Overtones, Intervals, and Generative Transformations in György Ligeti's Hamburg Concerto (draft)

Kris Shaffer
Yale University

György Ligeti's Hamburg Concerto opens with a canon. There are three voices: horns 1 and 2, horn 3, horn 4. The two upper voices proceed up by whole tone (E♭ to F, F to G), the lower voice down by whole tone (E♭ to D♭). Because all three of these voices are played on natural horns (horns 1 and 2 in F, 3 and 4 in E♭), the whole tone (interval category i2) is here manifest as three different pitch intervals: 8:7 ratio (230c with Ligeti's tuning approximations), 9:8 ratio (200c), and 10:9 ratio (185c). Further, the E♭ and F natural horns are tuned i2 apart, and this i2 can be considered a 9:8 or equal tempered whole-tone (200c in Ligeti's approximations). This variety of whole-tone intervals (the 11:10, 165c whole-tone also enters the picture in m. 6) raises the question: are the differences in interval motivic or structural?

This question extends beyond the whole-tones of the opening bars. Throughout the work, pitches and intervals produced on one or more of the natural horns not only conflict with analogous pitches or intervals on other natural horns in other keys or on other partials; they also conflict with the equal-tempered chromatic space in which the orchestra plays. Some of these conflicts are trivial, for example the difference between an equal-tempered whole tone (200c) and a just-tuned 9:8 whole tone (204c). Listeners are likely to hear such differences (if they are even perceived) as negligible differences between two musicians who are not 'in tune' with each other, not as intervals which are structural to the work or the musical language in which it is written. Performers are also likely to unconsciously 'correct' such pitch differences, despite Ligeti's instructions to the contrary. In the Hamburg Concerto, however, Ligeti does simplify matters in this regard by approximating tuning, compressing all possible tunings of a notated pitch into four degrees of pitch alteration: unaltered (equal-tempered pitches, fundamentals, 3rd and 9th partials, and octaves); 'just a bit lower'—15c lower (5th and 15th partials and octaves); 'lower'—30c lower (7th partial and octaves); and 'quarter-tone lower'—50c lower (11th and 13th partials and octaves).

pitch category:	C	C-sharp/D-flat				D				D-sharp/E-flat				E				F			
partial:	—	11°	7°	5°	—	11°	7°	5°	—	11°	7°	5°	—	11°	7°	5°	—	11°	7°	5°	—
pitch (cents):	0	50	70	85	100	150	170	185	200	250	270	285	300	350	370	385	400	450	470	485	500

etc.

Figure 1. Ligeti's Tuning Approximations

But even within this system of approximations, there are 49 potential pitches between any note and the octave above it, inclusive (four for each of the twelve pitch classes, plus the octave), and there are 133 potential intervals between the size of a unison and an octave, inclusive, with 11 possible interval sizes for each interval type (i.e. 11 major seconds, 11 minor thirds, etc.; c.f. figure 2).

Intervals of a given type (general)

Interval (cents) (x=ET int.)	Constituent partials (lower, upper)
x-50	(-, 11°) (11°, -)
x-35	(5°, 11°)
x-30	(-, 7°)
x-20	(7°, 11°)
x-15	(-, 5°) (5°, 7°)
x	(-, -) (5°, 5°) (7°, 7°) (11°, 11°)
x+15	(7°, 5°) (5°, -)
x+20	(11°, 7°)
x+30	(7°, -)
x+35	(11°, 5°)
x+50	(-, 11°) (11°, -)

Example: all possible whole-tone (i2) intervals

Interval (cents)	Constituent partials (lower, upper)
150	(-, 11°) (11°, -)
(11:10) 165	(5°, 11°)
170	(-, 7°)
180	(7°, 11°)
(10:9) 185	(-, 5°) (5°, 7°)
(9:8, ET) 200	(-, -) (5°, 5°) (7°, 7°) (11°, 11°)
215	(7°, 5°) (5°, -)
220	(11°, 7°)
(8:7) 230	(7°, -)
235	(11°, 5°)
250	(-, 11°) (11°, -)

Figure 2. Potential Intervals Within in *Hamburg Concerto* Tuning System

Considering Ligeti's instructions to the horn players not to alter the overtone pitches with their right hands (and Ligeti's alteration of the orchestral instruments' pitch in some places to match them), and considering the methodical way in which Ligeti conceptualizes and notates the altered pitches, is seems a far assumption that Ligeti intends the resulting issues of pitch and interval to be structural, or at least significant to the work. The first two bars of the work reinforce the idea that the difference in interval size is motivic or structural. Canon voices 2 and 3 (horns 3 and 4, in E♭) begin the movement on E♭ and F as a 9:8 (200c) whole-tone. Voice 1 (horns 1 and 2, in F) answers with E♭ and F as an 8:7 (230c) whole-tone. Thus, in the first four notes of the piece, Ligeti uses two pitches shared by the two series in play, and foregrounds the difference in tuning, leaving the impression that the tuning difference is structural.

But there is also something the same about each of these differently tuned whole-tones. The change in gestalt which occurs within each canonic voice is analogous to that of the other two, despite the difference in pitch distance traveled; i.e., each voice moves up or down by one partial in the overtone series. Elsewhere in the work (as will be seen in this paper), the difference in pitch distance traveled by a single gestalt shift is much greater and extends beyond the boundaries of a single interval category like i2 (in fact, each gestalt shift corresponds to an infinite number of pitch-space intervals). This duality whereby the same 'change in gestalt' produces differences in 'pitch distance traveled' suggests that an analysis based on Lewinian transformational theory may reveal something useful about the underlying structure of such passages. Indeed, this is the case. Underneath the surface of a number of passages in the Hamburg Concerto which exhibit complex pitch relationships resulting from the use of overtone-based motives and sonorities is a much simpler transformational structure. A number of complex passages in the natural horns are generated from a small set of simple transformations which act on the overtone series, and those transformations are members of a mathematical group. The transformations of this group are often arranged in a *product network*, a simple, yet highly organized structure that, together with one or more overtone series, can generate a complex pitch surface from as few as two underlying transformations.

