Analysing Gravitational Waveforms using Machine Learning

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Abstract

Machine Learning has emerged as one of the most relelvant tools with applications in several areas of study. In this work, we apply Machine Learning to the area of Gravitational wave astronomy. Detection of gravitational waves is extremely complicated and requires templates of waveforms. These waveforms depend on the masses and spins of black holes and in principle, can be used to capture the complete dynamics of black holes mergers. Here, we try to analyse gravitational waveforms using machine learning methods and try to extract information about its source types without introducing any additional physics. We find its more efficient to work in frequency domain compared with time domain. We consider several standard supervised machine learning classification methods and find good accuracy with random forest and KNN algorithms.

1 Introduction

Gravitational Wave Astronomy has emerged as a new way to explore the dark regions of universe. Predicted in 1905 by Henri Poincaré, the first theoretical proof of these waves came from Einstein's theory of General Relativity in 1915. According to this theory, the universe resides in four dimensional continuum of space and time called spacetime. In empty case (i.e. without energy), the spacetime remains flat. However, presence of massive objects creates curvature in spacetime which we feel as gravitational force. Further, when two massive objects move in this spacetime, it causes continuous change of curvature creating ripples, known as gravitational waves. Thus, gravitational waves are perturbations of spacetime.

There are numerous strong sources of gravitational waves such as binary black holes, binary stellar systems, supernova etc which emit in different frequency range. However, due to the extremely weak interaction of this radiation with matter and exceptionally small amplitudes of these waves, their detection is quite challenging. Further, detectors suffer from different kinds of noise in the surroundings whose ordersare similar to signal strength, which makes this task more challending. Hence, to ensure a definite detection requires a thorough data analysis using multiple methods.

Through several decades of hard work of hundreds of people, the first detection of gravitational waves happened in September 2015 by LIGO almost after 100 years of their prediction[1]. This signal was emitted by merger of two binary black holes of masses $35M_\odot$ and $30M_\odot$ about 1.4 billion light years ago. With more recent detections of GW from binary black holes, study of such systems has developed a newfound research interest. Many researchers are working on developing new methods for data analysis and decrypting the information about the sources to more astrophysical topics such as understanding merger of two black holes occur, black hole information paradox etc.

One of our topic of interest is developing methods to extract information about the original system which emitted these waves. This follows from several motivations, some of which are as follows [2-4].

- This is currently an important problem faced in the detection process. The usual parameter estimation methods rely of computing the match of detected signal with waveform templates, which, can have huge error bars and also take a long time to complete (from days to weeks). Doing this with Machine Learning may provide faster and more accurate results.
- For performing simulations of binary black holes, we need some initial configuration of black holes to start the simulation. We can use machine learning (regression methods) to find the gaps between current bank of waveforms where more simulations are required and also determine the initial data to begin the simulations.
- Though the waveforms contain relevant information about spins and masses of black holes, it is quite non-trivial to extract this information. For eg. many non-spinning waveforms have very similar match as spin-aligned waveforms. Hence, even when LIGO detects a signal, it is very difficult to confidently say the spin-type of the binary. Here, we aim to find the relevant features, which can distinguish between such configurations thus providing extremely relevant information.
- In general, machine learning can be used to directly distinguish and extract the signals from noise, extrapolate the signals and even classify different types of noise.

In this project, we try to pursue this problem of feature extraction using the methods of machine learning. More specifically, we want to understand which features in gravitational waves correlates with the particular characteristics of the binary black holes. A black hole is the most simple astrophysical object which can be completely classified by its mass and spins. Thus, a binary black hole configuration can be distinguished based on these eight features (two masses and six spin components). Since it's extremely difficult to deal with eight dimensional space using regression, we first simplify the problem as a classification problem. Our problem can be split into two parts:

- Classification of waveforms based on the Spin-type: Each binary black hole configuration can be divided into non-spinning, aligned spins and precessing based on type of spins. In non-spinning, neither of the two black holes are spinning. In aligned spins, each black hole is rotating along (or opposite to) the axis of orital plane. Finally, when the spins have any random directions then its a precessing case.
- Classification based on Mass Ratio: Instead of treating individual masses we change our parameters to total mass and ratio of the two masses. This is simply because the waveforms of different total masses just differ by a scaling factor and hence, can be accounted by choice of units which really leaves us with just one mass dependent parameter - mass ratio.

