1 Replicate Herkenhoff 2019

1.1 Model Timeline

(see p2614)

At the start of every period:

1. Household calculate present value functions and decide whether to look for credit (c_pol)
2. Lenders then view the entire state space of borrowers and send credit offers to maximize profits (find Q and $A\psi$)
3. Household chooses the optimal repayment decision r and asset to save/borrow bp
    - A household remains matched to a lender until the household defaults or the match is destroyed exogenously
4. After the credit market closed, the aggregate state is realized, the exogenous job loss is realized, and the unemployed agents enter the labour market where they look for jobs paying $\tilde{w} \in \mathbb{W}$
5. Before entering the next period, expense shock is realized

1.2 Exogenous Variables for Stationary Equilibrium

(see Calibration, p2621.)

1. $\gamma = 0.5$. Benefit replacement rate.
2. $\delta = 0.1$. Job destruction rate.
3. $\zeta = 1.6$. Labor matching elasticity parameter .
4. $r_f = 0.04$. APR.
    - $r_{f_q} = 1.04^{1/4} - 1$. Quarter percentage rate.
5. $\zeta_C = 0.37$. Credit matching elasticity parameter.
6. $\tau = 0.049$. Proportional minimum servicing fee for credit matching (per annum).
    - $\tau_q = 1.049^{1/4} - 1$. Quarter servicing fee for credit matching.
7. $\bar{s} = 0.01$. Credit separation rate
8. $K_C = 1.75e^{-6}$. Credit vacancy cost.
9. $\sigma = 2$. Risk aversion.
10. $T = 120$. The total household Life span in quarters.
11. $T_{economy} = 280$. The total length of the economy in quarters.

12. $x = 0.28$. Size of expense shock.

13. $p_x = 0.022$. Probability of experiencing unmodeled shocks.

14. $K_L = 0.021$ (Calibrated). Labor vacancy posting cost. $K_L = 0.0021$ (Data).

15. $\beta = 0.974$ (Calibrated). Quarterly household discount factor. $\beta = 0.0519$ (Data)

16. $K_D = 0.184$(Calibrated). The disutility of default. $K_D = 0.0106$ (Data).

17. $\chi_C = 0.21$ (Calibrated). Utility cost of obtaining credit. $\chi_C = 0.331$ (Data).

18. $\eta = 0.604$ (Calibrated). Flow utility of leisure. $\eta = 0.9360$ (Data).

19. $A_{1977} = 0.48203$. Credit matching efficiency.

20. $\epsilon_D = 0.001$ Default penalty.

21. $\epsilon = 2.2204e{-}16$. Break-point.

1.3 Value Functions and Notations

1. Firm value function $J_t(w; y)$ defined over $(w; y)$

$$J_t(w; \Omega) = y - w + \beta \mathbb{E}\left[(1-\delta) J_{t+1}(w; \Omega')\right] \tag{1}$$

- $\Omega = (\mu,\ A,\ y)$. Aggregate state.
  (a) $\mu:\ \{W, U\} \times \{C, N\} \times \mathcal{W} \cup \mathcal{Z} \times \mathcal{B} \times N_T \to [0,\ 1]$. Household distribution.
  A 5-D matrix (e.g. the distributions of the (un)employed, people w/o credit access, income, wage, and age)
    - $e \in \{W, U\}$. Current employment status, where $e = W$ if employed and $e = U$ if unemployed.
    - $a \in \{C, N\}$. Credit access status, where $a = C$ if has credit access and $a = N$ if no credit access.
    - $w \in \mathcal{W}$. Current wage if employed or unemployment benefits $z = \gamma w \in Z$ if unemployed.
    - $b \in \mathcal{B}$. Net asset.
    - $t \in N_T$. Age.

  (b) y: aggregate productivity.
  (c) A: aggregate credit matching efficiency
  (d) Law of motion for the distribution of household across all state variable $\mu' = \Phi(\Omega, A', y')$

2. Household value functions

(a) Unemployed household defined over (z, b, Ω) (see p2615.)

$$U_t(z, b, \Omega) = \max\{A\psi_{U,t}(z, b; \Omega)U_t^C(z, b; \Omega) + (1 - A\psi_{U,t}(z, b; \Omega))U_t^N(z, b; \Omega) - \chi_C, \\ U_t^N(z, b; \Omega)\} \tag{2}$$

i. With credit access

$$\begin{aligned} U_t^C(z, b; \Omega) = &\max_{b' \in \mathcal{B}, D \in [0,1]} u(c) - x(D) + \eta \\ &+ (1 - s(D)) \cdot \beta \mathbb{E}_{\Omega'} \left[\max_{\tilde{w} \in \mathcal{W}} p_{t+1}(\tilde{w}; \Omega')\hat{W}_{t+1}^C(\tilde{w}, b'; \Omega') + (1 - p_{t+1}(\tilde{w}; \Omega'))\hat{U}_{t+1}^C(z, b'; \Omega') \right] \\ &+ s(D) \cdot \beta \mathbb{E}_{\Omega'} \left[\max_{\tilde{w} \in \mathcal{W}} p_{t+1}(\tilde{w}; \Omega')\hat{W}_{t+1}(\tilde{w}, b'; \Omega') + (1 - p_{t+1}(\tilde{w}; \Omega'))\hat{U}_{t+1}(z, b'; \Omega') \right] \end{aligned} \tag{3}$$

ii. Without credit access

$$\begin{aligned} U_t^N(z, b; \Omega) = &\max_{b' \geq 0, D \in [0,1]} u(c) - x(D) + \eta \\ &+ \beta \mathbb{E}_{\Omega'} \left[\max_{\tilde{w} \in \mathcal{W}} p_{t+1}(\tilde{w}; \Omega')\hat{W}_{t+1}(\tilde{w}, b'; \Omega') + (1 - p_{t+1}(\tilde{w}; \Omega'))\hat{U}_{t+1}(z, b'; \Omega') \right] \end{aligned} \tag{4}$$

(b) Employed household defined over (w, b, Ω) (see Appendix A)

$$W_t(w, b, \Omega) = \max\{A\psi_{W,t}(w, b; \Omega)W_t^C(w, b; \Omega) + (1 - A\psi_{W,t}(w, b; \Omega))W_t^N(w, b; \Omega) - \chi_C, \\ W_t^N(w, b; \Omega)\} \tag{5}$$

i. With credit access

$$\begin{aligned} W_t^C(w, b; \Omega) = &\max_{b' \in \mathcal{B}, D \in [0,1]} u(c) - x(D) \\ &+ (1 - s(D)) \cdot \beta \mathbb{E}[(1 - \delta)\hat{W}_{t+1}(w, b'; \Omega') + \delta \max_{\tilde{w \in \mathcal{W}}}\{p_{t+1}(\tilde{w}; \Omega')\hat{W}_{t+1}(\tilde{w}, b'; \Omega') \\ &+ (1 - p_{t+1}(\tilde{w}; \Omega'))\hat{U}_{t+1}(\gamma w, b'; \Omega')\}] + s(D) \cdot \beta \mathbb{E}[(1 - \delta)\hat{W}_{t+1}^C(w, b'; \Omega') \\ &+ \delta \max_{\tilde{w \in \mathcal{W}}}\{p_{t+1}(\tilde{w}; \Omega')\hat{W}_{t+1}^C(\tilde{w}, b'; \Omega') + (1 - p_{t+1}(\tilde{w}; \Omega'))\hat{U}_{t+1}^C(\gamma w, b'; \Omega')\}] \end{aligned} \tag{6}$$

ii. Without credit access

$$\begin{aligned} W_t^N(w, b; \Omega) = &\max_{b' \geq 0, D \in [0,1]} u(c) - x(D) + \beta \mathbb{E}[(1 - \delta)\hat{W}_{t+1}(w, b'; \Omega') \\ &+ \delta \max_{\tilde{w \in \mathcal{W}}}\{p_{t+1}(\tilde{w}; \Omega')\hat{W}_{t+1}(\tilde{w}, b'; \Omega') + (1 - p_{t+1}(\tilde{w}; \Omega'))\hat{U}_{t+1}(\gamma w, b'; \Omega')\}] \end{aligned} \tag{7}$$