Over the course of this paper, I will explore several ways in which complexity on the pitch surface of the Hamburg Concerto is generated by simpler transformational processes within overtone space. This is by no means meant to be an exhaustive analysis of the work (or even the pitch elements of the work), nor am I proposing a system of analysis which may be applied across the board in order to reach a comprehensive analysis of the entire work. Rather, the interaction between pitch and transformational intervals in this work forms one of likely several narratives which are active in this work. But I hope the following analysis will go a long way in illuminating this conflict which is probably the most salient and the most unique feature of this work.

Ordered-pair Notation and the *f/p* Group

Because this conflict deals with the possibility of up to 48 different pitches per octave, the standard twelve-pitch-class and octave designation will not suffice for this analysis. Thus, before proceeding to a transformational analysis of relevant passages, we must establish a pitch nomenclature which allows for both the equality of identical gestalt shifts (such as those in the opening canon) and the difference in pitch-space intervals which results from these identical shifts in different environments.

I propose the use of ordered-pair notation for overtone series partials, where the first value in the pair is the fundamental of the series to which the pitch belongs (in standard pitch-class/octave designation), and the second value is the partial number. Thus, the opening E♭4 and F4 which open the movement would be notated as (E♭1, 8) and (E♭1, 9), respectively. Each ordered pair represents a unique object in pitch space, and the interval between any two ordered pairs can be measured as the distance between the two objects in pitch space or by the transformation which maps one of the objects onto the other (those transformations will be defined shortly). The ordered-pair pitch representations of the notes in the four natural horns in the four opening bars are given in figure 3 above the respective notes.

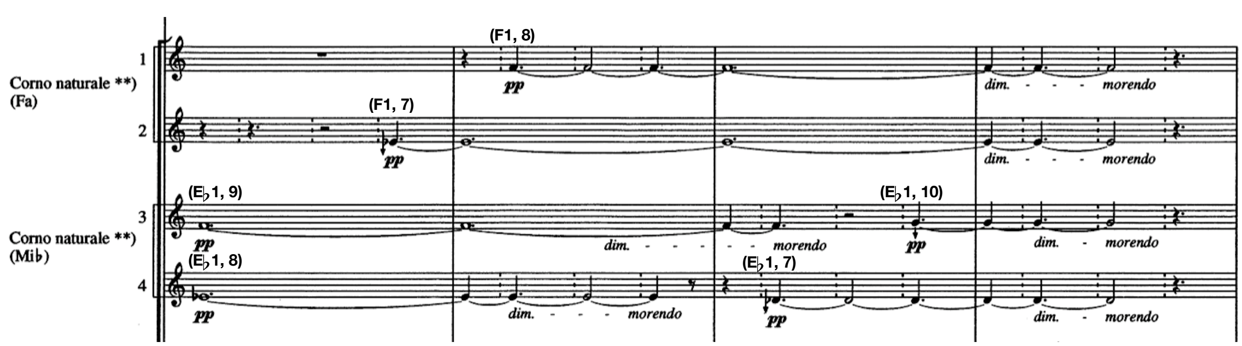

Figure 3. Ordered-pair Notation for mm. 1-4

This notation schema is particularly valuable when analyzing transformational distance between pitches. The 'stepwise' nature of the whole-tone movements within either of these two overtone series can be seen clearly in the ordered-pair notation. The difference (*cartesian* interval in overtone space) between any two adjacent pitches in an overtone series is (0,1); that is, the fundamental does not change, and the partials differ by 1. From this we derive our first two *transformational* intervals governing overtone-derived pitches: (0,+1) and (0,-1), ascending and descending by one partial, respectively. As an example from voice 1 of the opening canon, int((F1, 7), (F1, 8)) = (0,+1). Of course, the interval in *pitch space* which corresponds to these two moves varies depending on what partials are involved.

The ordered-pair notation provides us with a second cartesian measurement of overtone-space interval and a second pair of transformations. The difference (cartesian interval in overtone space) between (E♭1, 8) and (E, 8), for example, is (1,0); that is, the fundamental differs by 1 semitone, while the partial remains the same. From this we derive two more transformational intervals: (+1,0) and (-1,0). Such a transformation travels the same distance in pitch space no matter the register.

These four transformations—(+1,0), (-1,0), (0,+1), (0,-1)—along with an identity transformation—(0,0)—compose with each other to generate a mathematical group of infinite members. All operations resulting from the binary composition of these four transformations and the identity—such as (+3,-7) or (-4,-12)—are members of this group. We can further simplify our notation by denoting the operation (+1,0) which transposes the fundamental as *f* and denoting the operation (0,+1) which transposes the partial as *p*. The following expression, then, accounts for every operation in this group (which I will call the *f/p* group), where *m* and *n* are integers (positive, negative, or zero):

$$f^m * p^n$$

Thus the identity operation is f^0p^0; (+3,-7) is f^3p^{-7}; and (-4,-12) is $f^{-4}p^{-12}$.