In the following report, we describe our methodology and relevant results. Section 2 elaborates on the data and method of extraction. Section 3 describes the results of dimensionality reduction methods followed by analysis using supervised learning for each case in section 4 and 5. Here we employ five main classification algorithms - Decision Trees, Neural Networks, Boosting, KNN and SVM. Finally, we discuss the results and conclude with future possibilities.

2 Data

2.1 Data Description

As any other signal, gravitational waveform can be described using the amplitude and phase of the signal as [5]

$$h = Ae^{\iota\phi} = \sum_{l=2}^{n} \sum_{m=-l}^{l} A_{lm} e^{\iota\phi_{lm}} \tag{1}$$

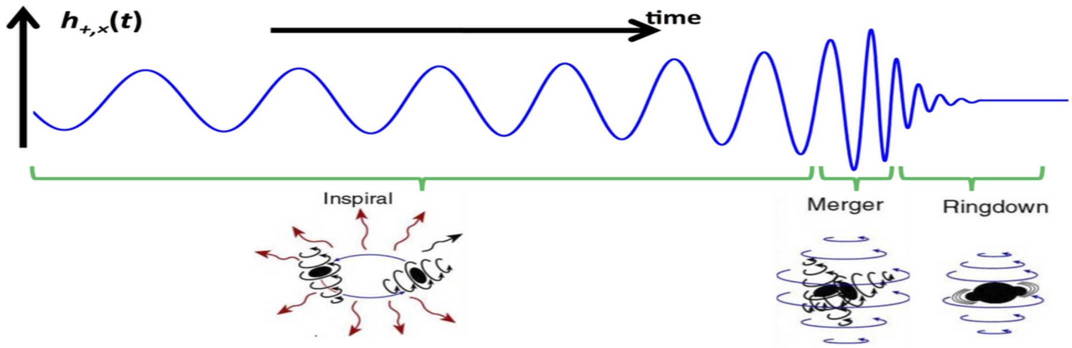

Figure 1: Gravitational Waveform template

Here, h is the strain measured in the detector [1]. A and ϕ are amplitude and phase of the gravitational wave. One can use spin-weighted spherical harmonics as basis to decompose this signal giving us the expression on the right. It has been shown that out of all l,m modes, l=2 and m=2 are the most dominant ones. Thus we restrict to the (2,2) mode and hence, our waveform can be written as:

$$h = A_{2,2}e^{\iota\phi_{2,2}} = h_+ + \iota h_\times$$

where the h_+ is the real part and $h_\times$ is the complex part of the (2,2) mode of waveform. In our dataset, our sample space consists of 486 waveform templates, each having information about time, h_+ and $h_\times$, spin-type and mass ratio. We consider each point of h_+ and $h_\times$ in time as our feature space. Thus, for a waveform of length n in time has n features in h_+ and n features in $h_\times$. This consitutes our x-dataset. The target or y set consists of either spin-types (non-spinning, aligned-spin, precessing) or mass ratio(1-10) of each waveform depending on the problem.

Physically, the double derivative of strain is a measure of curvature of spacetime. When massive objects move in spacetime, this causes continuous variation in curvature which can be tracked using this strain function. Thus strain can provide valuable information about the source.

2.2 Data Extraction

We use waveform catalog from two different Numerical Relativity groups - SXS and Georgia Tech [6-23]. These waveforms are generated by assuming some initial configuration of the binary black holes system and evolving them using Einstein's equations, the numerical methods of simulation varies between the two groups. The waveform is provided in form of h5 files, the format of which varies depending on the waveform which makes it difficult to extract the data. We initially started with h5py package of python which did not work very efficiently. After some search, we used the LalSuite package originally provided by LIGO (publically available at https://wiki.ligo.org/DASWG/LALSuite). Due to its complicated structure, we finally ended up with a using a mathematica scripts freely available by Georgia Tech NR group [http://www.einstein.gatech.edu/] and with some modifications, we were able to extract each waveform and relevant information about the initial configuration.

