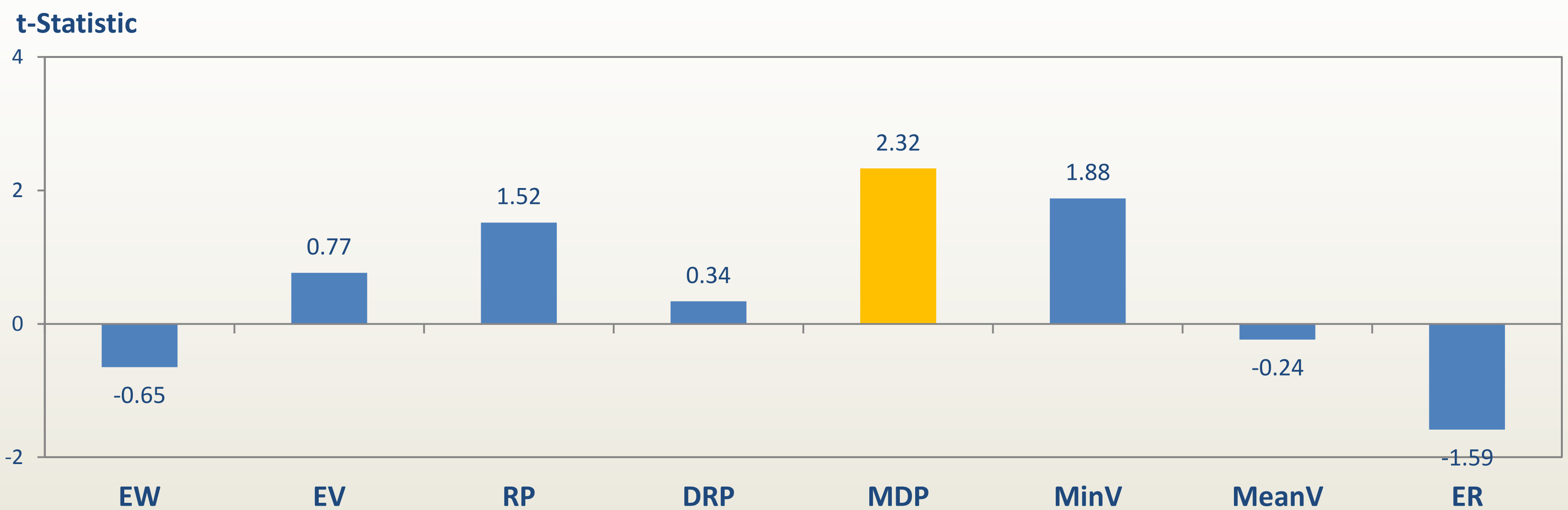
Robustness Testing:
TEST 3 – Bootstrapping Construction

INPUTS	Use 500 days of in-sample returns (ISR) to compute Mean (Mu) and Covariance (S). Testing over the period of 2000 to 2014.
FUNCTION	Conduct Portfolio Optimization (F) using the aforementioned methods.
OUTPUTS	Compute weights for each asset (W) under each method.
OUTSAMPLE	Apply weights to the next 60 days out-of-sample returns (OSR) to compute Sharpe Ratio, Drawdown and Risk Contribution.

1000 Iterations

INPUTS	Random sample 500 data points as in-sample returns (ISR).
FUNCTION	Conduct Portfolio Optimization (F) using the aforementioned methods.
OUTPUTS	Compute weights for each asset (W) under each method.
OUTSAMPLE	Random sample 60 data points as out-of-sample returns (OSR).

Robustness Testing:
TEST 3 – Bootstrapping Result



	EW	EV	RP	DRP	MDP	MinV	MeanV	ER
Sharpe Ratio (Base)	0.64	1.15	1.19	0.95	1.21	1.48	1.36	0.64
Sharpe Ratio (Sampling)	0.61	1.19	1.26	0.97	1.33	1.58	1.35	0.56
***t*-Statistic**	-0.65	0.77	1.52	0.34	2.32	1.88	-0.24	-1.59
***p*-value**	0.52	0.45	0.14	0.73	0.03	0.07	0.81	0.12
will reality outperform?	No	No	No	No	Yes	No	No	No

Robustness Testing:
Which is robust?