(c) Continuation value function for households $\hat{V}_{t+1} \in \{\hat{W}_{t+1},\ \hat{W}_{t+1}^C,\ \hat{U}_{t+1},\ \hat{U}_{t+1}^C\}$

$$\hat{V}_{t+1}(\tilde{w}, b'; \Omega') = p_x V_{t+1}(\tilde{w}, b' - x; \Omega') + (1 - p_x)V_{t+1}(\tilde{w}, b'; \Omega) \tag{8}$$

(d) Preference function $u(c) + x(D) + \eta(1-h)$ (see p.2620)

$$u(c) + x(D) + \eta(1-h) = \frac{c^{1-\sigma} - 1}{1-\sigma} - K_D \cdot \frac{D}{1 - D + \epsilon D} + \eta(1-h) \tag{9}$$

- $h = \begin{cases} 1, & e = U \\ 0, & e = W \end{cases}$
- $A\psi$: credit finding rate (the probability a household meets a lender).
- p_{t+1}: next period job finding probability.
- $u(c)$: utility over consumption.
- $D \in [0, 1]$. The fraction of debt defaulted upon.
  - $s(D)$: credit relationship breakup probability.
  - $-x(D)$: utility penalty of default.
- $\tilde{w}$: the wage that the household searches for in the next period.
- Π_y: the transitional probability of aggregate productivity $P(y'|y)$ for $y \in Y$, accounts for the expected utilities

1.4 Pseudocode

1. **Setup grids:**
   (a) Wage Grid: $w\_grid = \text{np.arange}(\text{start} = w_{min},\ \text{stop} = w_{max},\ \text{step} = w_{step})$
      i. First by:
         - $w_{1min} = 0.9$
         - $w_{1max} = 0.97$
         - $w_{1step} = 0.01$

      ii. Then by:
         - $w_{2min} = 0.971$
         - $w_{2max} = 1.01$
         - $w_{2step} = 0.001$

      iii. $Nw = 48$

   (b) Unemployment Benefit Grid: $z\_grid = \gamma \cdot w\_grid$
   (c) Asset Grid: $b\_grid = \text{np.linspace}(\text{start} = b_{min},\ \text{stop} = b_{max},\ \text{num} = N_b)$
      - $b_{min} = -0.4875$
      - $b_{max} = 1.75$
      - $N_b = 60$

- $step = 0.0375$
- $b\_step = round(\frac{x}{step},\ 1)$. Expense shock will reduce one's asset position by $bstep$

(d) Repayment Grid: $r\_grid =$ np.linspace(start $= r_{min},$ stop $= r_{max},$ num $= r_b$)

- $r = 1 - D$
- $r_{min} = 0$
- $r_{max} = 1$
- $N_r = 41$

(e) Aggregate Productivity (y) : a vector of values $(y \in Y)$ that follows an AR(1) process (see p2618)

- $ln(y') = \rho\ ln(y) + \epsilon_1,\ s.t.\ \epsilon_1 \sim N(0, \sigma)$
- $\rho = 0.89605$
- $\sigma = 0.0055249$
- $\mu = 0$
- $N_y = 9$
- Use Rouwenhorst estimation from *QuantEcon*

2. **Set:**

(a) $b' = b^*_T(\cdot) = 0$ (i.e. b_{T+1} or b'_T). Period T asset choice.

(b) $q_T(\cdot) = 0$. Period T bond price.

(c) $J^{old}(w; y) = J_{T+1}(w; \Omega) = 0$. Zero for the firm's continuation profit function at period T.

(d) $\hat{V}_{old}(\tilde{w}, b'; \Omega') \in \begin{cases} \hat{U}_{old}(\tilde{w}, b'; \Omega') = \hat{U}_{T+1} = 0 \\ \hat{U}^C_{old}(\tilde{w}, b'; \Omega') = \hat{U}^C_{T+1} = 0 \\ \hat{W}_{old}(\tilde{w}, b'; \Omega') = \hat{W}_{T+1} = 0 \\ \hat{W}^C_{old}(\tilde{w}, b'; \Omega') = \hat{W}^C_{T+1} = 0 \end{cases}$

Zero for each of the household continuation value functions at the period T.

(e) $Q_{old} = QW_T = QU_T = 0$. There is no lenders' profit in the last period because there is no contact rate between households and lenders in the last period.

3. **Solve for firm's value functions iterating backwards from $t = T$ to $t = 1$.**

(a) Given:

- Parameters $(K_L,\ \beta,\ \delta,\ \zeta)$
- $w \in w\_grid$
- $y \in Y$
- Firm's continuation profit function $J^{old}(w; y')$
- Submarket tightness $\theta_t(w; \Omega) = \frac{v_t(w;\Omega)}{u_t(w;\Omega)}$
  - $v_t(w; \Omega)$: number of job vacancies posted in the (w, t) submarket

- Wesubmarket the

- The constant returns to scale of the employment matching function $M(u, v) = \frac{u \cdot v}{(u^\varsigma + v^\varsigma)^{1/\varsigma}} \in [0, 1)$
  - $\frac{M(u,v)}{u} = \frac{\theta}{(\theta^\varsigma+1)^{1/\varsigma}}$, where $\frac{1}{u}$ is the scalar
  - $\frac{M(u,v)}{v} = \frac{1/\theta}{((1/\theta)^\varsigma+1)^{1/\varsigma}}$, where $\frac{1}{v}$ is the scalar
- Job finding probability $p_t(w; \Omega) = \frac{M(u_t(w;\Omega), v_t(w;\Omega))}{u_t(w;\Omega)} = \frac{\theta}{(\theta^\varsigma+1)^{1/\varsigma}}$
- Vacancy filling rate $f_t(w; \Omega) = \frac{M(u_t(w;\Omega), v_t(w;\Omega))}{v_t(w;\Omega)} = \frac{1/\theta}{((1/\theta)^\varsigma+1)^{1/\varsigma}}$
- Free entry condition $K_L = f_t(w; \Omega) J_t(w; \Omega)$
  - $f = \frac{K_L}{J} = \frac{1/\theta}{((1/\theta)^\varsigma+1)^{1/\varsigma}}$

(b) Proceed as follows:

- *Step 1:* compute $J_T(w; y) = y - w + \beta\mathbb{E}[(1 - \delta)J^{old}(w; y')]$ (see *Eq.1*) for each $w \in w\_grid$ and $y \in Y$
  - if $J_T < \epsilon$, set $J_T = \epsilon$ so that job finding probability is less than 1