Because of the closure rule (the result of the binary composition of any two elements in a group must be a member of the group) and the nature of the generative operations of this group, this group contains an infinite number of members, with both *m* and *n* extending infinitely in both positive and negative directions. The set of musical objects on which this group acts is also infinite in members: the set of all possible overtones of all possible equal-tempered fundamentals. However, due to the nature of the overtone series (there is no zeroth partial, nor are there negative partials), this group-set combination cannot form a *generalized interval system* (GIS) as defined by David Lewin (GMIT, p. 26). But this group-set combination does form a *simply transitive system*, which allows for the cartesian/transformational duality in overtone space, and this simply transitive system—in tandem with the overtone series—is the means by which a set of simple transformational intervals can generate a more diverse and complex set of intervals in pitch space.

Movement II. (Signale, Tanz, Choral) and Movement V. (Spectra): Stasis Begets Diversity

A straightforward, yet powerful case of the generative nature of the *f/p* transformations can be found in the second movement. In mm. 8-10, the solo horn (in F) and the fourth horn (in D) proceed in parallel motion in overtone space, and this motion generates a different pitch-space interval at each pitch level.

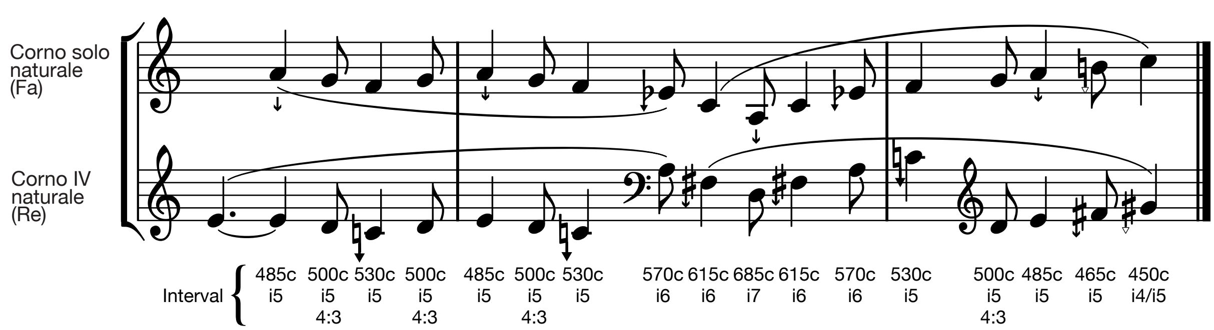

Figure 4. Solo Horn (F) and Horn 4 (D), mm. 8-10

As figure 4 demonstrates, no vertical pitch interval is repeated, except when the same two pitches are involved. Some intervals may still belong to the same interval category—e.g., the first three vertical intervals in the example all belong to the i5 category—but the precise intervals are all unique to the pair of pitches which generates them.

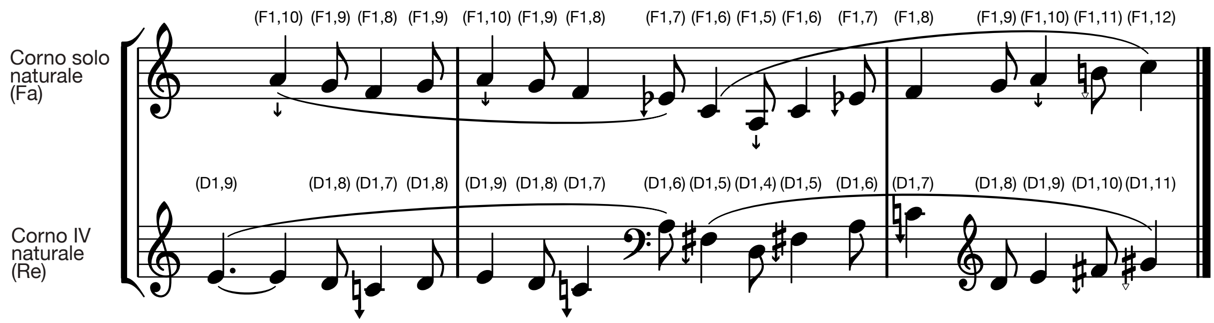

Figure 5. Solo Horn (F) and Horn 4 (D), mm. 8-10, Ordered-Pair Notation

However, by renotating the pitches as ordered pairs (figure 5), we can generalize this progression in a product network (as briefly presented in GMIT, p. 206, and further described by Cohn/Dempster 1992, p. 172ff.). Every dyad-to-dyad progression in this passage is a move left or right on the product network.

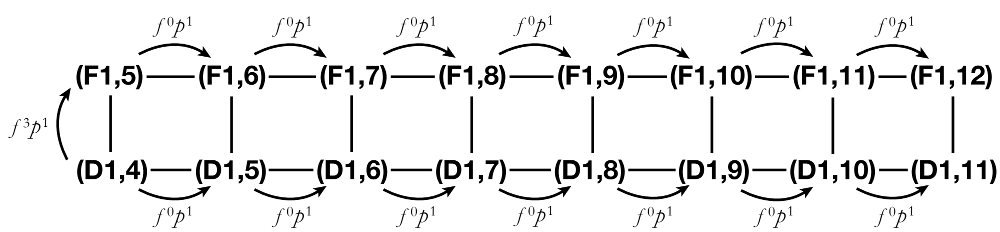

Figure 6. Solo Horn (F) and Horn 4 (D), mm. 8-10, Product Network

In this network, one can easily see that underlying this diverse (and theoretically infinite) set of pitch intervals are two generative intervals in overtone space: (3,1) and (0,1). Thinking transformationally, int(horn 4, solo horn), which governs the vertical relationship between the two voices, equals f^3p^1, and int(note x, note x+1), which represents the progression from note to note in a single voice, equals f^0p^1. Because of the nature of the overtone series, the static vertical interval f^3p^1 generates a different pitch interval at each new pitch level.