	EW	EV	RP	DRP	MDP	MinV	MeanV	ER
Test 1 Random Allocation	X	√	√	√	√	√	√	X
Test 2 Leverage Invariance	X	√	√	X	√	X	√	X
Test 3 Bootstrapping	X	X	X	X	√	X	X	X

- **MDP passed all three test and hence is the most robust approach**
  - It outperforms random allocation
  - It is immune to individual asset volatility
  - It outperforms after accounting for sample bias
- **RP, EV and MeanV are competitive approaches**
  - They add value to performance and are consistent through volatility shifts
  - They may underperform as the measurements are subject to sample bias
- **DRP and MinV are not quite so robust**
  - DRP is sensitive to the input data and hence reduced stability through time
  - MinV leads to concentration in low vol assets, and thus shows significant performance drop once return sample changes
- **EW and ER are not robust methods**
  - They do no better than roll a dice and decide where to put our money!

Performance and Robustness:
Methods recap, who is the winner?

Measurements	EW	EV	RP	DRP	MDP	MinV	MeanV	ER
Sharpe (rank best to worst)	7	5	4	6	3	1	2	7
DD/Vol (rank best to worst)	8	5	6	4	7	2	1	3
Turnover (rank best to worst)		1	1	4	2	2	3	4
Robust Test 1 Random Allocation	X	√	√	√	√	√	√	X
Robust Test 2 Leverage Invariance	X	√	√	X	√	X	√	X
Robust Test 3 Bootstrapping	X	X	X	X	√	X	X	X

- **From robustness stand point, MDP is a winner**
- **MDP requires performance enhancement to improve its risk characteristics i.e. DD/Vol**
- Can a trend overlay, help manage drawdown and enhance risk adjusted returns?

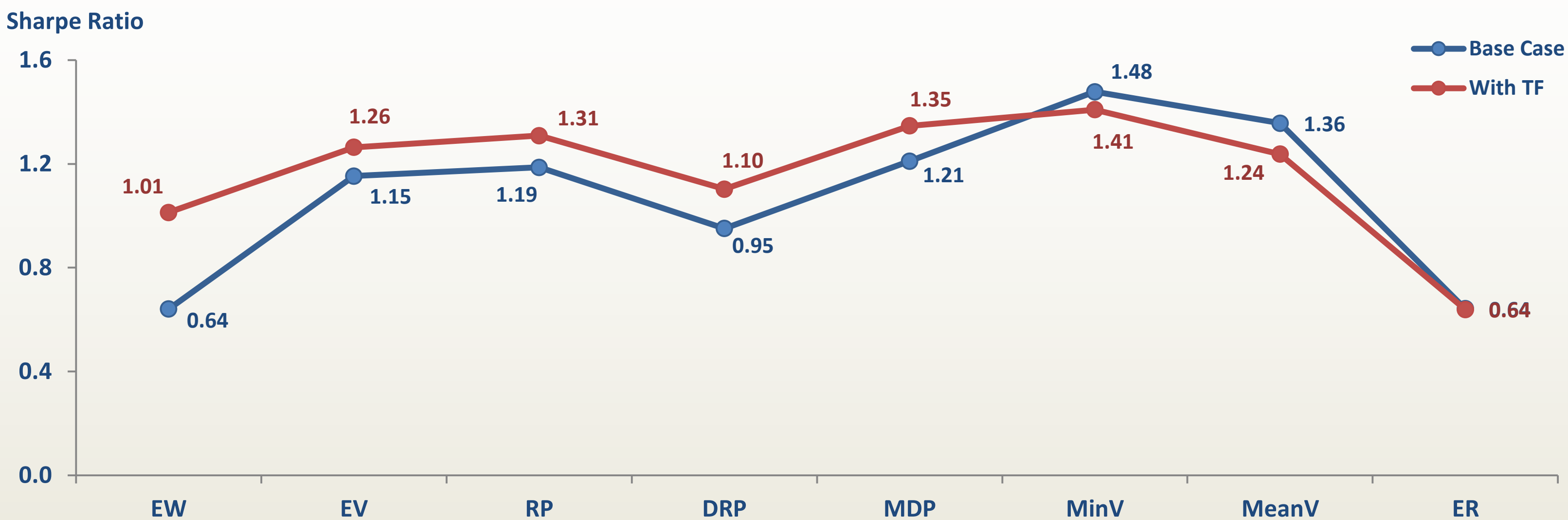
Ranking is based on rounding off metric to 2 decimal places

