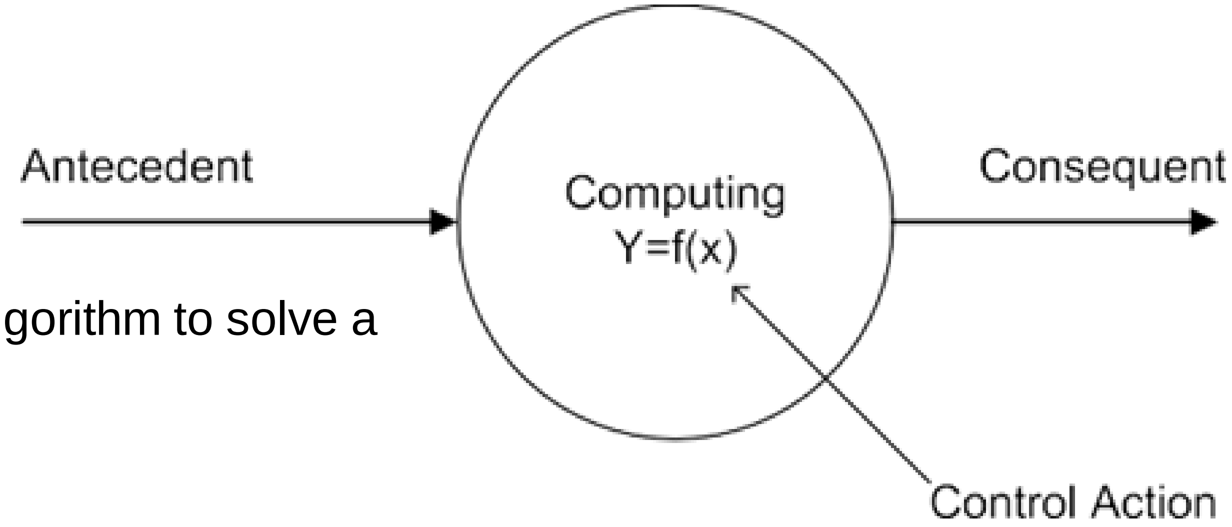
Introduction to Fuzzy Systems

CO542 - Neural Networks and Fuzzy Systems

Concept of Computing



y = *f* (*x*), f is a mapping function
f is also called a formal method or an algorithm to solve a problem.

In 1996, LA Zadeh (LAZ) introduced the term **hard computing**.

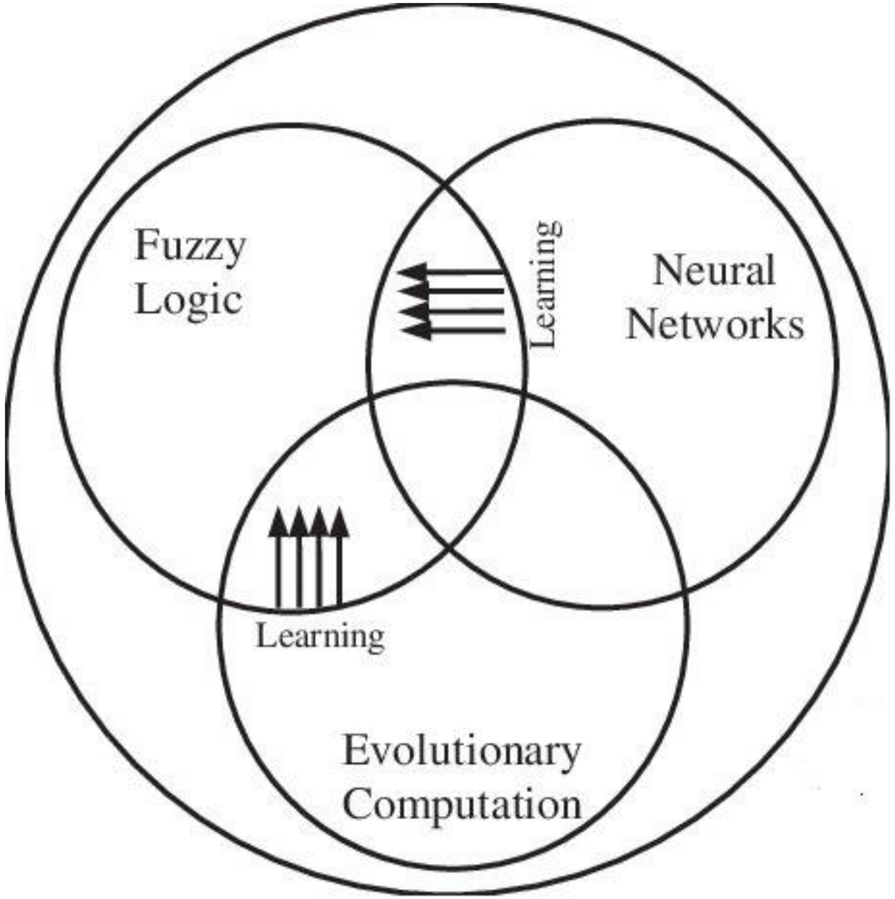
- Should provide precise solution.
- Control action should be unambiguous and accurate.
- Suitable for problem, which is easy to model mathematically.