(A similar parallel motion occurs between horn 1 (in F) and horn 2 (in E) in mm. 11-13, where int(horn 2, horn 1) = $f^{1}p^{-3}$. And in mm. 13-15, the solo horn and horn 4 pair up again, this time at int(horn 4, solo horn) = $f^{3}p^{2}$.)

The fifth movement (Spectra) provides some more complicated passages in pitch space which are generated (at least in part) by static or simple elements in overtone space. The movement opens with a homophonic, four-voice chorale in the natural horns (all in E), with an independent line in the solo horn. Applying the *f/p* group to the natural horns provides interesting insights into the structure of this passage. Here are the first four bars of the natural horns, notated from highest to lowest, in ordered-pair notation:

Horn 1 (E)	**(E1,9)**	**(E1,10)**	**(E1,9)**	**(E1,11)**	**(E1,12)**	**(E1,13)**	**(E1,14)**	**(E1,11)**
Horn 3 (E)	**(E1,7)**	**(E1,8)**	**(E1,7)**	**(E1,9)**	**(E1,11)**	**(E1,12)**	**(E1,13)**	**(E1,10)**
Horn 2 (E)	**(E1,6)**	**(E1,7)**	**(E1,6)**	**(E1,8)**	**(E1,10)**	**(E1,11)**	**(E1,11)**	**(E1,9)**
Horn 4 (E)	**(E1,5)**	**(E1,6)**	**(E1,4)**	**(E1,6)**	**(E1,8)**	**(E1,9)**	**(E1,10)**	**(E1,8)**

Figure 7. Movement 5 Opening Chorale, Ordered-pair Notation

This passage can be divided into two phrases, with parallel motion in overtone space governing almost entirely throughout. Though the horns are reoriented to begin the second phrase (i.e., the arrangement of vertical intervals between the voices change), overtone intervals between the horns are static for the most part within each of the two phrases. Thus a product network can represent the upper two voices (horns 1 and 3) for each phrase; horn 2 can participate in those networks with only one 'wrong note,' and horn 4 can participate in the product network for phrase 2 with only one 'wrong note.' The two 'wrong notes' in horn 4, phrase 1, actually boil down to one 'wrong interval' between the second and third chords, an early reorientation, so to speak, after which horn 4 follows the same pattern as the rest of the horns. That is, until another 'wrong note' on the last chord. Figure 8 shows the product network with 'wrong notes' in gray.

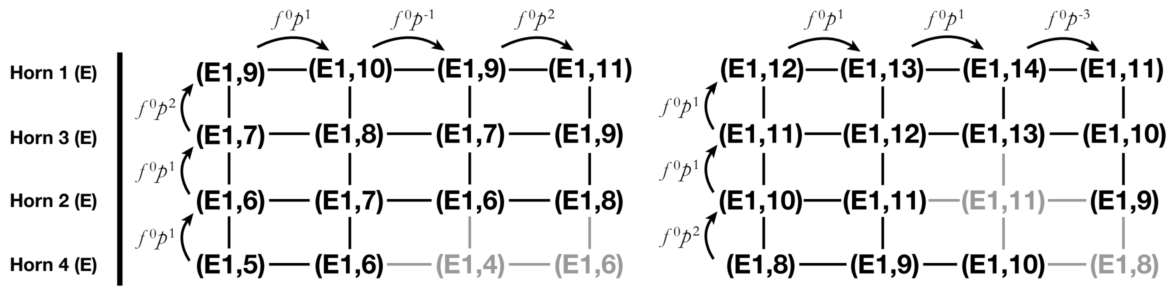

Figure 8. Product Network of Movement 5 Opening Chorale

Such an analysis raises the question: why the 'wrong notes'? Is Ligeti fudging the composition, or does this analysis miss the point? There are two aspects of this product network analysis that may help answer these questions. First, the inconsistencies of pattern in the product network generate a higher-level pattern themselves. The boundary intervals between highest and lowest voice change in each phrase as a result of the wrong notes. The first phrase begins spanning a distance of 4 partials (5-9) and ends spanning 5 (6-11); the second phrase begins spanning 4 (8-12) and ends spanning 3 (8-11). This expand-contract pattern can be seen in two other domains in this passage: the boundary intervals of the same four voices in pitch space (which, though not necessarily determined by the overtone pattern is nonetheless not surprising) and the pitch-space interval patterns of the solo horn. The soloist's two phrases each consist of two dyads. In the first phrase, the dyad-span contracts from i5 to i4; in the second phrase, the dyad span expands from i1 to i2. (The mid-phrase shifts in overtone fundamental in the solo horn make it difficult to analogize its line in the overtone domain with the chorale.) Thus, it is possible that the anomalies in the product network analysis reflect the desire to project this pattern of boundary interval.

The second way in which we might explain away these anomalies is by splitting the four voices into two pairs, by register (horn 1 with horn 3, horn 2 with horn 4). By doing so, other patterns emerge.

