Proof By Case Examples: When writing proof by classes you should include:

1 Begin with a clear written statement of the given facts or assumptions.

2 Provide a clear written statement of what is to be proven.

3 Consider all possible classes which must be proven in order to prove the mathematical statement.

4 Write the body of the proof. For each case, this must include a sequence of logical steps leading to the desired result. Provide clear reasoning for each step. finish your proof with a clear statement of that which was to be proven. At this stage you should avoid shortcuts.

Example 1. Prove that if x is a real number, then $|x-3|+x \geq 3$. In order to apply general algebraic techniques when we must find a way of getting ride of the absolute value. We can do this if we restrict our selves to two possibilities; $x-3 \geq 0$ and $x-3 \leq 0$. In doing so, we cover the entire universe.

Proof. Assume x is a real number. Show that $|x-3|+x \geq 3$. We will consider the following cases.
Case 1: Assume $x \geq 3$. Then $|x-3| = x-3$, so that $|x-3|+x = x-3+x = 2x-3 \geq 2(3)-3 = 3$. Thus $|x-3|+x \geq 3$.
Case 2: Assume $x \leq 3$. Then $|x-3| = 3-x$, so that $|x-3|+x = 3-x+x = 3 \geq 3$. Thus $|x-3|+x \geq 3$.
Therefore, for all possible cases, it has been proven that $|x-3|+x \geq 3$.

□

Example 2. Prove that if m is an integer, then $2m^2-$ is odd.

Proof. Assume that m is an integer. Show that $2m^2-1$ is odd, i.e. that $2m^2-1 = 2k+1$ for some integer k. We will consider the following cases:
Case 1: Assume that m is even, i.e., $m = 2n$ for some integer n. Then $2m^2-1 = 2(2n)^2-1 = 8n^2-1-1+1 = 8n^2-2+1 = 2(4n^2-1)+1 = 2k+1$, where $k = 4n^2-1$ is an integer. Thus, $2m^2-1$ is odd.
Case 2: Assume that m is even, i.e., $m = 2n+1$ for some integer n. Then $2m^2-1 = 2(2n+1)^2-1 = 2(4n^2+4n+1)-1 = 8n^2+8n+2-1 = 8n^2+8n+1 = 2(4n^2+4n)+1 = 2k+1$, where $k = 4n^2+4n$ is an integer. Thus, $2m^2-1$ is odd.
Therefore, for all possible cases, it has been proven that $2m^2-1$ is odd. □