Example:

- Solving numerical problems (e.g. Roots of polynomials, Integration etc.)
- Searching and sorting techniques

Soft Computing

- Neural networks and Fuzzy Logic Systems are often considered as a part of Soft Computing area:



- Soft Computing

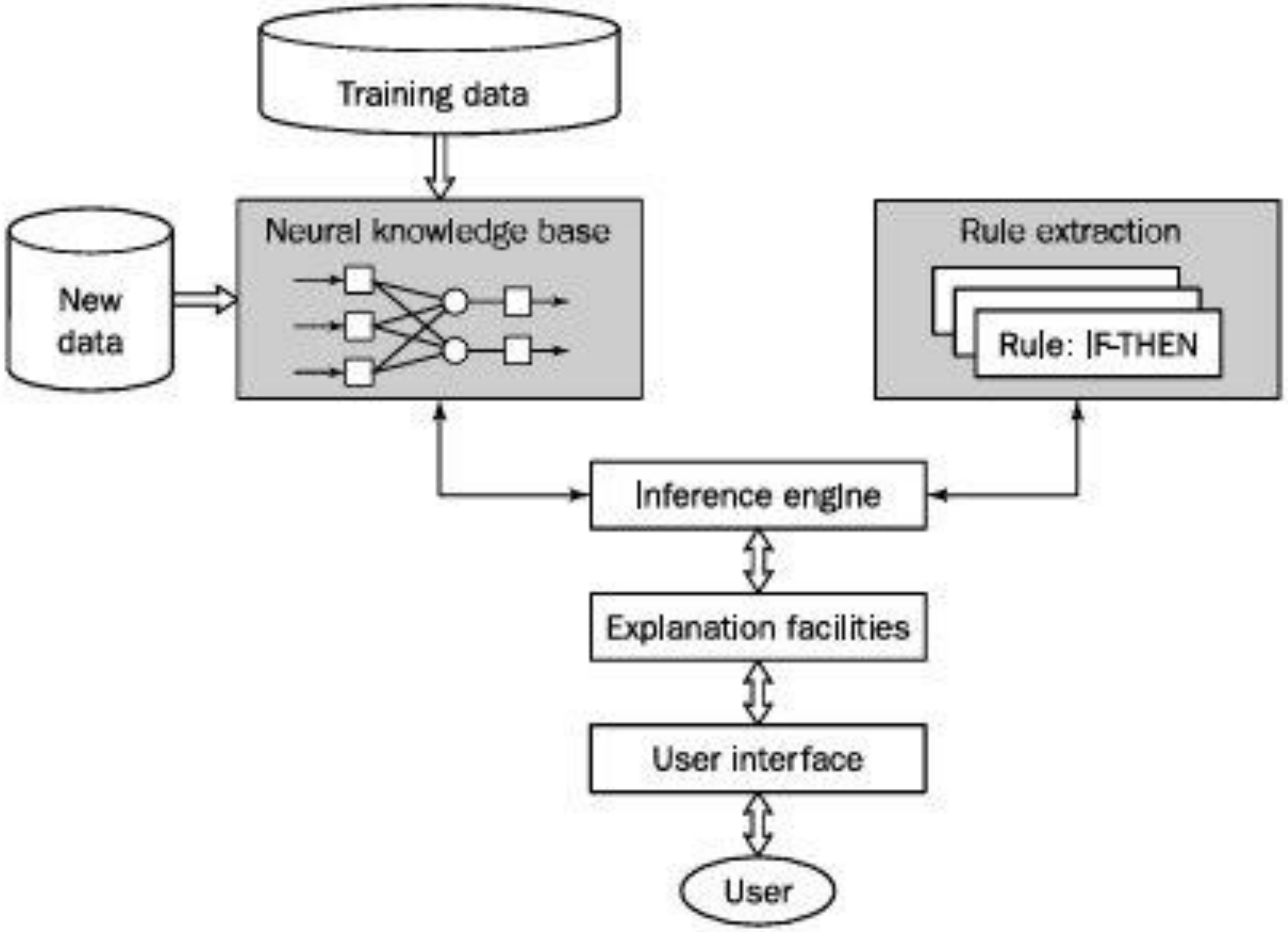
"Soft computing is the use of approximate calculations to provide imprecise but usable solutions to complex computational problems."

