Tutotrial Sheet 4

Practise Problems

① $T(n) = 3T(n/2) + n^2$

Using master method comparing with $T(n) = aT(n/b) + f(n)$

$a = 3 \quad b = 2 \quad f(n) = n^2$

$K = \log_b a = \log_2 3 \simeq 1.6$

as $f(n) > n^K$ & $n^2 > n^{1.6}$

so, $TC = \Theta(n^2)$

② $T(n) = 4T(n/2) + n^2$

Comparing with $T(n) = aT(n/b) + f(n)$

$K = \log_b a = \log_2 4 = 2$

As $f(n) = n^K$ & $n^2 = n^2$

So, $TC = \Theta(n^2 \log n)$

③ $T(n) = T(n/2) + 2^n$

Comparing with $T(n) = aT(\frac{n}{b}) + f(n)$

$K = \log_b a = \log_2 1 = 0$

As, $f(n) > n^K$ & $2^n > n^0$

so, $TC = O(2^n)$

(4) $T(n) = 2^n \, T\left(\frac{n}{2}\right) + n^n$

Compare it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$K = \log_2 2^n \Rightarrow n$

As, $f(n) = n^k \Rightarrow n^n = n^n$

$TC = \Theta(n^n \log n)$

(5) $T(n) = 16\,T\left(\frac{n}{4}\right) + n$

Compare it with $T(n) = 16\,T\left(\frac{n}{4}\right) + n$

$K = \log_4 16 = \log_4 4^2 = 2$

As, $f(n) < n^k \Rightarrow n < n^2$

$TC = \Theta(n^2)$

(6) $T(n) = 2T(n/2) + n\log n$

Compare it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$K = \log_2 2 = 1$

As $f(n) > n^k \Rightarrow n\log n > n$

$TC = O(n\log n)$

(7) $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

Compare it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$K = \log_2 2 = 1$

As $f(n) < n^k \Rightarrow \frac{n}{\log n} < n^1$

$\therefore TC = \Theta(n)$

Q8 $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Compare it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_b a = \log_4 2 = \frac{1}{2}$

As $f(n) > n^k \Rightarrow n^{0.51} > n^{0.5}$

$\therefore TC = \cancel{\Theta}\ O(n^{0.51})$

Q9 $TC = 0.5\,T(n/2) + 1/n$

Compare it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_b a = \log_2 0.5 = \log_2 \frac{1}{2}$

$= \log_2 1 - \log_2 2 = 0 - 1 = -1$

As $f(n) = n^k \Rightarrow \frac{1}{n} = \frac{1}{n}$

$\therefore TC = O\left(\frac{\log n}{n}\right)$

(10) $T(n) = 16T\left(\frac{n}{4}\right) + n!$

Compare it with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_4 16 = 2$

$f(n) > n^k \Rightarrow n! > n^2$

so, $TC = O(n!)$

Q11 $T(n) = 4T(\frac{n}{2}) + \log n$

Comparing with $T(n) = aT(\frac{n}{b}) + f(n)$

$K = \log_b a = \log_2 4 = 2$

As $f(n) < n^k \Rightarrow \log n < n^2$

~~TC = O~~

So, $TC = \Theta(n^2)$

Q12 $T(n) = \sqrt{n}\, T(\frac{n}{2}) + \log n$

As comparing with $T(n) = aT(\frac{n}{b}) + f(n)$

$a = \sqrt{n}$ So, master method Not applicable

Q13 $T(n) = 3T(\frac{n}{2}) + n$

Comparing with $T(n) = aT(\frac{n}{b}) + f(n)$

$K = \log_b a = \log_2 3 \simeq 1.6$

As $f(n) < n^k \Rightarrow n < n^{1.6}$

So, $TC = \Theta(n^{8/5})$

Q14 $T(n) = 3T(n/3) + \sqrt{n}$

Comparing with $T(n) = aT(\frac{n}{b}) + f(n)$

$K = \log_b a = \log_3 3 = 1$

As $f(n) < n^k \Rightarrow \sqrt{n} < n^1$

So, $TC = \Theta(n)$

Q15 $T(n) = 4T(\frac{n}{2}) + cn$

Comparing with $T(n) = aT(\frac{n}{b}) + f(n)$

$k = \log_2 4 = 2$

As $f(n) \leq n$ So, TC will depend on c

If $c \leq n \Rightarrow TC = \Theta(n^2)$

If $c > n \Rightarrow TC =$

Q16. $T(n) = 3T(\frac{n}{4}) + n\log n$

Comparing with $T(n) = aT(\frac{n}{b}) + f(n)$

$k = \log_4 3 = 1.6$

$f(n) > n^k$ & $n\log n > n^{1.6}$

$\therefore TC = O(n\log n)$

Q17 $T(n) = 3T(\frac{n}{3}) + n/2$

Comparing with $T(n) = aT(\frac{n}{b}) + f(n)$

$k = \log_3 3 = 1$

$f(n) < n^k$ & $\frac{n}{2} < n$

$\therefore TC = O(n)$

Q18 $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

Comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_3 6 \cong 1.6$

$f(n) \Rightarrow n^{1.6} \Rightarrow n^2 \log n > n^{1.6}$

$TC = O(n^2 \log n)$

Q19 $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$k = \log_2 4 = 2$

$f(n) < n^k \Rightarrow \frac{n}{\log n} < n^2$

$\therefore TC = \Theta(n^2)$

Q20 $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

Comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

~~$k = \log_8 64 = 2$~~

$k = \log_8 64 = 2$

$T(n) = 64T\left(\frac{n}{8}\right) + (-1)\, n^2 \log n$

$\Rightarrow 64T\left(\frac{n}{8}\right) + n^2 \log \frac{1}{n}$

$TC = \Theta\left(n^2 \log \frac{1}{n}\right)$

Q21 $T(n) = \cancel{78}\ 7T\left(\frac{n}{3}\right) + n^2$

Comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$K = \log_3 7 \simeq 1.7$

$\therefore f(n) > n^k$ as $n^2 > n^{1.7}$

$\therefore TC = O(n^2)$

Q22 $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

Comparing with $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$K = \log_2 1 = 0$

As, $\cos(n)$ is bounded above by 1 and below by -1 so, $n(2 - \cos n)$ is basically $O(n)$

As, $f(n) > n^0$

So, $TC = O(n)$