This was followed with data-cleaning and data-sampling. To apply machine learning algorithm, we need to ensure that each waveform as same number of features. As our feature space is dependent on length of waveforms, we first need to crop some of the waveforms to ensure that each waveform has same length of the data. A waveform can be described in three parts - Inspiral, Merger and Ringdown. Inspiral part of waveform corresponds to weak gravity region where black holes are still far apart. Merger part corresponds to the merger of two black holes into a more massive black holes. The ringdown region corresponds to the final black hole, which is still perturbed and hence emitting radiation. Since merger of the waveform is the most relevant part, as it shows how the information

[1] To detect gravitational waves, one uses advanced interferometric methods where intereference of light is used to detect the gravitational waves. In its default state, the light in two arms of detector interfere destructively. However, when gravitational wave passes, it causes stretch in one arm and compression in another leading to visible light signals. This stretch and compression is measured in form of strain.

about two black holes (8 features) is lost with final black hole formation (4 features), we choose this as reference point (i.e. t=0 at maximum amplitude). Since we are more concerned about this initial information, we crop the waveform in inspiral (beginning) and ringdown (ending) regions retaining complete merger information. Based on the shortest waveforms available, the length of each waveform is set to be 517 time points which translates to 1034 features in total. Clearly, our feature space is much larger than our sample size. To avoid the curse of dimensionality, we need to use some dimensionality reduction methods.

3 Preprocessing

Since the main goal of our project is to classify the black-hole systems on the basis of their spin-types, we divide the 487 simulations into three categories:

1. **Aligned-spins** When spins of the two black holes are aligned or anti-aligned with orbital angular momentum.
2. **Non-spinning** When both the black holes are not spinning.
3. **Precessing** When the two black holes have non-aligned spins.

3.1 Dimensionality Reduction

Efficient performance of machine learning algorithm relies on how much of feature space is spanned by training data. Thus, higher the dimensionality of feature space, more number of samples are required for training which follows from curse of dimensionality. Since we are limited by the number of samples, we need to extract out the most important features in this sample space. For this, we try various methods results of which are presented below:

3.1.1 Principal Component Analysis

As the first approach we start with principal component analysis. PCA relies of correlations between the features and rotates the data about the mean to align with axis along maximum variance. Choosing axis in this manner can capture most of the variance in the data with only a few components. As amplitude and phase of a signal at some time t is correlated with past iterations, PCA becomes a strong contender for dimensionality reduction. To perform PCA we first translate the data s.t. the mean of data is relocated at origin. This is important to ensure that we have correct basis that minimizes mean square error. This is followed with two cases, one in which we apply normalization on mean centered data and another where we directly use the mean centered data.

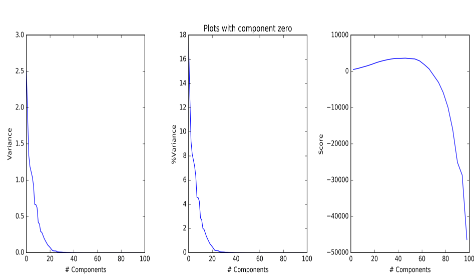

Figure 2: PCA for Mean Centered data

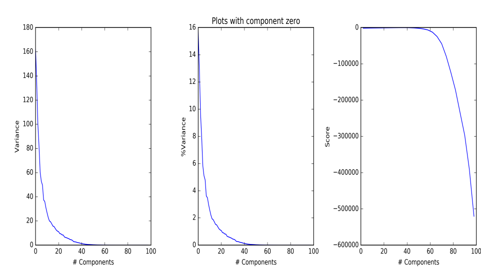

Figure 3: PCA for Mean-Centered Normalized data

Mean Centered Data: As we see in the figure 2, PCA is extremely effective with capturing the most variance of the data in only around 326 components thus reducing the dimensionality significantly. While the score improves till 40 components, it then saturates and finally drops down.

Mean Centered Normalized Data: In this case, after mean centering we also normalize the data. Normalization helps in standardising all the features to same reference. However, in this case, normalization has slightly negative impact as scales of original features carry relevant information and are not just artifacts of different units. This can be clearly seen in Figure 3, where it requires more then 40 components to capture same amount of variance as without normalization.

The effect of PCA can be seen in Figure 4. The I and II Principal Component captures most of the variance and are able to isolate precessing from non-precessing waveforms.

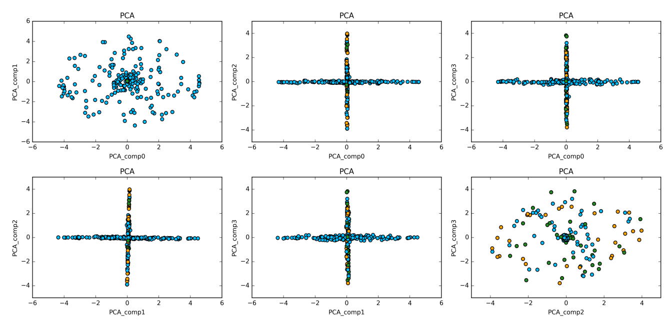

Figure 4: Plot of first few principal components. The orange color represents the non-spinning waveforms, green represents the aligned-spins and blue the precessing waveforms.