Goals of Soft Computing

- Zadeh, defined Soft Computing into one multidisciplinary system as the fusion of the fields of **Fuzzy Logic, Neuro-Computing, Evolutionary and Genetic Computing, and Probabilistic Computing.**
- The main goal of Soft Computing is to develop intelligent machines to provide solutions to real world problems, which are not modeled, or too difficult to model mathematically.
- Its aim is to exploit the tolerance for Approximation, Uncertainty, Imprecision, and Partial Truth in order to achieve close resemblance with human like decision making.
  - Approximation :here the model features are similar to the real ones, but not the same.
  - Uncertainty :here we are not sure that the features of the model are the same as that of the entity (belief).
  - Imprecision :here the model features (quantities) are not the same as that of the real ones, but close to them.

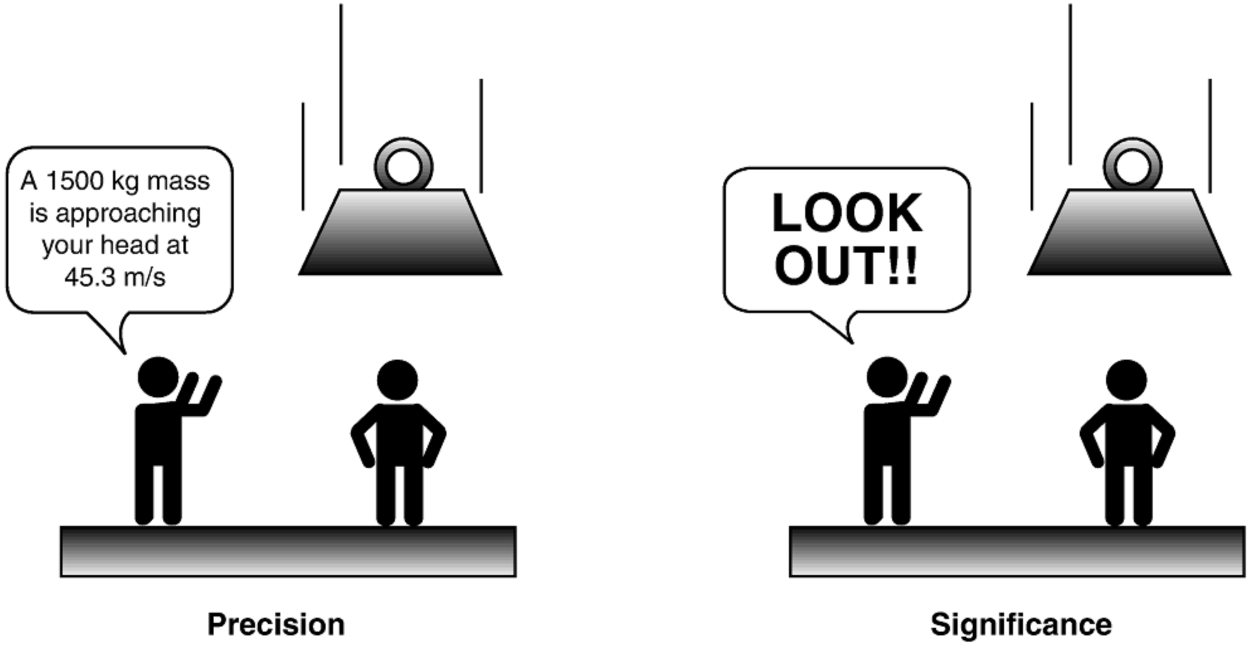
Neuro-Fuzzy systems

- We may say that neural networks and fuzzy systems try to emulate the operation of human brain.
- Neural networks concentrate on the structure of human brain, i.e., on the “hardware” emulating the basic functions, whereas fuzzy logic systems concentrate on “software”, emulating fuzzy and symbolic reasoning.



Foundations of Fuzzy Logic

- Processing of vague and imprecise data is our everyday experience
- Consider the problem of Precision and Significance in the real world:

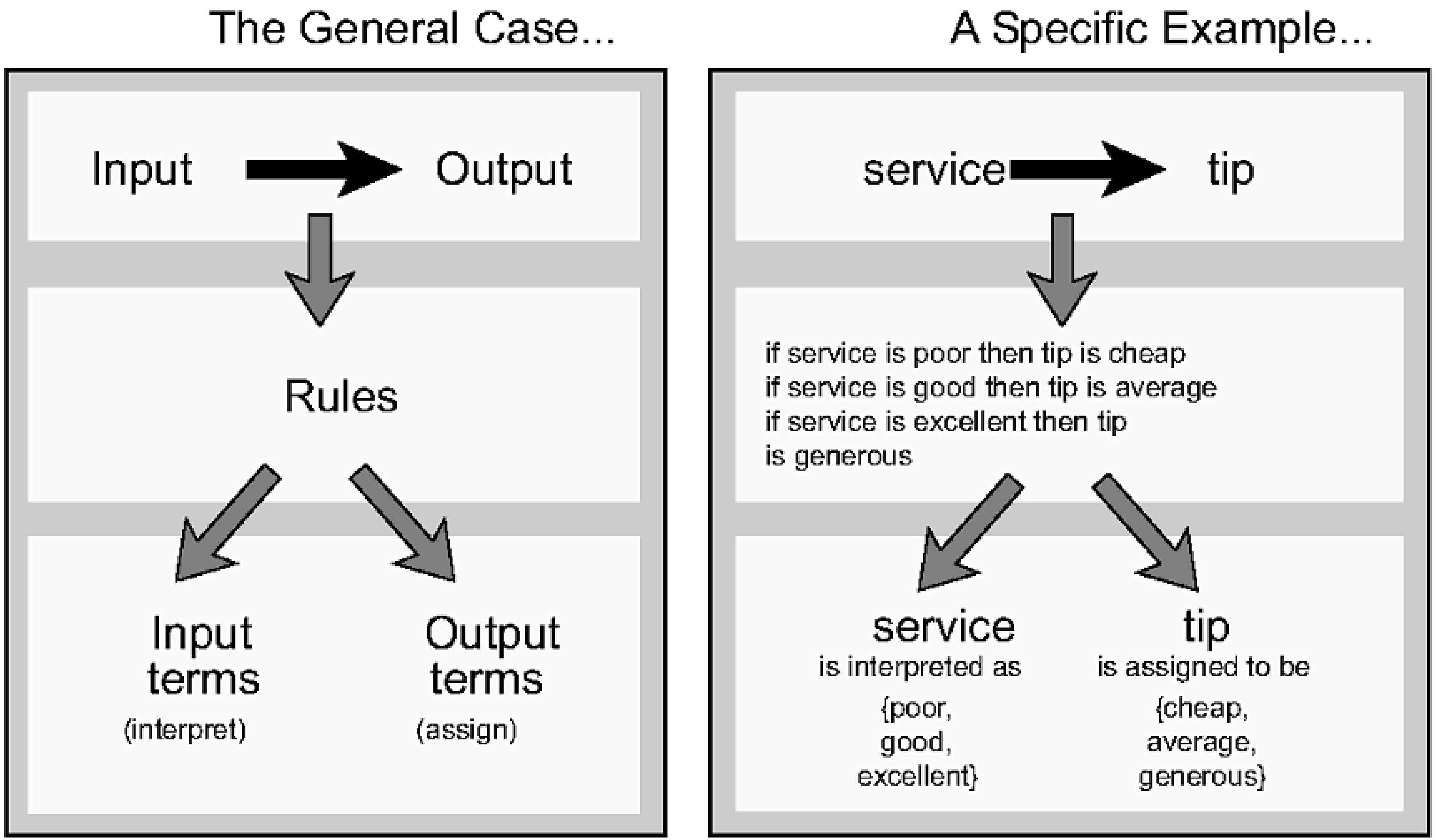


- Fuzzy logic is a convenient way to map an input space to an output space.

Fuzzy logic

- Fuzzy logic is about mapping an input space to an output space, and the primary mechanism for doing this is a list of if-then statements called rules.
- All rules are evaluated in parallel, and the order of the rules is unimportant
- The rules refer to variables and the adjectives that describe those variables.

Fuzzy logic



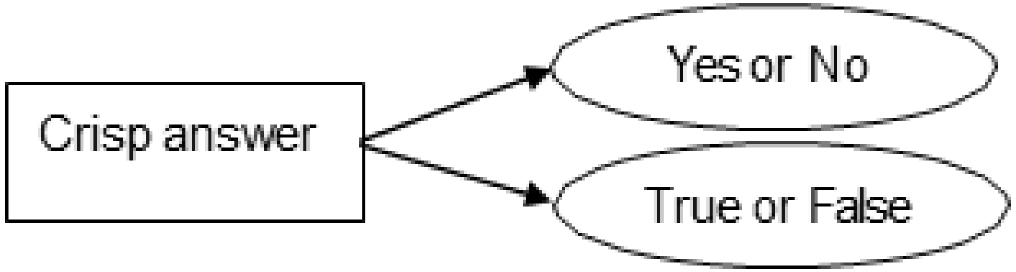
Fuzzy inference is a method that interprets the values in the input vector and,