- *Step 2:* compute market tightness in period T via:

$$\theta_T(w;\ y) = ((\frac{K_L}{J_T(w; y)})^{-\varsigma} - 1)^{1/\varsigma} \tag{10}$$

  - if $(\frac{K_L}{J_T(w;y)})^{-\varsigma} < 1$, set $(\frac{K_L}{J_T(w;y)})^{-\varsigma} = 1$ so that job finding probability is more than 0

- *Step 3:* compute job finding probability $p_T(w; \Omega) = \frac{\theta_T}{(\theta_T^\varsigma+1)^{1/\varsigma}}$ for period T

- *Step 4:* update $J^{old}(w; y') = J_T(w; y)$

- *Step 5:* repeat *Step 1-4* and iterate over period T-1,...,1 to compute $p_{T-1}, ..., p_1$ (p_t is a 3-D matrix: $Nw \times Ny$)

- Notes: In the Baseline Model, the expected firms' profit is given by solely the transitional probability of states (i.e. aggregate productivity). Nevertheless, in the case of On-the-Job Search (see Appendix. p.24), the expected value needs to consider other probabilities, such as job search and credit finding probabilities)

4. **Solve for household's Period T value functions and optimal repayment and asset choice policy function**

   Objective: (1) Calculate 8 value functions in $t = T$: $\{U_T^C,\ U_T^N,\ W_T^C,\ W_T^N,\ U_{T,es}^C,\ U_{T,es}^N,\ W_{T,es}^C,\ W_{T,es}^N\}$, (2) find 8 corresponding (r^*, b'^*) policy functions, and (3) find $\tilde{w}$ for each (r^*, b'^*) pair and productivity states y.

(a) Given:

- Parameter $(\sigma,\ K_D,\ \eta,\ \epsilon D,\ p_x,\ A)$
- Individual characteristic $\{W, U\} \times \{C, N\}$
- Exogenous state (productivity) y
- $w \in w\_grid$ & $z \in z\_grid$
- $b \in b\_grid$
- Last period asset choice b'
- Last period bond price q_T
- Household optimal value functions: $\hat{V}^{old}(\tilde{w}, b'; \Omega') \in \{\hat{U}_{old}(\cdot),\ \hat{U}^c_{old}(\cdot),\ \hat{W}_{old}(\cdot),\ \hat{W}^c_{old}(\cdot)\}$

(b) Proceed as follows: $e \in \{W, U\}$ and $a \in \{C, N\}$

- *Step 1:* For $w \in w\_grid$ (or $z \in z\_grid$), $r \in r\_grid$, and $b \in b\_grid$, solve for the present value functions $U_T^C,\ U_T^N,\ W_T^C,\ W_T^N$
  (Dim $= Nw \times Nb \times Nr \times Ny$)
  - $U_T^C(z, b; \Omega) = U_T^N(z, b; \Omega) = u(c_T) - x(D_T) + \eta$ (*Eq.3* & *Eq.4*)
  - $W_T^C(w, b; \Omega) = W_T^N(w, b; \Omega) = u(c_T) - x(D_T)$ (*Eq.6* & *Eq.7*)

  $$\text{where } c_T = \begin{cases} z + rb, & e = U,\ a \in \{C, N\}\ (\textit{see } p2616.) \\ w + rb, & e = W,\ a \in \{C, N\}\ (\textit{see Appendix A. } p3. \ \& \ p4.) \end{cases}$$

  \- Note: set marginal cases make sure c ≥ 0

- *Step 2:* use grid-search method to find the (r^*, b'^*) (in period T, $(r^*, 0)$) pair that maximizes the present utility functions V
  (Dim $= Nw \times Nb \times Ny$)
  find $r^*_{WC}, r^*_{WN}, r^*_{UC}$, and r^*_{UN} that maximizes $V^*_{e\in\{W,U\},\ a\in\{C,N\},t} \in \{U_t^C,\ U_t^N, W_t^C,\ W_t^N\}$

- *Step 3:* find $U_T^*(z, b; \Omega)$ and $W_T^*(w, b; \Omega)$ using *Eq.2* & *Eq.5*
  (Dim $= Nw \times Nb \times Ny$)
  - determine and store whether each household applies for credit in a dummy variable
    $$c\_pol = \begin{cases} 1, & if\ apply \\ 0, & if\ not \end{cases}$$
  - Since $A\psi_T = 0$ as there is no contact rate between the lender and household in the last period, households will not apply for credit in the last period (i.e., $c\_pol_T = 0$) for all households.

- *Step 4:* find $U_T(z, b-x; \Omega)$, $U_T^C(\cdot)$, $W_T(w, b-x; \Omega)$, $W_T^C(\cdot)$ and $r^*_{UN,es}$, $r^*_{UC,es}$, $r^*_{WN,es}$, and $r^*_{WC,es}$
  (Dim $= Nw \times Nb \times Nr \times Ny$)

  For all indexes ib in b_grid:
  - if $ib <= b\_step$, set V_{es} $[i\_inc,\ ib] = V$ $[i\_inc,\ 0]$
  - if $ib > b\_step$, set V_{es} $[i\_inc,\ ib] = V$ $[i\_inc,\ ib - b\_step]$, where i_inc is the index for w_grid or z_grid

- *Step 5:* calculate expected value accounting for external shock - use *Eq.8*
  (Dim $= Nw \times Nb \times Ny$)

$$\hat{V}_T^* \in \begin{cases} \hat{U}_T^* = p_x U_T^*(z, b-x; \Omega) + (1-p_x) U_T^*(z, b; \Omega) \\ \hat{U}_T^{C*} = p_x U_T^{C*}(z, b-x; \Omega) + (1-p_x) U_T^{C*}(z, b; \Omega) \\ \hat{W}_T^* = p_x W_T^*(w, b-x; \Omega) + (1-p_x) W_T^*(w, b; \Omega) \\ \hat{W}_T^{C*} = p_x W_T^{C*}(w, b-x; \Omega) + (1-p_x) W_T^{C*}(w, b; \Omega) \end{cases}$$

- *Step 6:* find the value (w^*) and index (iw^*) of w_grid that maximizes the expected period T value functions (i.e., *Eq.12* & *Eq.13*) via grid search method
  (Dim reduced from ($Nw \times Nb \times Nz \times Ny$) to ($Nb \times Nz \times Ny$))

  - UPchoice =

$$\hat{V}^*_{new} = \max_{w^* \in \mathcal{W}} p_t(w^*; \Omega) \hat{W}_t^*(w^*, b; \Omega) + (1 - p_t(w^*; \Omega)) \hat{U}_t^*(z_{t-1}, b; \Omega) \tag{11}$$

  - UPCchoice =

$$\hat{V}^{C*}_{new} = \max_{w^* \in \mathcal{W}} p_t(w^*; \Omega) \hat{W}_t^{C*}(w^*, b; \Omega) + (1 - p_t(w^*; \Omega)) \hat{U}_t^{C*}(z_{t-1}, b; \Omega) \tag{12}$$

