COMP 5411 Advanced Computer Graphics
Written Assignment

Problem 1. (20 points)

(a) Show the following identity of the Bernstein polynomials

$$\frac{dB_i^n(u)}{du} = n(B_{i-1}^{n-1}(u) - B_i^{n-1}(u)),$$

with $B_{-1}^{n-1}(u) \equiv B_n^{n-1}(u) \equiv 0$.

(b) Use the identity in (a) to derive the following general expression for the first derivative of an n-degree Bezier curve.

$$\frac{dC(u)}{du} = n\sum_{i=0}^{n-1} B_i^{n-1}(u)(P_{i+1} - P_i).$$

(c) Use the expression in (b) to obtain formulas for the end first derivatives of an n-degree Bezier curve.

(d) Differentiate the expression in (b) once more to derive a general expression for the second derivative of an n-degree Bezier curve (you may want to let Q_i be $P_{i+1} - P_i$).

(e) Use the expression in (d) to obtain formulas for the end second derivatives of an n-degree Bezier curve.

Problem 2. (20 points)

Many applications require all involved curves to have the same degree. A polynomial of degree n is also a polynomial of degree $n+1$; therefore there exists a set of $n+2$ control points Q_i that defines a degree n Bezier curve originally defined by $n+1$ control points P_i. This process is called degree elevation. We give the relationship between Q_i and P_i without proof here:

$$\begin{aligned} Q_0 &= P_0, \\ Q_i &= \frac{i}{n+1}P_{i-1} + (1 - \frac{i}{n+1})P_i, \qquad 1 \le i \le n, \\ Q_{n+1} &= P_n. \end{aligned}$$

(a) Sketch the control polygon of a cubic Bezier curve defined by $(1,1), (3,3), (4,0), (2,0)$. Sketch as accurate as possible the resulting control polygon after degree elevation.

(b) The corner-cutting procedure is similar to deCasteljiau algorithm. In what way are they different?

Problem 3. (20 points)

(a) Consider a degree n Bézier curve $C(u) = \sum_{i=0}^{n} B_i^n(u)P_i$. How many linear interpolations are involved to evaluate $C(u)$ at a fixed value of u using the deCasteljau algorithm?

(b) Given a $(m \times n)$-degree Bézier surface $S(u, v)$ defined as

$$S(u,v) = \sum_{i=0}^{m} \sum_{j=0}^{n} B_i^m(u) B_j^n(v) P_{ij}$$

How many linear interpolations are required to evaluate the surface at fixed parameter values $(\hat{u}, \hat{v})$ using the deCasteljau algorithm? Note that there are two different answers.

(c) Assuming $m < n$, how would you perform the evaluation in part (b) so as to minimize the number of linear interpolations needed?

Problem 4. (20 points)

Given a cubic Bezier curve defined by the control points P_i, $i = 0, 1, 2, 3$. Explain how you would find the four Bspline control points of the same polynomial curve.

Problem 5. (20 points)

The first quadrant of the unit circle can be represented by the following quadratic rational functions:

$$C(u) = \left(\frac{1-u^2}{1+u^2}, \frac{2u}{1+u^2} \right), \qquad 0 \le u \le 1.$$

Let us derive the quadratic rational Bezier representation of this circular arcs from the above functions.

(a) Based on the endpoint interpolation property and the first derivative of rational Bezier curves at $u = 0$ and $u = 1$, find P_0, P_1 and P_2.

(b) Noting that $W(u) = \sum_{i=0}^{2} B_i^2(u) w_i = 1 + u^2$. Derive the weights w_0, w_1, w_2.

(c) Write P_i as P_i^w in homogeneous space.

(d) Write the nonrational (polynomial) Bezier curve $C^w(u)$ in terms of P_i^w. Explicitly write out the Bezier basis functions and the coordinates of P_i^w.

(e) Write the coordinate functions $X(u)$ and $Y(u)$ of the rational Bezier curve $C(u)$ by performing a perspective mapping. Show that they indeed equal the rational functions given above.