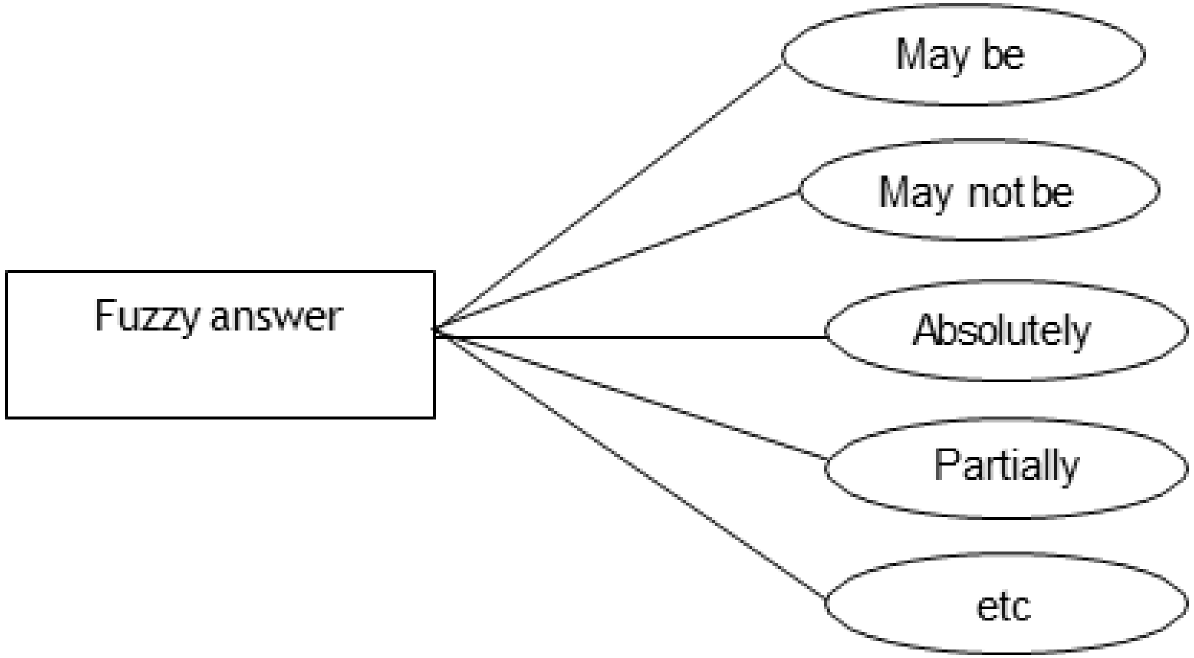
based on some set of rules,

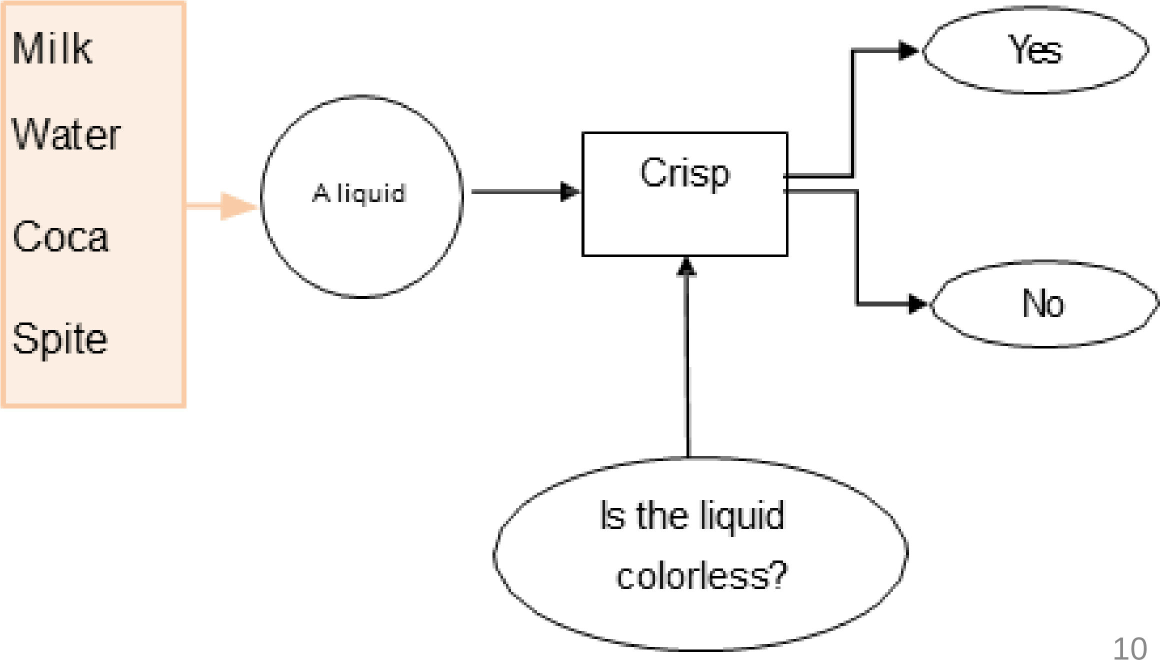
assigns values to the output vector.