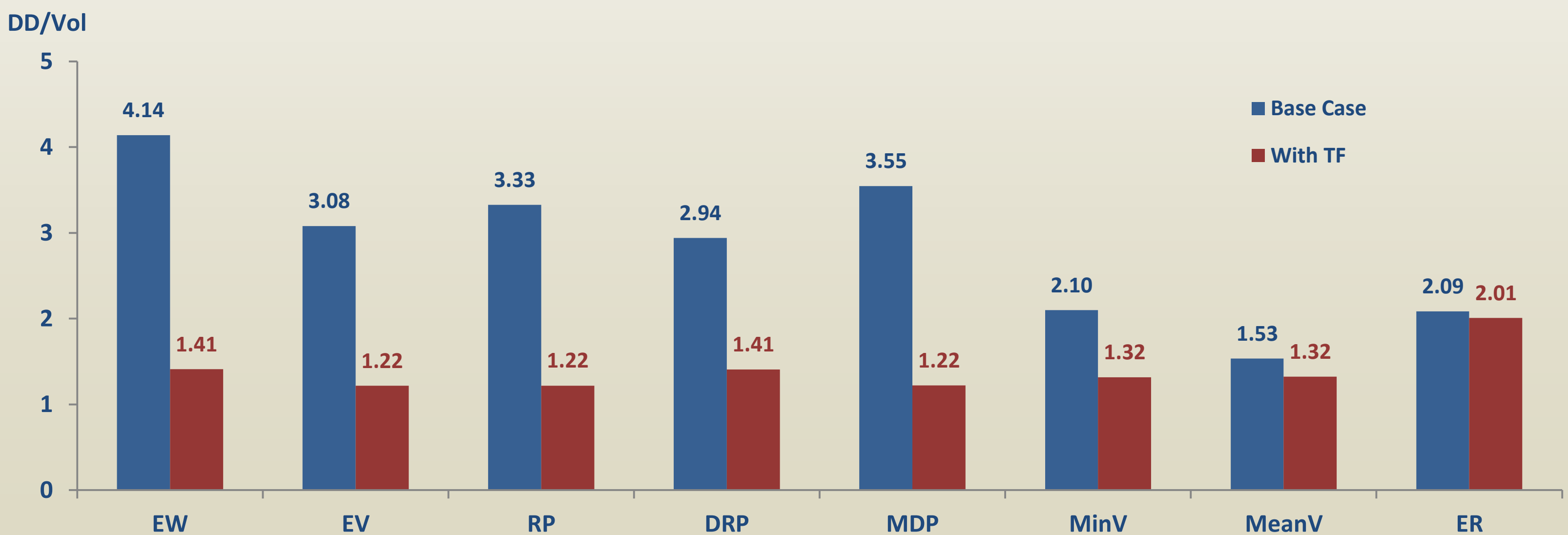
Can trend help?
Overlay construction

INPUTS	Use 500 days of in-sample returns (ISR) to compute Mean (Mu) and Covariance (S). Testing over the period of 2000 to 2014.
FUNCTION	Conduct Portfolio Optimization (F) using the aforementioned methods.
OUTPUTS	Compute weights for each asset (W) under each method.
OUTSAMPLE	Apply weights to the next 60 days out-of-sample returns (OSR) to compute Sharpe Ratio, Drawdown and Risk Contribution.

INPUTS	Use 500 days of in-sample returns (ISR) to compute Mean (Mu) and Covariance (S).
FUNCTION	Conduct Portfolio Optimization (F) using the aforementioned methods.
OUTPUTS	Compute weights for each asset (W) under each method.
TREND FILTER	**5-day SMA > 60-day SMA, 60-day SMA > 120-day SMA If asset failed the filter, allocate its capital to cash.**
OUTSAMPLE	Apply adjusted weights to the next 60 days out-of-sample returns (OSR) to compute Sharpe Ratio, Drawdown and Risk Contribution.

