CS 556 - Homework Probability Distribution

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Problem 1

Mopeds (small motorcycles with an engine capacity below) are very popular in Europe because of their mobility, ease of operation, and low cost. The article "Procedure to Verify the Maximum Speed of Automatic Transmission Mopeds in Periodic Motor Vehicle Inspections" (J. of Automobile Engr., 2008: 1615–1623) described a rolling bench test for determining maximum vehicle speed. A normal distribution with mean value 46.8 km/h and standard deviation 1.75 km/h is postulated. Consider randomly selecting a single such moped.

1. What is the probability that maximum speed is at most 50 km/h?

$$\begin{aligned}&\mu = 46.8 \quad \sigma = 1.75 \quad x = 50\\ &z = \frac{x-\mu}{\sigma} \to \frac{50-46.8}{1.75} \to \quad z = 1.82857\\ &P(x < 50) = 0.96627\end{aligned}$$

2. What is the probability that maximum speed is at least 48 km/h?

$$\begin{aligned}&\mu = 46.8 \quad \sigma = 1.75 \quad x = 48\\ &z = \frac{x-\mu}{\sigma} \to \frac{48-46.8}{1.75} \to \quad z = 0.68571\\ &P(x > 48) = 0.24645\end{aligned}$$

3. What is the probability that maximum speed differs from the mean value by at most 1.5 standard deviations?

$$z = 1.5 \quad \to P(-1.5 < x < 1.5) = 0.86639$$

Problem 2

Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with mean 8.8 and, standard deviation 2.8 as suggested in the article "Simulating a Harvester-Forwarder Softwood Thinning" (Forest Products J., May 1997: 36–41).

1. What is the probability that the diameter of a randomly selected tree will be at least 10 in.? Will exceed 10in.?

$$\begin{aligned}&\mu = 8.8 \quad \sigma = 2.8 \quad x = 10\\ &z = \frac{x-\mu}{\sigma} \to \frac{10-8.8}{2.8} \to \quad z = 0.42857\\ &P(x > 10) = 0.33412\end{aligned}$$

2. What is the probability that the diameter of a randomly selected tree will exceed 20in.?

$$
\begin{aligned}
&\mu = 8.8 \quad \sigma = 2.8 \quad x = 20 \\
&z = \frac{x-\mu}{\sigma} \to \frac{20-8.8}{2.8} \to \quad z = 4 \\
&P(x > 20) = 0.00003
\end{aligned}
$$

3. What is the probability that the diameter of a randomly selected tree will be between 5 and 10in.?

$$
\begin{aligned}
&\mu = 8.8 \quad \sigma = 2.8 \quad x_1 = 5 \quad x_2 = 10 \\
&z_1 = \frac{x-\mu}{\sigma} \to \frac{5-8.8}{2.8} \to \quad z_1 = -1.357 \\
&z_2 = \frac{x-\mu}{\sigma} \to \frac{10-8.8}{2.8} \to \quad z_2 = 0.42857 \\
&P(5 < x < 10) = 0.57851
\end{aligned}
$$

4. What value c is such that the interval includes 98% of all diameter values?

$$
\begin{aligned}
&z \text{ where } P(-z < x < z) = 0.98 \to \quad z = 2.326 \\
&z = 2.324 \quad \mu = 8.8 \quad \sigma = 2.8 \\
&z = \frac{x-\mu}{\sigma} \to \quad 2.326 = \frac{x-8.8}{2.8} \to \quad x = 15.3128 \\
&-z = \frac{x-\mu}{\sigma} \to \quad -2.326 = \frac{x-8.8}{2.8} \to \quad x = 2.2872 \\
&\mu \pm 6.5128
\end{aligned}
$$

5. If four trees are independently selected, what is the probability that at least one has a diameter exceeding 10in.?

$$
\begin{aligned}
&\text{For independent events, Probability all 4 have diameter less than 10in } = P(x < 10)^4 \\
&\mu = 8.8 \quad \sigma = 2.8 \quad x = 10 \\
&z = \frac{x-\mu}{\sigma} \to \frac{10-8.8}{2.8} \to \quad z = 0.42857 \\
&P(x < 10) = 0.66588 \to 0.66588^4 = 0.1966 \\
&P(\text{At least 1 has diameter greater than 10in}) = 1 - 0.1966 = 0.8034
\end{aligned}
$$