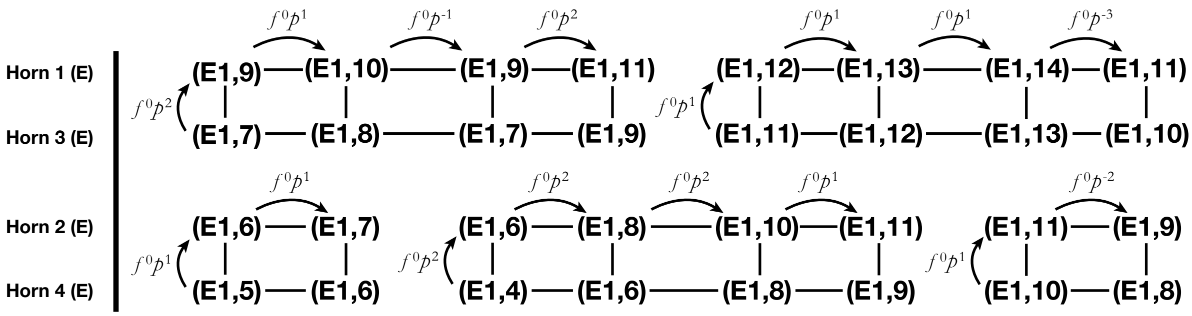

Figure 9. Product Networks in Horn Pairs in the Opening Chorale

The upper horns form a 'perfect' product network for each phrase, with a change in vertical interval coming between the two phrases. (The first four dyads are separated by 2 partials, the latter four by 1.) Taking into account the 'early reorientation' interpretation of the 'wrong interval' in horn 4, we can see the same pattern occurs in the lower two horns as in the upper two, albeit rotated by two chords. The lower two voices reorient between chords 2 and 3 and again between chords 6 and 7, rather than between 4 and 5 and (trivially) the last and the first. The middle product network of the lower horns is isomorphic to the first phrase of the upper horns (parallel motion at (0,2)); the outer network of the lower horns is isomorphic to the second phrase of the upper horns (parallel motion at (0,1)). Rather than anomalous, this passage turns out to be quite economical on the deeper layers of the overtone domain, in addition to bearing some structural resemblance to the independent solo line. (And given the patterns that obtain from breaking these two pairs of voices each into four dyad pairs—one per bar—I am suspicious that a Klumpenhauwer-network analysis will reveal further patterns of organization, but that is beyond the scope of this paper.)

Movement VII. (Hymnus): Product Networks in an Extended Structure

What follows is not quite a complete analysis of the tonal structure of the final movement of the Hamburg Concerto, as I will focus on the product networks governing the four natural horns. However, I will mention here that the double bass doubles a horn line throughout, with the exception of the last three notes. Also, the upper strings, which sustain G5, C6, and D6 throughout, fulfill a function similar to that of the solo horn in the opening movement (absent from the final movement). In the 'Praeludium,' the solo horn in B-flat provided a perfect fifth (3:2) bisection of the whole-tone (9:8) tuning between the E-flat and and F natural horns. In the 'Hymnus,' the tremolo G provides that same bisection between the sustained natural harmonics C and D. Also of interest in this movement is the bowed suspended cymbal. When bowed, the cymbal has a clearly discernible pitch, but Ligeti does not indicate what that pitch should be or any relationship it should have to other pitches in this movement. It is a curious contribution to the texture, and, in my mind, would warrant extended comment in any ‘complete’ analysis.

The final movement of Ligeti’s Hamburg Concerto thematicizes the directed motion of all four natural horns (1 and 3 in F, 2 and 4 in E) from various starting points toward their 16th partial (F5 or E5). This motion is clouded at first by undirected lines of ascending and descending motion within the horn lines before beginning their final ascents in mm. 8-9. It is also contrasted by stasis in the upper strings, which sustain the trichord [G5, C6, D6] throughout the movement. Both the varied texture of the opening bars and the communal ascent of the final bars exhibit coherence as the horns pair up at various moments and proceed—as in the opening of the fifth movement—in parallel motion in overtone space. Thus the variety of occurring sonorities (and the variety of ‘mistunings’ of those sonorities which do recur) can be explained by a series of (often overlapping) product networks. The networks are not as neat as those in the opening of the fifth movement, but the analysis will go a long way in reducing the complexity of the surface of this passage and demonstrating the generative nature of the transformational intervals in the overtone domain.

To aid in the analysis, the following is a renotation of the horn parts in ordered-pair notation. Because of the regular harmonic rhythm, only barlines are given, without rhythmic values.

	1		2		3		4		5		6		7		8	
Horn 1 (F)	(F1,12)	(F1,13)	(F1,14)	(F1,15)	(F1,16)	(F1,13)	(F1,12)	(F1,13)	(F1,7)	(F1,9)	(F1,8)	(F1,10)	(F1,9)	(F1,7)	(F1,6)	(F1,5)
Horn 2 (E)	(E1,7)	(E1,6)	(E1,9)	(E1,10)	(E1,7)	(E1,8)	(E1,9)	(E1,10)	(E1,11)	(E1,13)	(E1,14)	(E1,15)	(E1,16)	(E1,13)	(E1,10)	(E1,9)
Horn 3 (F)	(F1,8)	(F1,9)	(F1,10)	(F1,11)	(F1,12)	(F1,9)	(F1,10)	(F1,7)	(F1,6)	(F1,5)	(F1,6)	(F1,8)	(F1,6)	(F1,5)	(F1,4)	(F1,3)
Horn 4 (E)	(E1,5)	(E1,4)	(E1,6)	(E1,7)	(E1,5)	(E1,6)	(E1,7)	(E1,8)	(E1,9)	(E1,8)	(E1,10)	(E1,12)	(E1,11)	(E1,9)	(E1,7)	—