- *Step 7:*
  store
  - $r^*_{UN},\ r^*_{UC},\ r^*_{WN},\ r^*_{WC}$
  - $r^*_{UN,es},\ r^*_{UC,es},\ r^*_{WN,es},\ r^*_{WC,es}$
  - $\tilde{w}_{UP},\ \tilde{w}_{UPC}$

  update
  - $b' = b \in b\_grid$
  - $\hat{W}_{old}(w, b'; \Omega') = \hat{W}_T^*(w, b; \Omega)$
  - $\hat{V}^*_{old}(\tilde{w}, b'; \Omega') = \hat{V}^*_{new}(w^*, b; \Omega)$
  - $\hat{V}^{C*}_{old}(\tilde{w}, b'; \Omega') = \hat{V}^*_{new}(w^*, b; \Omega)$

5. **Solve for household's value functions and optimal wage policy function iterating backward from $t = T - 1$ to $t = 1$**

   Objectives in each period: Solve for (1) bond price and lender's profits, (2) 8 value functions in $t = T$: $\{U_T^C,\ U_T^N,\ W_T^C,\ W_T^N,\ U_{T,es}^C,\ U_{T,es}^N,\ W_{T,es}^C,\ W_{T,es}^N\}$, (3) 8 corresponding $(r^*,\ b'^*)$ policy functions, (4) $\tilde{w}$ for each $(r^*,\ b'^*)$ pair and productivity states y.

   (a) Given:
   - Parameter $(r_{f_q},\ \tau_q,\ \sigma,\ K_D,\ \eta,\ \epsilon D,\ \chi_C,\ \beta,\ x,\ p_x,\ K_C,\ \delta,\ \bar{s},\ \zeta_C,\ A)$
   - Individual characteristic $\{W, U\} \times \{C, N\}$
   - Exogenous state & transitional matrix y & Π
   - Job finding probability $p_t(w; y)$
   - $w \in w\_grid$ & $z \in z\_grid$
   - $b \in b\_grid$
   - Optimal wage that job-searching household seeks $\tilde{w}$
   - Next period repayment decision and asset choices
     $rp^* \in \{r_{UN}^*,\ r_{UC}^*,\ r_{WN}^*,\ r_{WC}^*, r_{UN,es}^*,\ r_{UC,es}^*,\ r_{WN,es}^*,\ r_{WC,es}^*\}$
   - Next period asset choices b'
   - Expected value for employed households who keep their jobs that pay $w \in w\_grid$: $\hat{W}_{old}(w, b'; \Omega')$
   - Household **optimized** expected value functions $\hat{V}_{old}^*(\tilde{w}, b'; \Omega')$, $\hat{V}_{old}^{C*}(\tilde{w}, b'; \Omega')$

   (b) Proceed as follows: for $e \in \{W, U\}$, $a \in \{C, N\}$, and $t \in [0, T-1]$:
   Assume we are at $t = T - 1$:
   - Step 1: compute bond price $q_{T-1}(\cdot)$ using *Eq.13* (see p2617.)
     (Dim $= Nw(z) \times Nb' \times Ny'$)

$$q_{e,t}(w, b', D; \Omega) = \begin{cases} \frac{\bar{s}\mathbb{E}\left[1-\hat{D}^{a'}_{e',t+1}(w',b';\Omega')\right]+(1-\bar{s})\mathbb{E}\left[1-\hat{D}^{C}_{e',t+1}(w',b';\Omega')\right]}{1+r_f+\tau}, & b' \in \mathcal{B}_{\text{-}},\ D = 0 \\ 0, & b' \in \mathcal{B}_{\text{-}},\ D > 0 \\ \frac{1}{1+r_f}, & b' \in \mathcal{B}_{+} \end{cases} \tag{13}$$

     - Solve for both employed and unemployed cases, assuming there is no on-the-job search for employed households.
     - See section 1.5.2 for complete bond price calculation and this link for intuitions

   - *Step 2:* calculate lender's profit at $t = T - 1$
     (Dim $= Nw \times Ny$)
     - First calculate EQW and EQU. For $t = T-1$, $EQW = EQU = 0$ since $Q_T = 0$

– Employed:

$$QW_t = \begin{cases} 0, & if\ \mathbb{D}\ >\ 0 \\ \frac{b'^*}{1+r_f} + \frac{1-\bar{s}}{1+r_f} * \mathbb{E}QW, & if\ b'^* \in \mathbb{B}_+\ (i.e.\ D\ =\ 0) \\ \frac{\tau}{1+r_f} * b'^* * q_{W,t} + \frac{1-\bar{s}}{1+r_f} * \mathbb{E}QW, & if\ b'^* \in \mathbb{B}_\text{-},\ D\ =\ 0 \end{cases} \tag{14}$$

– Unemployed:

$$QU_t = \begin{cases} 0, & if\ \mathbb{D}\ >\ 0 \\ \frac{b'^*}{1+r_f} + \frac{1-\bar{s}}{1+r_f} * \mathbb{E}QU, & if\ b'^* \in \mathbb{B}_+\ (i.e.\ D\ =\ 0) \\ \frac{\tau}{1+r_f} * b'^* * q_{U,t} + \frac{1-\bar{s}}{1+r_f} * \mathbb{E}QU, & if\ b'^* \in \mathbb{B}_\text{-},\ D\ =\ 0 \end{cases} \tag{15}$$

- See section 1.5.3 and this link to simplify the calculations of lender's profit and $\mathbb{E}QU_t,\ \mathbb{E}QW_t$

- *Step 3:* find $A\psi_{e,T-1}(w, b; \Omega)$ the probability a household meets a lender (Dim $= Nw \times Nb \times Ny$)

  i. plug Q_{t-1} into the free entry condition (see *Eq.16*) to obtain the contact rate between lenders and household (i.e. $\theta_{C,T-1} = \frac{v_{C,T-1}}{u_{C,T-1}}$)

  $$K_C = A\phi_{e,t}(w, b; \Omega)Q_t(e, w, b; \Omega) \tag{16}$$

  – $v_C(\mathbf{x})$: the number of credit offers sent to households in credit submarket $\mathbf{x}$, where $\mathbf{x} = (e, w, b, \Omega)$

  – $u_C(\mathbf{x})$: the number of households searching for credit with state vector $\mathbf{x}$

  – $M_C(u_C, v_C) = \frac{u_C \cdot v_C}{(u_C^{\zeta_C} + v_C^{\zeta_C})^{1/\zeta_C}} \in [0, 1)$ is a CRS credit matching function

  – $A\phi_{e,t}(w, b; \Omega) = \frac{A \cdot M_C(v_{C,t}(\mathbf{x}), u_{C,t}(\mathbf{x}))}{v_{C,t}(\mathbf{x})}$: the probability a lender matches with a household

  $$\theta_{C,t} = ((\frac{K_C}{AQ_t})^{-\zeta_C} - 1)^{1/\zeta_C} \tag{17}$$

  * if $Q_t < \epsilon$, set $Q_t = \epsilon$ so that credit finding probability is less than 1

  * if $(\frac{K_C}{AQ_t})^{-\zeta_C} < 1$, set $(\frac{K_C}{AQ_t})^{-\zeta_C} = 1$ so that credit finding probability is more than 0

  ii. compute $A\psi_{e,t}(w, b; \Omega) = \frac{A \cdot M_C(v_{C,t}(\mathbf{x}), u_{C,t}(\mathbf{x}))}{u_{C,t}(\mathbf{x})} = A * \frac{\theta_C}{(\theta_C{}^{\zeta}_C + 1)^{1/\zeta_C}}$: the probability a household meets a lender

