

Assignment - 9

1) Explain how first order logic overcomes shortcomings of propositional logic.

→ In first order logic user can use quantifiers as it expresses generalization, specialization & pattern, while propositional logic do not use quantifiers so cannot express generalization, specialization & pattern. Moreover, talking about propositional logic we are allowed to use and, or, etc, while for first order logic we are allowed to use forall & exist over variables which makes the logic more expressive.

2) Express universal quantifiers quantification in terms of existential quantification.

→ The two quantifiers are actually intimately connected with each other, through negation. Asserting that everyone dislikes parsnips is the same as asserting there does not exist someone who likes them,

$$\forall x \neg \text{Likes}(x, \text{Parsnips}) \iff \neg \exists x \text{ Likes}(x, \text{Parsnips}).$$

As \forall is really a conjunction over the universe of objects & \exists is a disjunction, it should not be surprising that they obey De Morgan's rules.

$$\forall x \neg P \equiv \neg \exists x P$$

$$\neg \forall x P \equiv \exists x \neg P$$

$$\forall x P \equiv \neg \exists x \neg P$$

$$\neg \forall x \neg P \equiv \exists x P$$

3) Translate the following into first-order logic.

a) Some students took CS411 in Spring 2020.

$$\exists x \text{ Student}(x) \wedge \text{Takes}(x, \text{CS411}, \text{Spring 2020}).$$

b) Some students wear a hoodie with VIC logo on it.

$$\exists x \exists y \text{ Student}(x) \wedge \text{wears}(x, y) \wedge \text{Hoodie}(y) \wedge \text{PrintedOn}(y, \text{VIC logo}).$$

c) Something that glitters is not always gold, whereas gold always glitters.

$$\exists x (\text{glitters}(x) \wedge \neg \text{gold}(x)) \wedge \forall y (\text{gold}(y) \rightarrow \text{glitters}(y))$$

d) No one can win with everyone all the time.

$$\forall x \exists y \exists t (\neg \text{Win}(x, y, t))$$

e) All CS courses are difficult, except two.

$$\exists x \exists y \text{ CS Course}(x) \wedge \text{CS Course}(y) \wedge \neg(x=y) \wedge \neg \text{Difficult}(x) \wedge \neg \text{Difficult}(y) \wedge (\forall z \text{ CS Course}(z) \wedge \neg(y=z) \wedge \neg(x=z) \rightarrow \text{Difficult}(z))$$