Fuzzy logic vs. Crisp logic

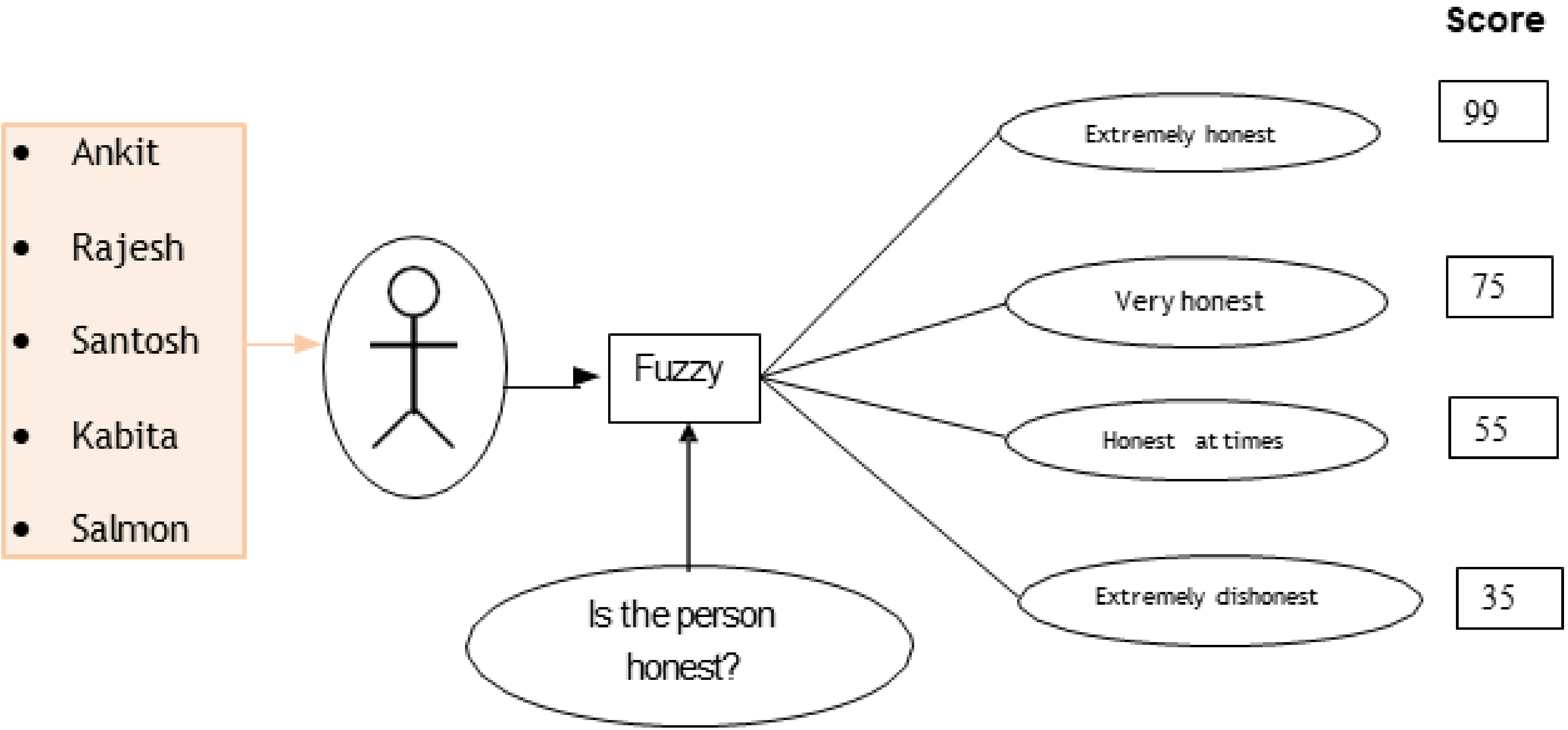
- Dictionary meaning of **fuzzy** is not clear, noisy etc. Example: Is the picture on this slide is fuzzy?
- Antonym of fuzzy is **crisp**
- Example: Are the chips crisp?