- *Step 4:* For $w \in w\_grid$ (or $z \in z\_grid$), $r \in r\_grid$, and $b \in b\_grid$ solve for the present value functions $U^C_{T-1},\ U^N_{T-1},\ W^C_{T-1},\ W^N_{T-1}$ (*Eq.3, Eq.4, Eq.6, Eq.7*)
(Dim $= Nw \times Nb \times Nrb' \times Ny$)

$$
\begin{cases}
U_t^C(z,b;\Omega) = u(c_t) - x(D_t) + \eta + s(D)\cdot \beta \mathbb{E}_{\Omega^*}\hat{V}^*_{old}(\tilde{w}) + (1-s(D))\cdot \beta \mathbb{E}_{\Omega^*}\hat{V}^{C*}_{old}(\tilde{w}), & b' \in \mathbb{B} \\
U_T^N(z,b;\Omega) = u(c_t) - x(D_t) + \eta + \beta \mathbb{E}_{\Omega'}\hat{V}^*_{old}(\tilde{w}), & b' \geq 0 \\
W_t^C(w,b;\Omega) = u(c_t) - x(D_t) & \\
\qquad\qquad + (1-s(D))\cdot \beta \mathbb{E}_{\Omega'}\left[(1-\delta)\hat{W}_{old}(w) + \delta \hat{V}^*_{old}(\tilde{w})\right] & \\
\qquad\qquad + s(D)\cdot \beta \mathbb{E}_{\Omega'}\left[(1-\delta)\hat{W}^C_{old}(w) + \delta \hat{V}^{C*}_{old}(\tilde{w})\right], & b' \in \mathbb{B} \\
W_t^N(w,b;\Omega) = u(c_t) - x(D_t) + \beta \mathbb{E}_{\Omega'}\left[(1-\delta)\hat{W}_{old}(w) + \delta \hat{V}^*_{old}(\tilde{w})\right], & b' \geq 0
\end{cases}
$$

where:

  - credit relationship breakup probability $s(D) = \begin{cases} 1, & if\ 1-r > 0 \\ \bar{s}, & if\ 1-r = 0 \end{cases}$

  - $c_t = \begin{cases} z + (1-D)b - q_{U,t}(z,b',D;\Omega)b', & e=U,\ a=C \\ z + (1-D)b - \frac{1}{1+rf}b', & e=U,\ a=N \\ w + (1-D)b - q_{W,t}(w,b',D;\Omega)b', & e=W,\ a=C \\ w + (1-D)b - \frac{1}{1+rf}b', & e=W,\ a=N \end{cases}$

- *Step 5:* find $U_T^*(z,b;\Omega)$ and $W_T^*(w,b;\Omega)$ using *Eq.2* & *Eq.5*
(Dim $= Nw \times Nb \times Ny$)

  - determine and store whether each household applies for credit in a dummy variable
    $c\_pol = \begin{cases} 1, & if\ apply \\ 0, & if\ not \end{cases}$

- *Step 6:* use grid-search method to find the (r^*, b'^*) pair that maximizes the present utility functions V
(Dim $= Nw \times Nb \times Ny$)

  for ibp in bp_grid and for ir in r_grid, find (r^*_{WC}, bp^*_{WC}), (r^*_{WN}, bp^*_{WN}), (r^*_{UC}, bp^*_{UC}), and (r^*_{UN}, bp^*_{UN}) that maximizes $V^*_{e\in\{W,U\},\ a\in\{C,N\},t} \in \{U_t^C,\ U_t^N, U_t,\ W_t^C,\ W_t^N,\ W_t\}$

  - Also store q, $A\psi$, c_pol, and Q that correspond to the optimal (r^{WC*}, b'^{WC*}) and (r^{UC*}, b'^{UC*}) selection since q, $A\psi$, c_pol, and Q only matter to household with credit access.

- *Step 7:* find $U_T(z, b-x;\Omega)$, $U^C_T(\cdot)$, $W_T(w, b-x;\Omega)$, $W^C_T(\cdot)$ and $(r^*_{WC,es}, bp^*_{WC,es})$, $(r^*_{WN,es}, bp^*_{WN,es})$, $(r^*_{UC,es}, bp^*_{UC,es})$, and $(r^*_{UN,es}, bp^*_{UN,es})$
(Dim $= Nw \times Nb \times Nr \times Ny$)

  For all indexes ib in b_grid:

- if $ib <= b\_step$, set $V_{es}\ [i\_inc,\ ib] = V\ [i\_inc,\ 0]$
- if $ib > b\_step$, set $V_{es}\ [i\_inc,\ ib] = V\ [i\_inc,\ ib - b\_step]$, where i_inc is the index for w_grid or z_grid

- *Step 8:* account for external shocks - calculate *Eq.8*
(Dim = $Nw \times Nb \times Ny$)

$$\hat{V}_t^* \in \begin{cases} \hat{U}_t^* = p_x U_t^*(z, b - x; \Omega) + (1 - p_x) U_t^*(z, b; \Omega) \\ \hat{U}_t^{C*} = p_x U_t^{C*}(z, b - x; \Omega) + (1 - p_x) U_t^{C*}(z, b; \Omega) \\ \hat{W}_t^* = p_x W_t^*(w, b - x; \Omega) + (1 - p_x) W_t^*(w, b; \Omega) \\ \hat{W}_t^{C*} = p_x W_t^{C*}(w, b - x; \Omega) + (1 - p_x) W_t^{C*}(w, b; \Omega) \end{cases}$$

- *Step 9:* find the value (w^*) and location (iw^*) of w_grid that maximize the expected period $T - 1$ value functions (specified below in *Eq.12* & *Eq.13*) via grid search method
(Dim reduced from ($Nw \times Nb \times Nz \times Ny$) to ($Nb \times Nz \times Ny$))
  - UPchoice =
$$\hat{V}_{new}^* = \max_{w^* \in \mathcal{W}} p_t(w^*; \Omega) \hat{W}_t^*(w^*, b; \Omega) + (1 - p_t(w^*; \Omega)) \hat{U}_t^*(z_{t-1}, b; \Omega) \tag{18}$$
  - UPCchoice =
$$\hat{V}_{new}^{C*} = \max_{w^* \in \mathcal{W}} p_t(w^*; \Omega) \hat{W}_t^{C*}(w^*, b; \Omega) + (1 - p_t(w^*; \Omega)) \hat{U}_t^{C*}(z_{t-1}, b; \Omega) \tag{19}$$
  - Also store q, $A\psi$, c_pol, and Q that correspond to the optimal w^* selection based on both UPchoice and UPCchoice

  - store the pt, r, b', A_ψ, and c_pol corresponding to the index iw* from optimizing UPchoice and UPCchoice

- *Step 10:* update
store
  - r^*, r_{es}^*
  - bp^*,
  - $r^*(UP(C))$, $r_{es}^*(UP(C))$
  - $bp^*(UP(C))$, $bp_{UN,es}^*(UP(C))$
  - $\tilde{w}_{UP},\ \tilde{w}_{UPC}$

  update
  - $b' = b \in b\_grid$
  - $c\_pol_{old} = c\_pol(r^*, bp^*)$, $A\psi_{old} = A\psi(r^*, bp^*)$, and $Q_{old} = Q(r^*, bp^*)$
  - $c\_pol_{UP(C),old} = c\_polUP(C)$, $A\psi_{UP(C),old} = A\psi_{UP(C)}$, and $Q_{UP(C),old} = Q_{UP(C)}$
  - $\hat{W}_{old}(w, b'; \Omega') = \hat{W}_T^*(w, b; \Omega)$
  - $\hat{V}_{old}^*(\tilde{w}, b'; \Omega') = \hat{V}_{new}^*(w^*, b; \Omega)$
  - $\hat{V}_{old}^{C*}(\tilde{w}, b'; \Omega') = \hat{V}_{new}^*(w^*, b; \Omega)$

  and follow *Step* 1 $\sim$ *Step* 10 to find the optimal asset and wage policy function for $t = T - 2,\ T - 3,\ ....,\ 1$.