Can trend help?
Overlay testing results: Sharpe and Drawdown-to-Vol

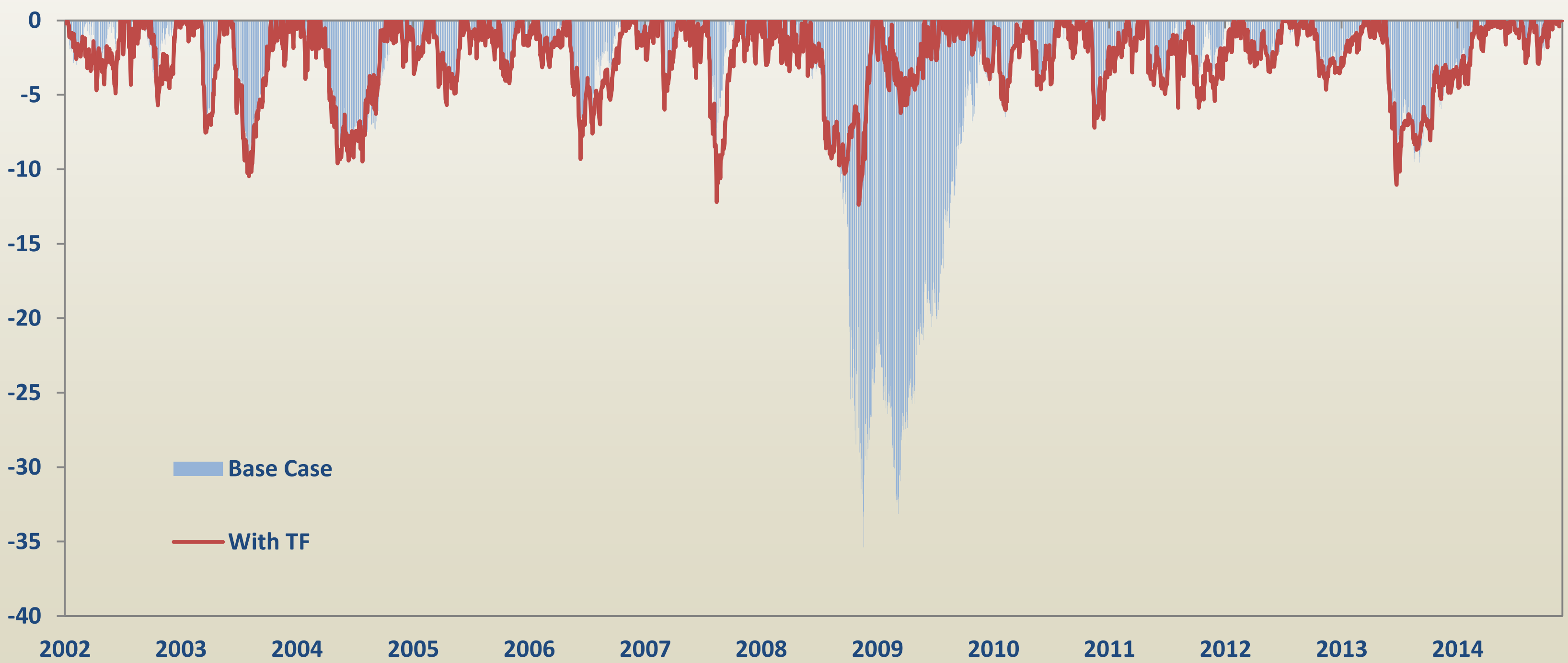




Can trend help?
Overlay testing results: Maximum Drawdown

- **Sharpe ratio improved across the board, except for MinV and MeanV**
- **DD/Vol improved for all the methods significantly**
  - The trend overlay equalized DD/Vol across the board
- **The simple trend overlay is able to manage the Drawdown effectively**

