CS 556 - Homework Derivates

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Problem 1

Top 7 cards are drawn from a well-shuffled standard 52-card deck. Find probability that:

1. The 7 cards include exactly 3 aces. Usign combinations since order does not necessarily matter.

$$\frac{\binom{4}{3}\binom{48}{4}}{\binom{52}{7}}$$

$$\binom{4}{3} = \frac{4!}{3!(4-3)!} = \frac{24}{6} = 4$$

$$\binom{48}{4} = \frac{48!}{4!(48-4)!} = \frac{48*47*46*45}{4!} = 194580$$

$$\binom{52}{7} = \frac{52!}{7!(52-7)!} = \frac{52*51*50*49*48*47*46}{7!} = 133784560$$

$$\text{Putting it all together: } \frac{4(194580)}{133784560} = 0.0058 = 0.58\%$$

2. The 7 cards include exactly 2 kings

$$\frac{\binom{4}{2}\binom{48}{5}}{\binom{52}{7}}$$

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4*3}{2} = 6$$

$$\binom{48}{5} = \frac{48!}{5!(48-5)!} = \frac{48*47*46*45*44}{5!} = 1712304$$

$$\text{Putting it all together: } \frac{6(1712304)}{133784560} = 0.07679 = 7.679\%$$

3. The 7 cards include exactly 3 aces. or exactly 2 kings, or both

$$\frac{\binom{4}{3}\binom{48}{4} + \binom{4}{2}\binom{48}{5} - \binom{4}{3}\binom{4}{2}\binom{44}{2}}{\binom{52}{7}}$$

$$\binom{44}{2} = \frac{44!}{2!(44-2)!} = \frac{44*43}{2} = 946$$

$$\text{Putting it all together: } \frac{4(1904580) + 6(1712304) - (4)(6)(946)}{133784560} = \frac{11029440}{133784560} = 0.08244 = 8.244\%$$

Problem 2

Alice and Bob have $2n + 1$ coins, each coin with probability of heads equal to $1/2$. Bob tosses $n + 1$ coins, while Alice tosses the remaining n coins. Assuming independent coin tosses, show that the probability that after all coins have been tossed, Bob will have gotten more heads than Alice is $1/2$.

$B =$ Bob has more, $\quad A =$ Alice has more, $\quad S =$ Bob and Alice have same # of heads

$P(\text{coin} = \text{head}) = 1/2, \quad P(\text{coin} = \text{tail}) = 1 - 1/2 = 1/2$

Since events B, A, and S make up all of the possible outcomes then $P(B) + P(A) + P(S) = 1$

At the point 2n coins have been tossed, $P(B) = P(A), and P(S) = 1 - 2P(A or B)$

Now for Bob's last coin, he can win if he is already ahead, or if they are tied and he gets a head

$P(B) + P(S) * P(coin = head) \quad = P(B) + 1 - 2P(B) * (1/2) \quad = P(B) + 1/2 - P(B) \quad = 1/2$

Problem 3

We are given three coins: one has heads in both faces, the second has tails in both faces, and the third has a head in one face and a tail in the other. We choose a coin at random, toss it, and the result is heads. What is the probability that the opposite face is tails?

$$C1 = 2\text{heads} \quad C2 = 2\text{tails} \quad C3 = 1\text{head, }1\text{tail}$$

$$P(C1) = P(C2) = P(C3) = 1/3$$

$$P(H|C1) = 1 \quad P(H|C2) = 0 \quad P(H|C3) = 1/2$$

$$P(C3|H) = \frac{P(H|C3) \cdot P(C3)}{P(H|C3) \cdot P(C3) + P(H|C2) \cdot P(C2) + P(H|C1) \cdot P(C1)}$$

$$P(C3|H) = \frac{1/2 \cdot 1/3}{1/2 \cdot 1/3 + 0 \cdot 1/3 + 1 \cdot 1/3} = \frac{1/6}{1/6 + 1/3} = \frac{1}{3}$$

Problem 4

Each of k jars contain m white and n black balls. A ball is randomly chosen from jar 1 and transferred to jar 2, then a ball is randomly chosen from jar 2 and transferred to jar 3, etc. Finally a ball is randomly chosen from jar k. Show that the probability that the last ball is white is the same as probability that the first ball is white, i.e. it is m/(m+n).

Could not figure out...

Problem 5

A power utility can supply electricity to a city from n different power plants. Power plant i fails with probability pi (pi, i need not be the same), independent of the others.
a) Suppose that any one plant can produce enough electricity to supply the entire city. What is the probability that the city will experience a black-out?
b) Suppose that two power plants are necessary to keep the city from a blackout. Find the

probability that the city will experience a blackout.

$a)P(\text{all power plants fail}) = p1 \cdot p2 \cdot p3 \ldots \cdot p_n$
$b) \quad P(\text{a power plant works }) = 1 - p_i$
$P(1 \text{ power plant left}) =$
$(1-p1) \cdot p2 \cdot p3 \ldots \cdot p_n + p1 \cdot (1-p2) \cdot p3 \ldots \cdot p_n + p1 \cdot p2 \cdot (1-p3) \ldots \cdot p_n + \ldots + p1 \cdot p2 \cdot p3 \ldots \cdot (1-p_n)$
$P(\text{all power plants fail or 1 is left }) = P(\text{all fail}) + P(1 \text{ left})$ as defined above