1.5 Simulate Sample Path

1. **Initialization**

   (a) Store all policy functions based on the results from solving the firm and household problems

   (b) Create a probability distribution function $j = pdfsim(x, P)$ so that given a uniform random number $x \in [0, 1]$, $pdfsim$ spits out the corresponding state j given the pdf P

   - find the index i in cumsum P where $x \geq cumsum(P)[i]$ and $x < cumsum(P)[i + 1]$

   (c) Create a probability distribution function $j = markovsim(x, P, i)$ so that given a uniform random number $x \in [0, 1]$, a transition matrix Π, and a previous state i, $markovsim$ spits out the next state j

   - The process is similar to $pdfsim$

   (d) Set size of economy:

      i. $N = 60,0000$ (cross-section of households)
      ii. $T_{\text{economy}} = 280$ (number of periods)

   (e) Initialize random draws to determine the transition of aggregate states:

      i. $ydraw = rand(T_{\text{economy}}, 1)$ – aggregate productivity

   (f) Initialize random draws to determine the transition of individual states:

      i. $wdraw = rand(T_{\text{economy}}, N)$ – draw for endogenous job-finding rate:
      ii. $fdraw = rand(T_{\text{economy}}, N)$ – draw for exogenous job-separation rate:
      iii. $cdraw = rand(T_{\text{economy}}, N)$ – draw for endogenous credit-finding rate:
      iv. $dcdraw = rand(T_{\text{economy}}, N)$ – draw for exogenous credit-separation rate:
      v. $esdraw = rand(T_{\text{economy}}, N)$ – draw for expense shock

   (g) Initialize random draws to determine initial state of economy/newborns:

      i. $edraw = rand(T_{\text{economy}}, N)$ – draw for initial employment status
      ii. $adraw = rand(T_{\text{economy}}, N)$ – draw for initial age
      iii. $wdraw$ – draw for initial wage/unemployment compensation. Note that we can use the same draw as for the job-finding rate ($wdraw$) (there is no jog-finding rate for newborns / in the first period of the economy)

   (h) Set distributions for the initial state of economy/newborns:

      i. $astart \sim$ uniform over $[1, T_{\text{span}}]$ – Age
      (Individuals live for $T_{\text{span}} = 120$ periods)

ii. $estart \sim [p_e, 1-p_e]$ – Employment status
($p_e = 0.00001$ is the probability of being employed)

iii. $wstart \sim$ uniform over $[w_{min}, w_{max}]$ – Wage/unemployment compensation

(i) Initialize the matrices to store simulated paths:

i. $Y(T_{economy})$ – aggregate productivity
ii. $age(T_{economy}, N)$ – age
iii. $b(T_{economy}, N)$ – asset holdings
iv. $r(T_{economy}, N)$ – repayment decision
v. $a(T_{economy}, N)$ – credit access
vi. ...

(j) Simulate exogenous aggregate states.

i. Aggregate productivity:
- Arbitrarily set initial state: $Y(1) = y\_grid\ [round(\frac{Ny}{2})]$.
- For $t = 2:280$, use *markovsim* to compute the transition.
$Y(t) = \text{markovsim}(ydraw(t), P_y, Y(t-1))$

(k) Simulate initial period ($T_{economy} = 1$).

Loop over individuals (for $i = 1:N$)

i. Set:
- $b(1, i) = 0$
- $a(1, i) =$ No access
- $r(1, i) = 1$

ii. Use *pdfsim* to assign:
- $age(1, i) = \text{pdfsim}(adraw(1, i), astart)$
- $w(1, i) = \text{pdfsim}(wdraw(1, i), wstart)$
- $e(1, i) = \text{pdfsim}(edraw(1, i), estart)$

2. **Simulate the lifelong decisions for household** i

Period $T_{economy} = 2$ to $T_{economy} = 280$

(a) Update and set
- Set $age_{old} = age\ _{T_{economy}=1, i}$
- Calculate $age_{new} = age_{old} + 1$

- If $age_{new} \ = \ T+1 \ = \ 120+1$:

$$\begin{cases} age_{new} \ = \ 1 \\ \text{Use } pdfsim \text{ to redraw an initial income } income_{old} \text{ repeat 1.5.5} \\ \text{Use } pdfsim \text{ to redraw an initial employment status } e_{old} \text{ repeat 1.5.6} \\ b_{old} = b \ _{T_{economy}=1,i} \\ c_{old} = c \ _{T_{economy}=1,i} \\ r_{old} = r \ _{T_{economy}=1,i} \end{cases}$$

- If $age_{new} \ \neq \ T+1$:

$$\begin{cases} age_{new} \ = \ age_{new} \\ income_{old} = income_{T_{economy}=1,i} \\ e_{old} = e_{T_{economy}+1,i} \\ b_{old} = b_{T_{economy}+1,i} \\ c_{old} = c \ _{T_{economy}+1,i} \\ r_{old} = r \ _{T_{economy}+1,i} \end{cases}$$

(b) If household i found a job last period (i.e., $e_{old} = W$), decide whether this household i experiences job loss

Update $e_{(new|e_{old}=W)} =$

$$\begin{cases} W, \ if \ (1-\delta) \ < \ Pr(fdraw) \\ U, \ if \ (1-\delta) \ > \ Pr(fdraw) \end{cases}$$

Set $e_{new} = W$ if $e_{(new|e_{old}=W)} = W$

- If employed individual is not fired, set $income_{new} = income_{old}$

(c) If the individual is unemployed (whether it is separated from their job exogenously or unemployed in the last period), given y, $income \ _{old}$, and b_{old}, find the corresponding optimal wage $\tilde{w}$ household searched for and job finding probability $pt(\tilde{w}; y)$

- Then, decide whether this household will find a job that is paying $\tilde{w}$ in the next period
  Set
  $$e_{new} = \begin{cases} W, \ if \ pt(\tilde{w}; y) \ < \ Pr(wdraw) \\ U, \ if \ pt(\tilde{w}; y) \ > \ Pr(wdraw) \end{cases} \quad \text{, if } e_{old} = U \text{ or } e_{(new|e_{old}=W)} = U$$
  - Use the same random number that is used to decide the initial income (see step 1.5.5)
  - If the individual finds job, income next period is $income_{new} = \tilde{w}$
  - If the individual does not find credit, income next period is $income_{new} = z_{old}$ (or $\gamma * w_{old}$)

