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CS 513

HW 1 Probability

1. P(J) = 0.2   P(S) = 0.3 $P(J \cap S) = 0.08$

   a. Using conditional probability:

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ => $P(J|S) = \frac{P(J \cap S)}{P(S)}$ => $P(J|S) = \frac{0.08}{0.3} = 0.2667$

   b. Using conditional probability:

$P(A|B') = \frac{P(A) - P(A \cap B)}{P(B')}$ => $P(J|S') = \frac{P(J) - P(J \cap S)}{1 - P(S)}$ => $P(J|S) = \frac{0.2 - 0.08}{0.7} = 0.1714$

   c. $P((A \cap B)|(A \cup B)) = \frac{P(A \cap B) \cap P(A \cup B)}{P(A \cup B)}$ => $\frac{P(A \cap B)}{P(A \cup B)}$ => $\frac{0.08}{P(A) + P(B) - P(A \cap B)}$ =>

$\frac{0.08}{0.2 + 0.3 - 0.08} = 0.1905$

2. P(H) = 0.8   P(S) = 0.9 $P(H \cup S) = 0.91$

   a. $P(H \cap S') = P(H) - P(H \cap S)$ => $P(H \cap S) = P(H) + P(S) - P(H \cup S)$

   $P(H \cap S) = 0.8 + 0.9 - 0.91 = 0.79$

   $P(H \cap S') = 0.8 - 0.79 = 0.01$

   b. $P(S \cap H') = P(S) - P(H \cap S)$ => $0.9 - 0.79 = 0.11$

   c. $P(H' \cap S') = 1 - P(H \cap S) = 1 - 0.91 = 0.09$

3. No, the events "Jerry is at the bank" and "Susan is at the bank" are not independent. As we know, for independent events, either P(A|B)=P(A) or P(A∩B)=P(A)P(B) need to be true. From the calculations in Q1, we know that neither of these hold true for the Jerry/Susan at the bank situation, meaning that these events are not independent.

4. 36 possible combinations of rolls on 2 dice

   a. P(Sum is 6) = 5/36 (1+5, 2+4, 3+3, 4+2, 5+1)

      P(2nd die shows 5) = 6/36 = ⅙

      P(Sum is 6 ∩ 2nd die shows 5) = 1/36

      P(Sum is 6 | 2nd die shows 5) = $\frac{1/36}{1/6} = 1/6$

As we know, for independent events, either P(A|B)=P(A) or P(A∩B)=P(A)P(B) needs to be true. P(Sum is 6 | 2nd die shows 5) = 1/6 but P(Sum is 6) = 5/36, which are not equal. Additionally, P(Sum is 6 ∩ 2nd die shows 5) = 1/36, but P(Sum is 6)*P(2nd die shows 5)=1/36*⅙ = 1/216, which are also not equal. Therefore, the events "The sum is 6" and "The second die is 5" are not independent.

   b. P(Sum is 7) = 6/36 = 1/6 (1+6, 2+5, 3+4, 4+3, 5+2, 6+1)

      P(1st die shows 5) = 6/36 = 1/6

      P(Sum is 7 ∩ 1st die shows 5) = 1/36

      P(Sum is 7 | 1st die shows 5) = $\frac{1/36}{1/6} = 1/6$

As we know, for independent events, either P(A|B)=P(A) or P(A∩B)=P(A)P(B) needs to be true. P(Sum is 7 | 1st die shows 5) = 1/6 and P(Sum is 7) = 1/6, which are equal. For independence, we only need to prove one of the 2 conditions to be true, therefore, the events "The sum is 7" and "The first die shows 5" are independent.

5. P(TX) = 0.60 P(NJ) = 0.10 P(O|TX) = 0.30 P(O|AK) = 0.20 P(O|NJ) = 0.10

   P(AK) = 1 - P(TX) - P(NJ) = 1 - 0.6 - 0.1 = 0.3

   a. Using Law of Total Probability:

P(O) = P(O|TX)P(TX) + P(O|NJ)P(NJ) + P(O|AK)P(AK) = 0.3*0.6 + 0.1*0.1 + 0.2*0.3 = 0.25

b. Using Conditional Probability:

P(TX|O) = $\frac{P(TX \cap O)}{P(O)}$ => P(TX∩O) = P(O|TX)P(TX) = 0.3*0.6 = 0.18 =>

P(TX|O) = $\frac{0.18}{0.25}$ = 0.72

6.

a. P(Did Not Survive) = 1490/2201 = 0.677

b. P(First Cabin) = 325/2201 = 0.1477

c. P(First Cabin | Survived) = $\frac{P(First\ Cabin \cap Survived)}{P(Survived)}$ =

$\frac{(203/2201)}{(711/2201)} = \frac{0.092}{0.323} = 0.2855$

d. No, for the two events (Survival and Staying in the First Class) to be independent, P(A|B)=P(A) or P(A∩B)=P(A)P(B) need to be true. As we can see from the calculations above, P(First Cabin | Survived) and P(First Cabin) are not equal. Additionally, $P(First\ Cabin\ \cap\ Survived)$ = 0.092 but P(First Cabin)*P(Survived) = 0.1477*0.323 = 0.0477, which are again not equal. This proves that they are not independent events.

e. $P((First\ Cabin\ \cap\ Child)\ |\ Survived)$ = $\frac{P((First\ Cabin \cap Child) \cap Survived)}{P(Survived)}$ =

$= \frac{(6/2201)}{(711/2201)} = \frac{0.0027}{0.323} = 0.0084$

f. P(Adult | Survived) = $\frac{P(Adult \cap Survived)}{P(Survived)} = \frac{(654/2201)}{(711/2201)} = \frac{0.2971}{0.323} = 0.9198$

g. As we can see from the calculations above, given that a passenger survived, age and staying in the first class are not independent. For 2 events (A and B) to be conditionally independent, $P((A\ \cap\ B)\ |\ C)\ =\ P(A|C)P(B|C)$. Using the example of $P((First\ Cabin\ \cap\ Child)\mid Survived)$ = 0.0084, while $P(A|C)P(B|C)$ = 0.2855*(1-0.9198) = 0.0229, and are not equal.

7.

	AI-Generated	Human Generated	Total
Predicted AI	970 (TP)	70 (FP)	1040
Predicted Human	30 (FN)	930 (TN)	960
Total	1000	1000	2000

Accuracy = $\frac{TP+TN}{TP+FP+FN+TN} = \frac{970+930}{2000} = \frac{1900}{2000} = 0.95$

Precision = $\frac{TP}{TP+FP} = \frac{970}{970+70} = \frac{970}{1040} = 0.9327$

Recall = $\frac{TP}{TP+FN} = \frac{970}{970+30} = \frac{970}{1000} = 0.97$

F1 = $\frac{2*Precision*Recall}{Precision+Recall} = \frac{2*0.9327*0.97}{0.9327+0.97} = 0.951$