Problem 3

Consider babies born in the "normal" range of 37–43 weeks gestational age. Extensive data supports the assumption that for such babies born in the United States, birth weight is normally distributed with mean 3432 g and standard deviation 482 g. [The article "Are Babies Normal?" (The American Statistician, 1999: 298–302) analyzed data from a particular year; for a sensible choice of class intervals, a histogram did not look at all normal, but after further investigations it was determined that this was due to some hospitals measuring weight in grams and others measuring to the nearest ounce and then converting to grams. A modified choice of class intervals that allowed for this gave a histogram that was well described by a normal distribution.]

1. What is the probability that the birth weight of a randomly selected baby of this type exceeds 4000g? Is between 3000 and 4000g?

$$\mu = 3432 \quad \sigma = 482 \quad x = 4000$$
$$z = \frac{x-\mu}{\sigma} \to \frac{4000-3432}{482} \to \quad z = 1.17842$$
$$P(x > 4000) = 0.11931$$

$$\mu = 3432 \quad \sigma = 482 \quad x = 3000$$
$$z = \frac{x-\mu}{\sigma} \to \frac{3000-3432}{482} \to \quad z = -0.89627$$
$$P(3000 > x > 4000) = 0.69563$$

2. What is the probability that the birth weight of a randomly selected baby of this type is either less than 2000g or greater than 5000g?

$$\mu = 3432 \quad \sigma = 482 \quad x = 2000$$
$$z = \frac{x-\mu}{\sigma} \to \frac{2000-3432}{482} \to \quad z = -2.97095$$
$$P(x < 2000) = 0.0014844$$

$$\mu = 3432 \quad \sigma = 482 \quad x = 5000$$
$$z = \frac{x-\mu}{\sigma} \to \frac{5000-3432}{482} \to \quad z = 3.25311$$
$$P(x > 5000) = 0.00057074$$

$$P(x < 2000 or x > 5000) = 0.0014844 + 0.00057074 = 0.00205514$$

3. What is the probability that the birth weight of a randomly selected baby of this type exceeds 7 lb?

$$7lbs = 3175.15g$$
$$\mu = 3432 \quad \sigma = 482 \quad x = 3175.15$$
$$z = \frac{x-\mu}{\sigma} \to \frac{3175.15-3432}{482} \to \quad z = -0.53288$$
$$P(x > 3175.15) = 0.70294$$

4. How would you characterize the most extreme .1% of all birth weights?

$$z \text{ where } P(-z < x < z) = 0.999 \to \quad z = 3.291$$
$$z = 2.576 \quad \mu = 3432 \quad \sigma = 482$$
$$z = \frac{x-\mu}{\sigma} \to \quad 3.291 = \frac{x-3432}{482} \to \quad x = 1845.738$$
$$-z = \frac{x-\mu}{\sigma} \to \quad -3.291 = \frac{x-3432}{482} \to \quad x = 5018.262$$

The most extreme 0.1% of all birth weights is under 1845.738g and over 5018.262g

5. If X is a random variable with a normal distribution and a is a numerical constant ($a = 0$), then $Y = aX$ also has a normal distribution. Use this to determine the distribution of birth weight expressed in pounds (shape, mean, and standard deviation), and then recalculate the probability from part (c). How does this compare to your previous answer?

   $1\text{lb} = 453.592\text{g} \rightarrow \mu = 3432/453.592 = 7.566271 \quad \sigma = 482/453.592 = 1.06262897$

   $\mu = 7.566271 \quad \sigma = 1.06262897 \quad x = 7$

   $z = \dfrac{x - \mu}{\sigma} \rightarrow \dfrac{7 - 7.566271}{1.06262897} \rightarrow \quad z = -0.5329$

   $P(x > 7) = 0.70295$

   The recalculed answer is almost exactly equal to the previous,

   with a minor difference likely attributed to a rounding difference