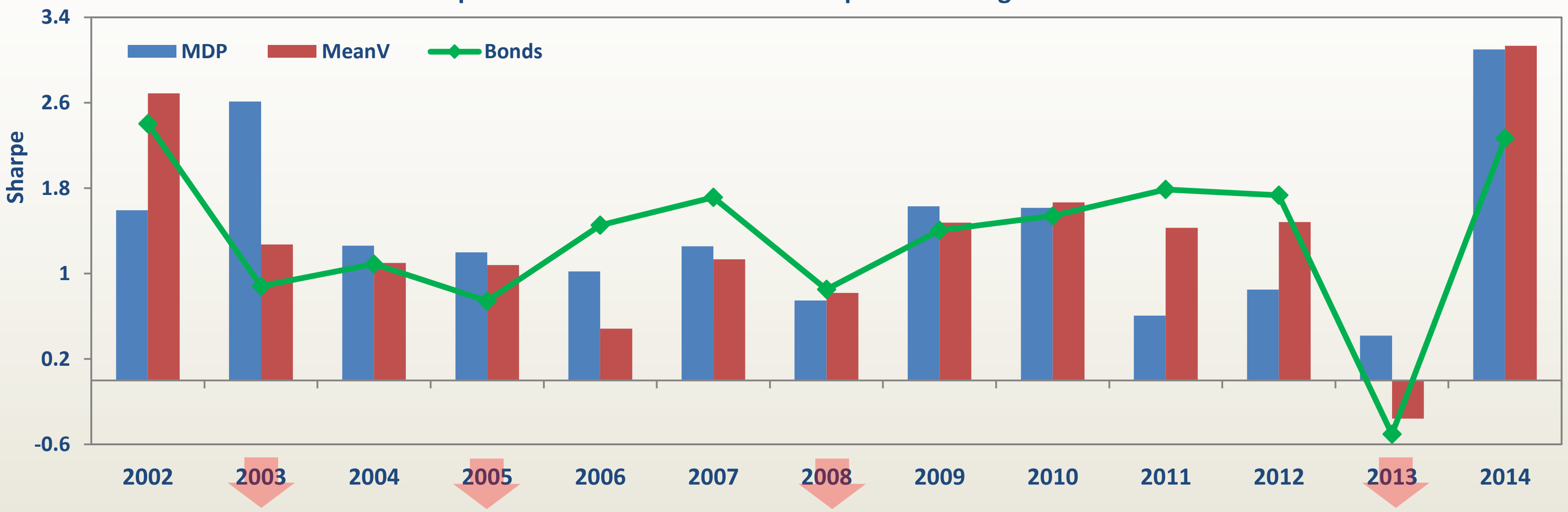
MDP Maximum Drawdown Comparison



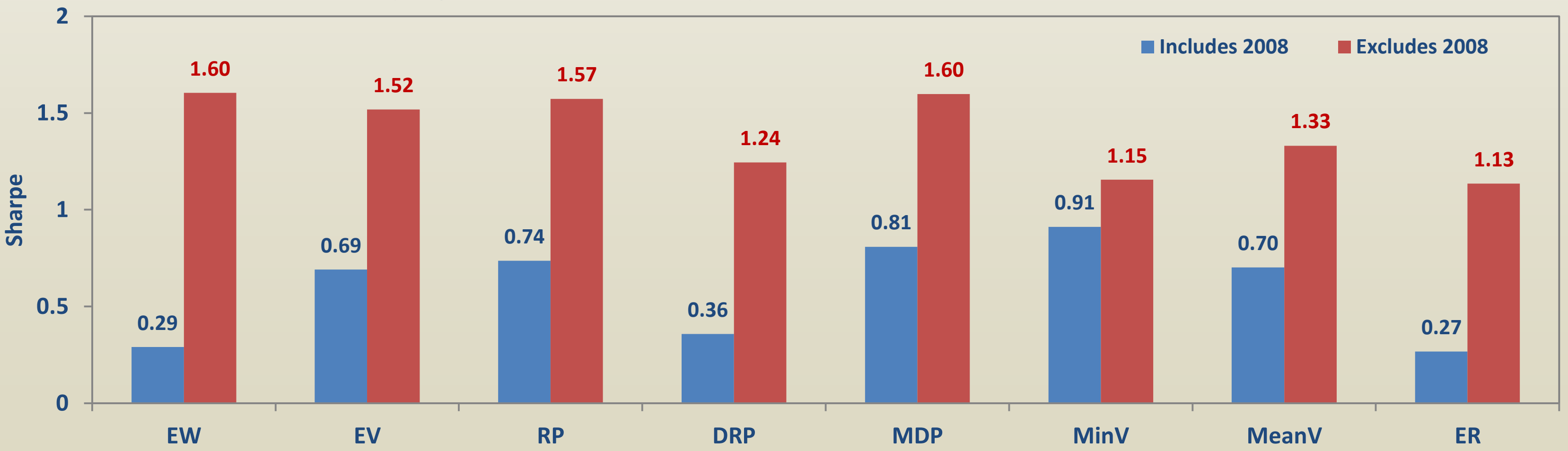
Allocation with Rising Rates

- MDP out-performs when bonds performs poorly

Sharpe Ratio of MDP and MeanV Comparison during 2002 to 2014



Sampling returns from bond down years (2003, 05, 08, 13) where bond SR <1



Performance and Robustness 2015
Method recap, who is the FINAL winner?