	9		10		11		12		13		14		15	
Horn 1 (F)	(F1,6)	(F1,7)	(F1,8)	(F1,9)	(F1,10)	(F1,11)	(F1,12)	(F1,13)	(F1,14)	(F1,15)	(F1,16)	—	—	—
Horn 2 (E)	(E1,10)	(E1,12)	(E1,13)	—	(E1,14)	—	(E1,16)	—	(E1,14)	(E1,15)	(E1,16)	—	—	—
Horn 3 (F)	(F1,4)	(F1,5)	(F1,6)	(F1,7)	(F1,8)	(F1,9)	(F1,10)	(F1,11)	(F1,12)	(F1,13)	(F1,14)	(F1,15)	(F1,14)	(F1,16)
Horn 4 (E)	(E1,8)	(E1,9)	(E1,10)	(E1,11)	(E1,12)	(E1,13)	(E1,15)	(E1,13)	(E1,12)	(E1,13)	(E1,14)	(E1,15)	(E1,16)	—

Figure 10. Movement 7 Horn Quartet, Ordered-pair Notation

The ‘Hymnus’ opens with parallel motion between the two horns in F (1 and 3) and between the two horns in E (2 and 4). I will begin by examining the relationships between these two pairs throughout the movement and then proceed on to other pairs and finally to a few sticky moments in the structure of the movement.

Horns 1 and 3 operate in parallel motion for almost the entire movement. As shown in the product networks of figure 11, they maintain an interval of (0,4) for the first six chords, and after a brief break they move in parallel at an interval of (0,2) for the remainder of the movement. The only exceptions are the downbeat of bar 7—(0,3)—and the last three pitches in horn 3, after horn 1 has reached its goal and sustains for the last four chords.

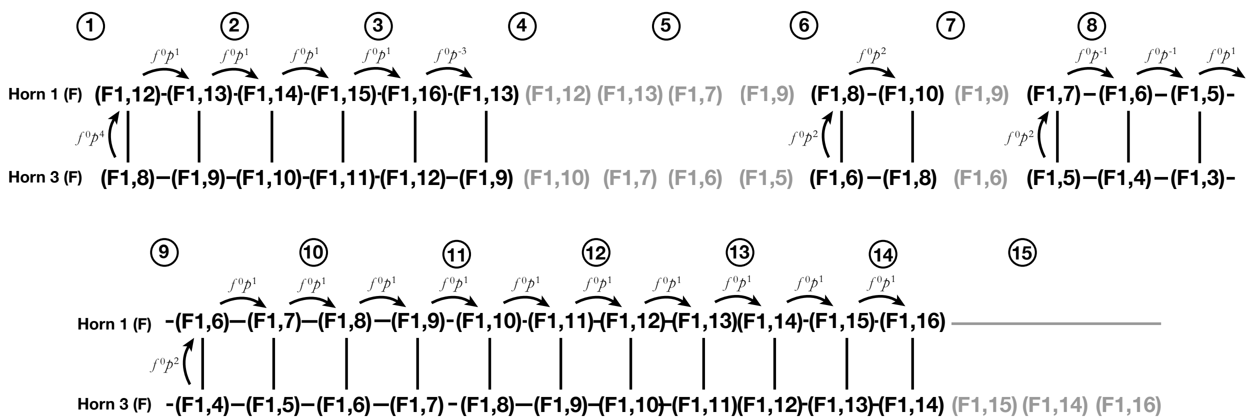

Figure 11. Movement 7, Horns 1 and 3

As figure 12 demonstrates, horns 2 and 4 are not quite so neat. Mm. 1, 2, and 3-5 demonstrate patterns of parallel motion (at (0,2), (0,3), and (0,2), respectively), as do mm. 8-10 (at (0,2) and (0,3)). Mm. 11-13 present an interesting pattern. A symmetrical progression (E1, 14) - (E1, 16) - (E1, 14) occurs in horn 2 over a symmetrical progression (E1, 12) - (E1, 13) - (E1, 15) - (E1, 13) - (E1, 12) in horn 4. We can consider this as a non-product-network pattern in these two voices which generates an interesting pitch structure, or we can consider it a single unit which, in a sense, prolongs the interval (0,2) which begins and ends the unit. In the latter case, mm. 10-14 then are governed by parallel motion at interval (0,2). In the former case, an extended product network does not hold, but mm. 11-13 are no longer 'anomalous' bars, in spite of the sustained pitches in horn 2, given the symmetrical structure they exhibit. (I will return to mm. 5-8 later.)