Independent Component Analysis: Next, we try independent component analysis. Unlike PCA, ICA works on minimizing the mutual information or maximizing the non-gaussianity under the assumptions that actual sources are independent and non-gaussian. In our case, each simulation is performed independent of each other with a completely different configuration. To check the non-gaussianity, we evaluate the kurtosis and skewness of the dataset (after performing pca).

Kurtosis measures the tailedness of the distribution and skewness measures the assymetry of a distribution about its mean. These provide a good measure of gaussianity. The plots in figure 5 shows variation of kurtosis and skewness for each component. This shows that there is significant gaussianity in first fifteen components for which kurtosis varies between 2-5 and skewness is close to zero. Clearly, ICA would not be very effective in this case. This can be clearly seen in Figure 6 where we have plots of first few ICA components. Most of the datapoints overlap over each other as compared with PCA where we see clear segregation of a class of points.

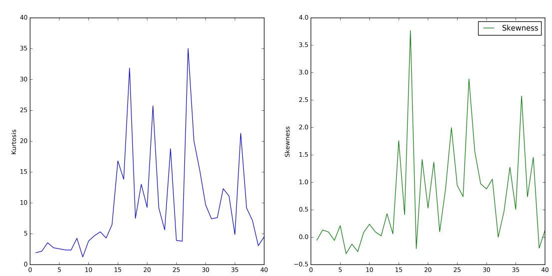

Figure 5: Plot on LHS shows the variation of Kurtosis and Skewness

RCA: The effect of RCA were even worse than PCA and ICA. Further these effects were variable with change of random seeding and hence not very trustable.

3.1.2 Fourier Transformation

Since our data is made of a finite time-series of a gravitational wave, it will be more useful to look at the fourier modes of each wave rather than the amplitudes of these waves. The discrete fourier transform of the data is a complex vector that has fourier components on the horizontal axis and their amplitudes on the vertical axis, see figure 7. One advantage of the fourier mode representation is

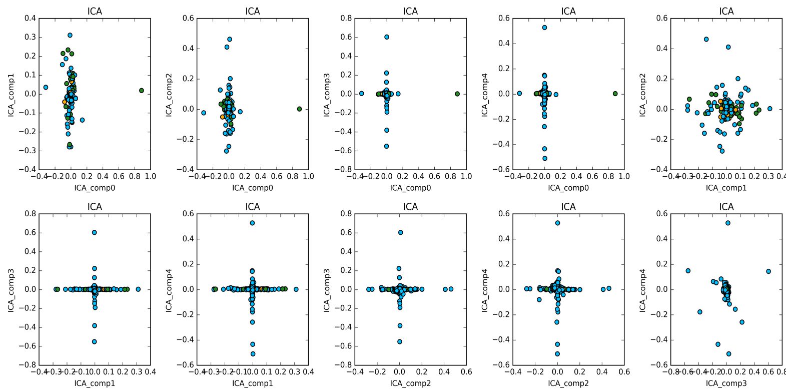

Figure 6: Plot of first independent components. The orange color represents the non-spinning waveforms, green represents the aligned-spins and blue the precessing waveforms.

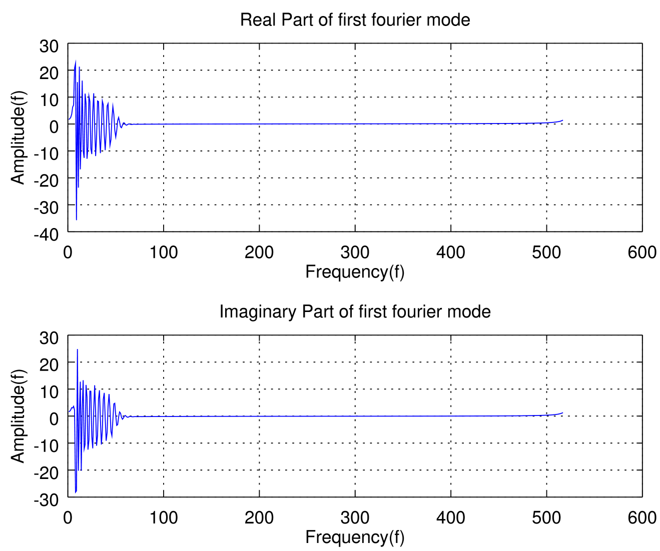

Figure 7: Discrete Fourier transform of the raw strain data. Observe how the dynamics is mostly confined to lower fourier modes.

that the frequencies in our data are a better representation of the dynamics of the black hole system and is more useful in classification problem.