Measurements	EW	EV	RP	DRP	MDP	MinV	MeanV	ER
Sharpe w. Overlay (rank best to worst)	7	5	3	6	2	1	4	8
DD/Vol w. Overlay (rank best to worst)	3	1	1	3	1	2	2	4
Turnover (rank best to worst)		1	1	4	2	2	3	4
Robust Test 1 Random Allocation	X	√	√	√	√	√	√	X
Robust Test 2 Leverage Invariance	X	√	√	X	√	X	√	X
Robust Test 3 Bootstrapping	X	X	X	X	√	X	X	X
Sharpe w. Overlay w. rising rates (rank best to worst)	1	3	2	5	1	6	4	7

- **With a simple trend overlay, we are able to reduce the drawdown for MDP**
- **MDP is a preferred method especially with rising interest rates expectations**
- **Risk based approaches RP and EV can also prove to be good choices**

Ranking is based on rounding off metric to 2 decimal places

Conclusion

Thought Process	Methods	Robustness	Tactical Tools	Allocation 2015
• Confidence level on Parameter Estimation • Choices to estimate Risk, Co-Risk and Returns	• Naïve Based • Risk Based • Return Based	• Tests ○ Random Allocation ○ Leverage Invariance ○ Bootstrapping • Risk based approaches are more robust • MDP the best	• Trend filter improves DD/Vol	• Risk based approaches do better with rising rates • MDP and RP are the top choices

References

- Introduction to Risk Parity and Budgeting, T. Roncalli
- Introducing Expected Returns into Risk Parity Portfolios: A New Framework for Tactical and Strategic Asset Allocation, T. Roncalli
- The Trend is Our Friend: Risk Parity, Momentum & Trend Following in Global Asset Allocation, Clare. A, Seaton J
- Toward Maximum Diversification, Choueifaty, Yves and Yves Coignard
- Parity Strategies and maximum diversification, Putnam Investments
- Diversifying Risk Parity, Lohre Harold, Opfer Heiko

Thank You
&
Questions

Appendix

More Comparisons

Exhibit 4. The Parity Strategies Family

Parity Strategy	Volatility Parity	Risk Parity	Correlation Parity
a.k.a.	Minimum Concentration	Equal Risk Contribution	Most Diversified Portfolio
When does it provide maximum diversification? (in-sample)	When all pairwise asset correlations are equal	When all pairwise asset correlations are equal	Always
What is being equalized?	Asset volatility contributions	Asset risk contributions	Correlation-weighted asset risk contributions
	$\mathbf{I} \bullet (\mathbf{W} \bullet \underbrace{(\Omega \bullet \mathbf{I} \bullet \Omega \bullet \omega)}_{\text{risk contributions}})$	$\mathbf{I} \bullet (\mathbf{W} \bullet (\Omega \bullet \mathbf{C} \bullet \Omega \bullet \omega))$	$\mathbf{C} \bullet (\mathbf{W} \bullet (\Omega \bullet \mathbf{C} \bullet \Omega \bullet \omega))$
	weights risk contributions		

where: $\mathbf{I}$ = identity matrix $\mathbf{C}$ = asset correlation matrix

ω = asset weights $\mathbf{W}$ = diagonal matrix of asset weights

Ω = diagonal matrix of asset volatilities $\bullet$ indicates matrix multiplication

$$DR = \frac{weighted\ \ average\ \ volatility}{portfolio\ volatility} = \frac{\sum \omega_i \sigma_i}{\sigma_P}$$

where

DR = portfolio Diversification Ratio

ω_i = the weight of asset i

σ_i = the volatility of asset i

σ_P = the volatility of the portfolio

$$DR = (\rho(1 - CR) + CR)^{-\frac{1}{2}}$$

where

DR = portfolio Diversification Ratio

CR = portfolio Concentration Ratio

ρ = portfolio volatility-weighted average correlation

$$\mathbf{CR}(\boldsymbol{w}) = \frac{\sum_i (w_i \sigma_i)^2}{(\sum_i w_i \sigma_i)^2}.$$

$$\rho(\boldsymbol{w}) = \frac{\sum_{i \neq j} (w_i \sigma_i w_j \sigma_j) \rho_{i,j}}{\sum_{i \neq j} (w_i \sigma_i w_j \sigma_j)},$$

Diversification Ratio (intuition)