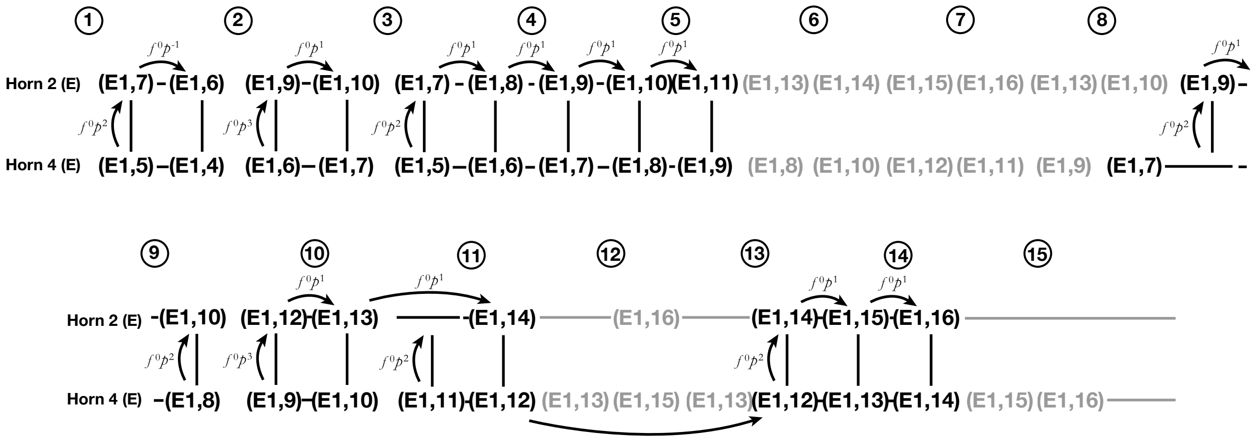

Figure 12. Movement 7, Horns 2 and 4

The horns also pair up across key boundaries. Horns 3 and 4 (in F and E, respectively) progress in parallel motion in mm. 8-11 at int(horn 4, horn 3) = f^1p^{-4}, and in mm. 13-15 at int(horn 4, horn 3) = f^1p^0. (Where both f and p relationships are involved, I will use transformational int(x, y) notation to avoid confusion.)

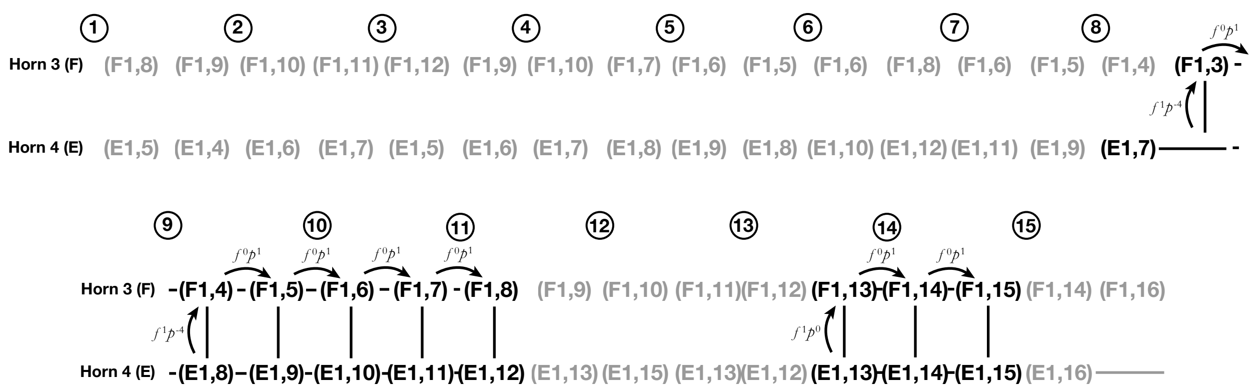

Figure 13. Movement 7, Horns 3 and 4

Horns 1 and 2 (also in F and E, respectively) have the same relationship in mm. 13-14 as horns 3 and 4 in mm. 13-15. Horns 1 and 2 also have an int(horn 4, horn 3) = f^1p^{-4} relationship in mm. 8-12, but the pattern is offset for two chords. Between chords 1 and 2 of m. 9, horn 2 jumps the gun, so to speak, moving up two partials (from 10 to 12) instead of 1. After moving up another partial to 13 in bar 10, it sustains an extra chord in order for horn 1 to catch up. On the second beat of m. 10, the two horns are back in sync. The sustained (E1, 14) in m. 11 does not throw off the pattern, since horn 2 takes one move of f^0p^2, in the time that horn one takes two moves of f^0p^1. After the sustained (E, 16) in m. 12, though, the pattern resets at int(horn 2, horn 1) = f^1p^0 at which it remains for the rest of the movement.

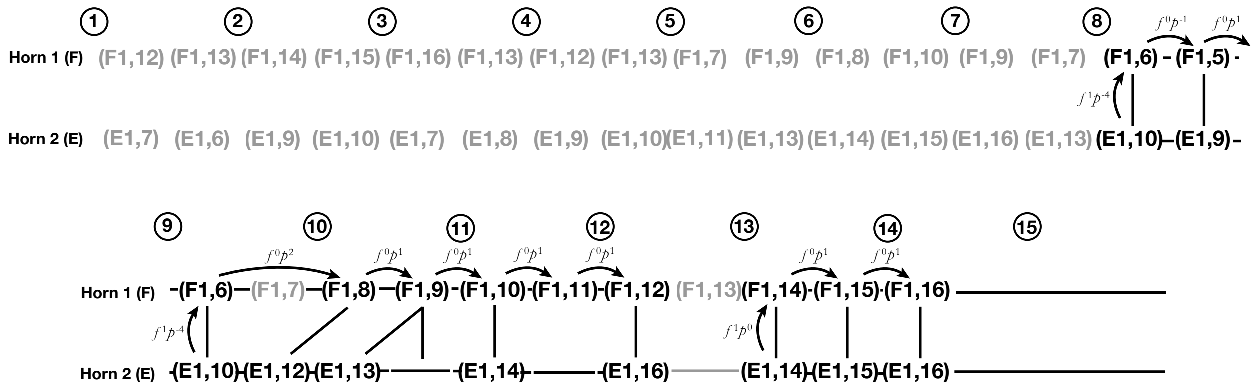

Figure 14. Movement 7, Horns 1 and 2

In mm. 5-7, horns 1 and 4 (in F and E, respectively) pair up briefly, progressing in parallel motion at int(horn 4, horn 1) = f^1p^{-2}, although the pattern skips the second chord in m. 5. (They also, less significantly, move in tandem in mm. 8-11 and 13-14, where all four horns are moving in parallel.)