Another useful representation of the data that is most commonly used in spectral analysis is the power spectral density. Power spectral density basically shows us how much weightage does a particular frequency of data hold. This can easily be calculated by finding the absolute value of the amplitude of each fourier component in the data. Figure 8 shows a plot of the spectral power density (PSD). This is a very powerful method and is used widely in various physical dynamics problems, identification of elements in far-off planetary and star atmospheres, remote sensing applications for mining and agriculture industries, etc. The spectral power density can be used to identify and rank the most important features (fourier modes). Furthermore, the information can be used to perform dimensional reduction based on the important modes. We, however, did not manually choose the leading fourier modes but rather used PCA on them (with 95% retained variance) so that we can condense even more variance in the resulting new features.

4 Classification - Spin Type

To explore how much better the algorithms perform with all these transformations we performed the classifications using some known classifiers at each step of the preprocessing. The order of these operations are as follows:

1. *Classifications based on raw time series data* We used Boosting, Bagging, KNN, and Decision Trees for classification. We did not train any neural network on the entire data as the training time involved (> 8 hours) did not justify it.
2. *Classifications based on PCA reduced raw data*- We performed classification of sources using fourier modes as data fed to the learning algorithms namely J48 decision tree, random forest, kNN (k=1), support vector machine with , Ada Boost with random forest on all fourier modes.
3. *Classification based on PCA and random projections reduced spectral data*-Reducing the dimensionality of the fourier modes by using Principal Component decomposition and Random Projections.

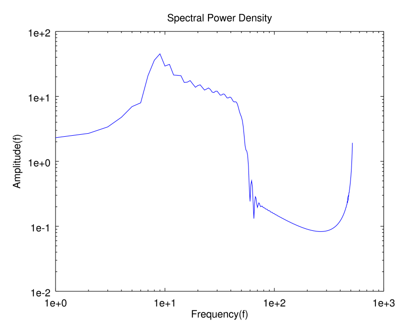

Figure 8: A logarithmic plot of spectral power density. It is this data that gave the best performance on classification.

The performance of the above mentioned supervised learning algorithms is summarized in figure 9

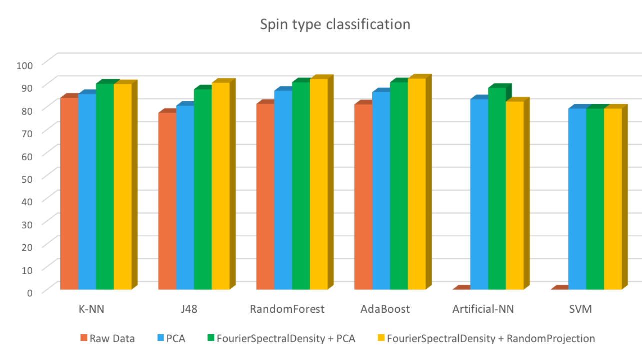

Figure 9: Summary of various classifiers' performance on raw data and data after dimensionality reduction.

4.1 Discussion of observations for spin-type classification

First of all, since there is a lot of raw data (468×1034), the runtime for the algorithms was significantly larger for raw data as compared to the dimensionality reduced data. The classification accuracy for raw data ranges from 77% to 83%. The PCA reduced data performs well with accuracy ranging from 79% to 85%. Figure 9 shows significant improvement in the performance came when we used the power spectral density with PCA and Random projections dimensional reduction.

Surprisingly, in this case, the random projections proved to be the best and fastest classification algorithm with accuracy ($> 93\%$). We investigated this seemingly contradictory result by choosing the same number of attributes ($= 10$) for PCA and Random Projections but Random Projection still won. We still expect that if we increase the number of components for both PCA and RCA, PCA will start to dominate because PCA sets a upper limit to retained variance in one run while random projection can have any distribution of variance.

Out of the classifier algoriths, random forest performs best in almost all types of preprocessing. K-nearest neighbor performs almost as well as random forest because both of them are weighted neighborhood algorithms[24] . The neighborhood of a point in random forest is adapted according to the importance of that point. This is reflected in the weights assigned to the various decision trees in the random forest algorithm. This is qualitatively similar to the kNN algorithm. We tried multilayer perceptrons (ANN) but due to the large number of attributes (1035), the ANN was taking too much time as compared to other algorithms that finished in just about a second. We decided to stop the algorithm.