(d) Decide whether this individual experiences expense shock and update b_{old} accordingly

$$b_{old} = \begin{cases} b_{old} - x, \ if \ px > \ Pr(esdraw) \\ b_{old}, \ if \ px < \ Pr(esdraw) \end{cases}$$

(e) If $c_{old} = N$, given y, $income_t$, b_{old}, and employment status, find whether one applies for credit c_pol_t. If so, use A_psi_t to decide whether this individual gets credits if it applies

$$\begin{cases} C, \ if \ A\_psi\_u_0 \ < \ Pr(cdraw) \\ N, \ if \ A\_psi\_u_0 \ > \ Pr(cdraw) \end{cases}$$

(f) Given y, $income_t$, b_{old}, employment status, and the credit access status, find the corresponding policy function r_t and b_t

- Given the updated employment and credit access status, use one of the $r^* \in \{r^*_{WC}, \ r^*_{WN}, \ r^*_{UC}, \ r^*_{UN}\}$ and one of the $bp^* \in \{bp^*_{WC}, \ bp^*_{WN}, \ bp^*_{UC}, \ bp^*_{UN}\}$

(g) Given default decision r^*_t and given the household currently has credit access, decide whether this household is separated from its credit matching status

- recall:
  $$s(D) = \begin{cases} \bar{s}, \text{ if } \mathrm{r}^*_t = 1 \\ 1, \text{ if } \mathrm{r}^*_t < 1 \end{cases}$$
- Determine whether this household would experience exogenous credit loss
  $$\begin{cases} C, \ if \ s(D) < \ Pr(dcdraw|C) \\ N, \ if \ s(D) > \ Pr(dcdraw|C) \end{cases}$$

(h) Given y, $income_t$, b_{old}, employment status, and the credit access status, find the corresponding bond price q (i.e., q_w and q_u)

(i) Update:

- $age_{old} \ = \ age_{new}$
- $income_{old} = income_{t,i}$
- $e_{old} = e_{t,i}$
- $b_{old} = b_{t,i}$
- $r_{old} = r_{t,i}$

(j) Repeat Step (a) to (j) for 280 periods.

3. **Repeat this process for $N = 60000$ individuals and simulate the life-long decisions of N individuals for $R = 10$ times**

1.6 Notes

1. Bond Price

   Let

   - r^* & bp^* ($irbp^*$) be the optimal repayment and asset choice pair (r^*, bp^*) from optimizing present utility function for ny*nc in period $t+1$
   - w (iw) be current wage in w_grid for ny*nc
   - z (iz - equal to iw) be the unemployment benefit in z_grid given current wage for ny*nc
   - w_1^* (iw_1^*) be the optimal wage from optimizing the *expected* $UPchoice$ for ny*nc for period t
     - Optimizing $UPchoice$ already accounts for whether or not a person applies for credit
   - w_2^* (iw_2^*) be the optimal wage from optimizing the *expected* $UPCchoice$ for ny*nc for period t
   - w_3^* (iw_3^*) be the optimal wage from optimizing the *expected* $WPchoice$ for ny*nc for period t
   - w_4^* (iw_4^*) be the optimal wage from optimizing the *expected* $WPCchoice$ for ny*nc for period t
     - With constant credit matching efficiency, $ny * nc = ny * 1$

   If r < 1, where r is the current choice of repayment decision r_grid [ir]:

   - if bp < 0:

     $q_{W,d,t} = q_{U,d,t} = 0$
   - if bp >= 0:

     $q_{W,d,t} = q_{U,d,t} = \frac{1}{1+r_f}$

   If r = 1, where r is the current choice of repayment decision r_grid [ir]:

   - For the people who are currently employed:

$$\mathrm{q}_{W,t}(w, b', D; \Omega) = \overline{s} * \frac{1}{1+r_f+\tau} * \Pi' * \{(1-\delta) * \{(1-px) * ((1 - c\_pol\_w_{t+1}(irbp^*, iw) * A\psi^C_{W,t+1}(irbp^*, iw))\,(1 - D^N_{W,t+1}(irbp^*, iw)) + c\_pol\_w_{t+1}(irbp^*, iw) * A\psi^C_{W,t+1}(irbp^*, iw)\,(1 - D^C_{W,t+1}(irbp^*, iw))) + px * ((1 - c\_pol\_w_{t+1}(irbp^*, iw) * A\psi^C_{W,t+1}(irbp^*, iw, es))\,(1 - D^N_{W,t+1}(irbp^*, iw, es)) + c\_pol\_w_{t+1}(irbp^*, iw) * A\psi^C_{W,t+1}(irbp^*, iw, es)\,(1 - D^C_{W,t+1}(irbp^*, iw, es)))\} + \delta * (1 - p_{t+1}(irbp^*, iw_1^*)) * \{(1-px) * ((1 - c\_pol\_u_{t+1}(irbp^*, iz) * A\psi^C_{U,t+1}(irbp^*, iz))\,(1 - D^N_{U,t+1}(irbp^*, iz)) + c\_pol\_u_{t+1} * A\psi^C_{U,t+1}(irbp^*, iz)\,(1 - D^C_{U,t+1}(irbp^*, iz))) + px * ((1 - c\_pol\_u_{t+1}(irbp^*, iz) * A\psi^C_{U,t+1}(irbp^*, iz, es))\,(1 - D^N_{U,t+1}(irbp^*, iz, es)) + c\_pol\_u_{t+1} * A\psi^C_{U,t+1}(irbp^*, iz, es)\,(1 - D^C_{U,t+1}(ir_1^*, es)))\} + \delta * p_{t+1}(irbp^*, iw_1^*) * \{(1-px) * ((1 - A\psi^C_{W,t+1}(irbp^*, iw_1^*))\,(1 -$$

$D^N_{W,t+1}(irbp^*, iw_1^*)) + A\psi^C_{W,t+1}(irbp^*, iw_1^*)\ (1 - D^C_{W,t+1}(irbp^*, iw_1^*))\ ) + px * (\ (1 - A\psi^C_{W,t+1}(irbp^*, iw_1^*, es))\ (1 - D^N_{W,t+1}(irbp^*, iw_1^*, es)) + A\psi^C_{W,t+1}(irbp^*, iw_1^*, es)\ (1 - D^C_{W,t+1}(irbp^*, iw_1^*, es))\ )\ \}\ \} + (1 - \bar{s}) * \frac{1}{1+r_f+\tau} * \Pi' * \{ (1-\delta) * (\ (1-px) * (1 - D^C_{W,t+1}(irbp^*, iw)) + px * (1 - D^C_{W,t+1}(irbp^*, iw, es))\ ) + \delta * (1 - p_{t+1}(irbp^*, iw_2^*)) * (\ (1-px) * (1 - D^C_{U,t+1}(irbp^*, iz)) + px * (1 - D^C_{U,t+1}(irbp^*, iz, es)\ ) + \delta * p_{t+1}(irbp^*, iw_2^*) * (\ (1-px) * (1 - D^C_{W,t+1}(irbp^*, iw_2^*)) + px * (1 - D^C_{W,t+1}(irbp^*, iw_2^*, es))\ )\ \}$
(20)

- For the people who are currently unemployed:

$\text{q}_{U,t}(z, b', D; \Omega) = \bar{s} * \frac{1}{1+r_f+\tau} * \Pi' * \{ p_{t+1}(irbp^*, iw_1^*) * \{ (1-px) * (\ (1 - A\psi_{W,t+1}(irbp^*, iw_1^*))\ (1 - D^N_{W,t+1}(irbp^*, iw_1^*)) + A\psi_{W,t+1}(irbp^*, iw_1^*)\ (1 - D^C_{W,t+1}(irbp^*, iw_1^*))\ ) + px * (\ (1 - A\psi_{W,t+1}(irbp^*, iw_1^*, es))\ (1 - D^N_{W,t+1}(irbp^*, iw_1^*, es)) + A\psi_{W,t+1}(irbp^*, iw_1^*, es)\ (1 - D^C_{W,t+1}(irbp^*, iw_1^*, es))\ )\ \} + (1 - p_{t+1}(irbp^*, iw_1^*)) * \{ (1-px) * (\ (1 - c\_pol\_u(irbp^*, iz) * A\psi_{U,t+1}(irbp^*, iz))\ (1 - D^N_{U,t+1}(irbp^*, iz)) + c\_pol\_u(irbp^*, iz) * A\psi_{U,t+1}(irbp^*, iz)\ (1 - D^C_{U,t+1}(irbp^*, iz))\ ) + px * (\ (1 - c\_pol\_u(irbp^*, iz) * A\psi_{U,t+1}(irbp^*, iz, es))\ (1 - D^N_{U,t+1}(irbp^*, iz, es)) + c\_pol\_u(irbp^*, iz) * A\psi_{U,t+1}(irbp^*, iz, es)\ (1 - D^C_{U,t+1}(irbp^*, iz, es)) + (1 - \bar{s}) * \frac{1}{1+r_f+\tau} * \Pi' * \{ (\ p_{t+1}(irbp^*, iw_2^*) * \{ (1 - px) * (1 - D^C_{W,t+1}(irbp^*, iw_2^*) + px * (1 - D^C_{W,t+1}(irbp^*, iw_2^*, es)\ \}\ ) + (\ (1 - p_{t+1})(irbp^*, iw_2^*)) * (\ (1 - px) * (1 - D^C_{U,t+1}(irbp^*, iz)) + px * (1 - D^C_{U,t+1}(irbp^*, iz, es))\ )\ \}$
(21)

2. Lender's profits

   In the paper (*Online Appendix p.4*):

$$Q_t(e, w, b; \Omega) = \begin{cases} q_{e,t}(w, b', D; \Omega)b' \\ \qquad -\frac{\bar{s}}{1+r_f}\mathbb{E}\left[\left(1 - \left(p_x D^{a'}_{e',t+1}(w', b' - x; \Omega') + (1 - p_x)D^{a'}_{e',t+1}(w', b'; \Omega')\right)\right) \cdot b'\right] \\ \qquad -\frac{(1-\bar{s})}{1+r_f}\mathbb{E}\left[\left(1 - \left(p_x D^{C}_{e',t+1}(w', b' - x; \Omega') + (1 - p_x)D^{C}_{e',t+1}(w', b'; \Omega')\right)\right) \cdot b'\right] \\ \qquad +\frac{(1-\bar{s})}{1+r_f}\mathbb{E}\left[p_x Q_{t+1}(e', w', b' - x; \Omega') + (1 - p_x)Q_{t+1}(e', w', b'; \Omega')\right], & \text{if } D_t = 0 \\ 0, & \text{if } D_t > 0 \end{cases} \quad (22)$$

   In other words,

$$Q_t(e,w,b;\Omega)=\begin{cases} q_{e,t}(w,b',D;\Omega)b' \\ \qquad -\frac{\bar{s}}{1+r_f}\mathbb{E}\left[\left(1-\hat{D}^{a'}_{e',old}\right)\cdot b'\right]-\frac{(1-\bar{s})}{1+r_f}\mathbb{E}\left[\left(1-\hat{D}^{C}_{e',old}\right)\cdot b'\right] \\ \qquad +\frac{(1-\bar{s})}{1+r_f}\mathbb{E}\left[p_x Q_{t+1}(e',w',b'-x;\Omega')+(1-p_x)Q_{t+1}(e',w',b';\Omega')\right], & \text{if } D_t=0 \\ 0, & \text{if } D_t>0 \end{cases}$$

where $q_{e,t}=\frac{\bar{s}\mathbb{E}\left[1-\hat{D}^{a'}_{e',t+1}(w',b';\Omega')\right]+(1-\bar{s})\mathbb{E}\left[1-\hat{D}^{C}_{e',t+1}(w',b';\Omega')\right]}{1+r_f+\tau}$, When $b'\in\mathbb{B}_{-}$, $D\ =\ 0$

Therefore:

- Employed:

$$QW_t=\begin{cases} 0, & if\ \mathbb{D}\ >\ 0,\ b'\in\mathbb{B}_+ \\ \frac{\mathbb{E}QW*(1-\bar{s})}{1+r_f}, & if\ D\ =\ 0,\ b'\in\mathbb{B}_+ \\ |\frac{\tau}{1+r_f}*b'*q_{W,d}|, & if\ D\ >\ 0,\ b'\in\mathbb{B}_- \\ |\frac{\tau}{1+r_f}*b'*q_W|+\frac{\mathbb{E}QW*(1-\bar{s})}{1+r_f}, & if\ D\ =\ 0,\ b'\in\mathbb{B}_- \end{cases} \tag{23}$$

EQW $=\Pi*\{(1-\delta)\ *\ ((1-px)Q_{W,t+1}(irbp^*,iw)\ +\ pxQ_{W,t+1}(irbp^*,iw,es))\ +\ \delta\ *\ (1-p_{t+1}(irbp^*,iw_2^*))\ *\ ((1-px)Q_{U,t+1}(irbp^*,iz)\ +\ pxQ_{U,t+1}(irbp^*,iz,es))\ +\ \delta\ *\ p_{t+1}(irbp^*,iw_2^*)\ *\ ((1-px)Q_{W,t+1}(irbp^*,iw_2^*)\ +\ pxQ_{W,t+1}(irbp^*,iw_2^*,es))\}$(24)

- Unemployed:

$$QU_t=\begin{cases} 0, & if\ \mathbb{D}\ >\ 0,\ b'\in\mathbb{B}_+ \\ \frac{\mathbb{E}QU*(1-\bar{s})}{1+r_f}, & if\ D\ =\ 0,\ b'\in\mathbb{B}_+ \\ |\frac{\tau}{1+r_f}*b'*q_{U,d}|, & if\ D\ >\ 0,\ b'\in\mathbb{B}_- \\ |\frac{\tau}{1+r_f}*b'*q_U|+\frac{\mathbb{E}QU*(1-\bar{s})}{1+r_f}, & if\ D\ =\ 0,\ b'\in\mathbb{B}_- \end{cases} \tag{25}$$

EQU $=\Pi*\{p_{t+1}(irbp^*,iw_2^*)\ *\ ((1-px)*Q_{U,t+1}(irbp^*,iw_2^*)\ +\ px*(irbp^*,iw_2^*,es))\ +\ p_{t+1}(irbp^*,iw_2^*)\ *\ ((1-px)*Q_{U,t+1}(irbp^*,iz)\ +\ px*(irbp^*,iz,es))\}$(26)