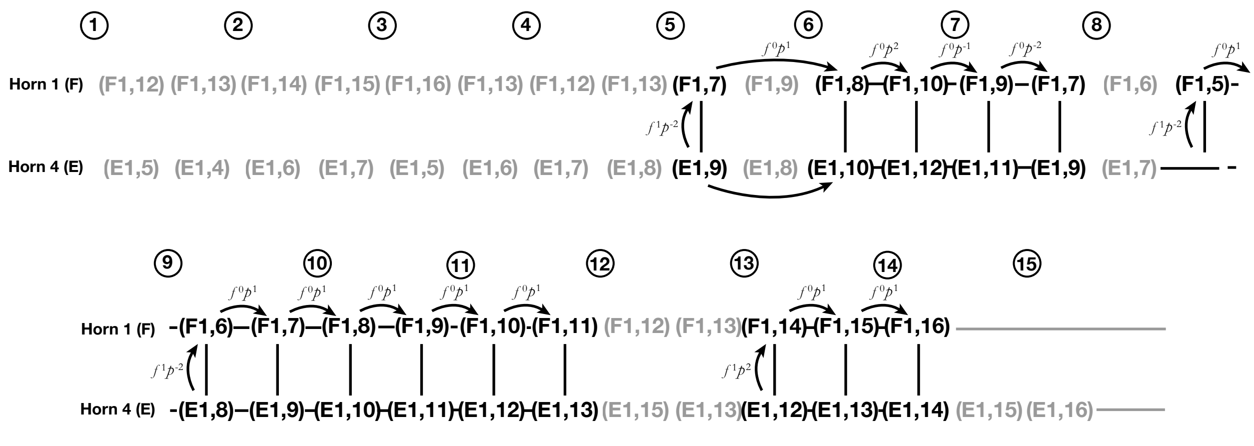

Figure 15. Movement 7, Horns 1 and 4

The second chord in m. 5 is an interesting anomaly in this movement. It is omitted by more than just the horn 1 and 4 parallel progression. In fact, it is the only chord in the movement which does not participate in any of the parallel progressions (assuming that one considers the second chord in m. 12 as part of the symmetrical unit in horns 2 and 4 in mm. 11-13). While this chord is not a particularly salient moment in the movement, which would warrant unique treatment in the structure, it does initiate an interesting background progression in horns 2 and 4, where there is a significant break in parallel motion between those two horns. As figure 12 above shows, horns 2 and 4 project the intervallic pattern (0,5) - (0,4) - (0,3) twice, beginning with the second chord in m. 5. This pattern is governed by a single hyper-interval, <0,-1>. That is, the interval between horns 2 and 4 decreases by a value of 1 partial with each successive chord.

	5		6		7		8	
Horn 2 (E)	(E1,11)	(E1,13)	(E1,14)	(E1,15)	(E1,16)	(E1,13)	(E1,10)	(E1,9)
Horn 4 (E)	(E1,9)	(E1,8)	(E1,10)	(E1,12)	(E1,11)	(E1,9)	(E1,7)	——
intervals:		(0,5)	(0,4)	(0,3)	(0,5)	(0,4)	(0,3)	
			<0,-1>	<0,-1>		<0,-1>	<0,-1>	

Figure 16. Movement 7, Horns 2 and 4, Hyper-Interval Progression

In addition to this deeper-level governing principle, the decrease in interval size in this pattern is indicative of the process which governs the horn quartet throughout the movement, as they all gradually progress toward their 16th partials. And just as the odd-chord-out at the end of m. 5 highlights the beginning of this hyper-intervallic progression, the end of this hyper-intervallic progression highlights the beginning of the communal ascent in m. 8. So while the second chord of m. 5 does not stand out aurally in any way, its absence from the product network structure is an anomaly which highlights the beginning of a pattern which is significant both symbolically (the decrease in interval size) and structurally (initiating the ascent in the four horns). Thus the entire pitch structure of the horn quartet is generated in some aspect from the underlying structure of parallel motions in overtone space.

Final Thoughts

The use of natural horns in multiple keys and in combination with a predominately equal-tempered orchestra poses interesting problems for both the composer and the analyst. Though this paper leaves much about Ligeti's Hamburg Concerto yet to be analyzed (including many other elements of pitch organization), the ordered-pair representation of overtone pitches, the *f*/*p* group, the simply transitive system, and the product network make a significant dent in uncovering the underlying structural elements of key passages in this idiosyncratic work. In particular, they provide a methodology for relating the generative elements in overtone space to the resulting musical elements in pitch space and accounting for the great diversity of pitches, intervals, and chords via simple principles, without reducing out differences which are aurally salient or structurally significant—such as register or intonation—and without forcing unintuitive choices—such as to which of the twelve chromatic pitch classes a quarter-tone-tuned pitch belongs. While this is by no means an exhaustive analysis (even of the last movement) of the Hamburg Concerto, hopefully the tools and principles presented here will help lay the groundwork for subsequent analyses of other domains of this work.

Bibliography

Cohn, Richard and Douglas Dempster. 'Hierarchical Unity, Plural Unities: Toward a Reconciliation.' In *Disciplining Music: Musicology and its Canons*, ed. Katherine Bergeron and Philip V. Bohlman. Chicago: University of Chicago Press, 1992.

Lewin, David. *Generalized Musical Intervals and Transformations*. New Haven: Yale University Press, 1987.

Satyendra, Ramon. 'An Informal Introduction to Some Formal Concepts From Lewin's Transformational Theory,' in *Journal of Music Theory* 48/1 (2004). [pre-publication manuscript.]