5 Classification - Mass Ratio

Here we try to classify the mass ratios of the black hole binary systems based on the same data as before. The only difference is that instead of nomial class lables, we now have numeric class values in the range $m1/m2 = [1, 10]$.

5.1 Preprocessing and Supervised learning

Here we are directly using the spectral power density as input data to the algorithm as the data is exactly the same.

Mass ratio as nominal class attribute For most of the previously used algorithms to work, we had to convert the mass ratio class attribute to a nominal attribute. We choose ten equally spaced bins for the mass ratios. We use all the previous supervised learning algorithms for mass ratios and summarize the results in the form of figure 10.

5.2 Supervised Learning

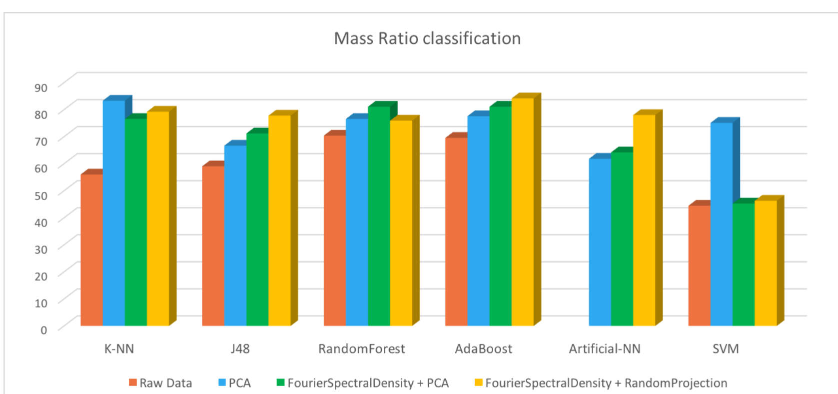

Figure 10: Summary of various classifiers' performance for mass ratio classification.

5.3 Regression

We also tried using the numeric class attribute and used Artifical Neural networks after performing dimensionality reduction using random projections on the spectral power density data. The relative absolute error comes out to be about 30%.

5.4 Unsupervised Learning

Although we know the class labels in our case, we were interested in investigating whether the unsupervised learning techniques can tell us something about the the data that we get. We used *Expectation Maximization* clustering on the label-less data and found that the optimum number of clusters came out to 10. Although, it matches the number of bins used as nominal labels for mass ratios, we must keep in mind that we have arbitrarily chosen this number of bins. We believe clustering might help us identifying the number of qualitatively different mergers taking place but this still neds to be investigated.

6 Summary and Conclusion

In this work, we performed the classification of gravitational waveforms based on the spin and mass ratio properties of black holes. These waves carry the information about the source and hence we try to classify them using supervised machine learning algorithms. As our features we use the real and imaginary part of the strain at every time iteration. After the initial preprocessing and cleaning, we used the methods of dimensionality reduction to avoid the curse of dimensionality arising from larger number of features than sample points. We find that in time-domain, PCA works out the best due to high correlations in our feature space. This is because wave properties at current time would depend on its past. This can also be looked from the binary black hole picture - position of black holes in their trajectory would depend on their past. We also adopt a second strategy to perform our analysis in fourier domain. After performing fourier transformation, we again perform dimensionality reduction to choose the most relevant frequencies. Here, both PCA and RCA perform quite well.

Following this, we apply methods of Supervised Learning for classification based on spin type and mass ratio. We find that most non-linear algorithms perform fairly well which is because the theory behind these waves itself is non-linear. Further, k-NN and Ensemble Learning performs much better than ANN and SVM. While good results from these algorithms were expected, however, the poor accuracy of ANN was surprising especially since ANN have also been used in past to analyse such waveforms[25]. These results suggest that in the eight dimensional space of spin-mass of two black holes, there is a natural clustering of waveforms at least based on spins (and hence, nearest neighbour search is quite powerful).

We repeat this classification for Mass Ratio case. However, compared to spins, we find that results for mass ratio are not quite accurate. This is particularly because, while information about low mass ratio is properly captured in (2,2) mode of waveform, for higher values (q¿3), we need higher order modes. This directs our future direction of work. We would first like to include the higher order mode information (which is not quite trivial) and perform the classification based on mass ratio. We would also like to move from classification and try to predict the spins and mass ratio using regression models. This would help us in finding the initial data required for generating the simulations. Future extensions can include use of Gaussian Process to identify regions of parameter space which lack sufficient number of waveforms and also develop methods to create waveform templates by using current bank of waveforms as features set.

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