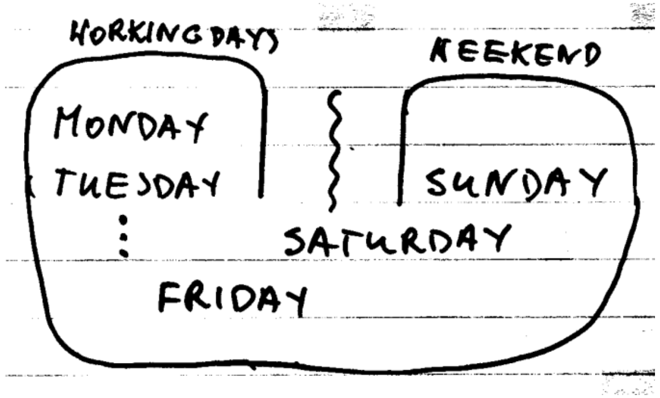


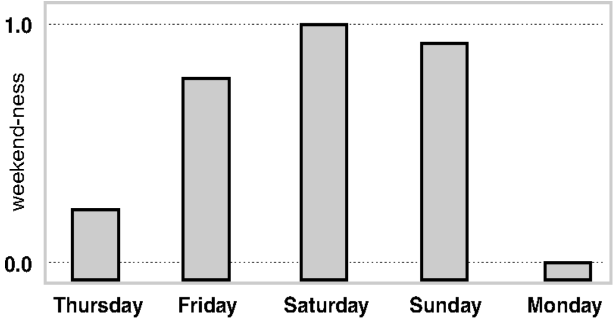
Fuzzy logic : Example



Fuzzy sets

- A fuzzy set, as opposed to the “classical set” does not have crisp, clearly defined boundary.
- In fuzzy logic, the truth of any statement becomes a matter of degree.



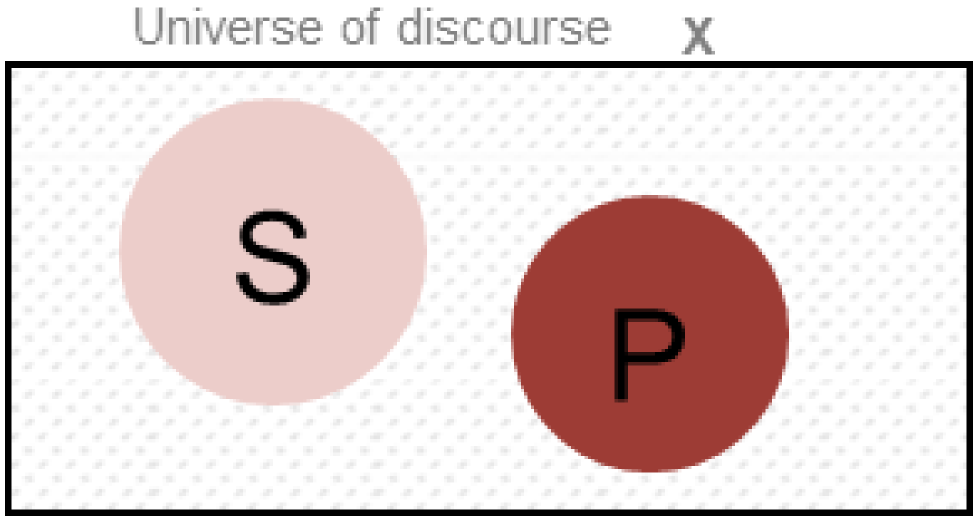


Days of the weekend multivalued membership

- Reasoning in fuzzy logic we give a degree of true to each statement, for example
- a degree of weekend-ness for every day of the week
- The plot shows how much a particular day can be classified as a weekend
- The function that defines the weekend-ness of any instant in time maps the input space (time of the week) to the output space (weekend-ness).
- It is known as a **membership function**.

Concept of fuzzy set

- To understand the concept of fuzzy set it is better, if we first clear our idea of crisp set.
- X = The entire university system of SL.
- H = All State Univ.= { s_1, s_2, s_3, ... , s_L }
- M = All Non State Univ.= { p_1, p_2, p_3, ... , p_N }



Here, All are the sets of finite numbers of individuals. Such a set is called crisp set.

Example of fuzzy set

- Let us discuss about fuzzy set.
- X = All students in CO542.
- S = All **Good students**.
- S = { (*s*, g) | s $\in$ X } and g(s) is a measurement of goodness of the student *s*.
- **Example:**
- S = { (Andy, 0.8), (Bob, 0.7), (Clara, 0.1), (Dylan, 0.9) }

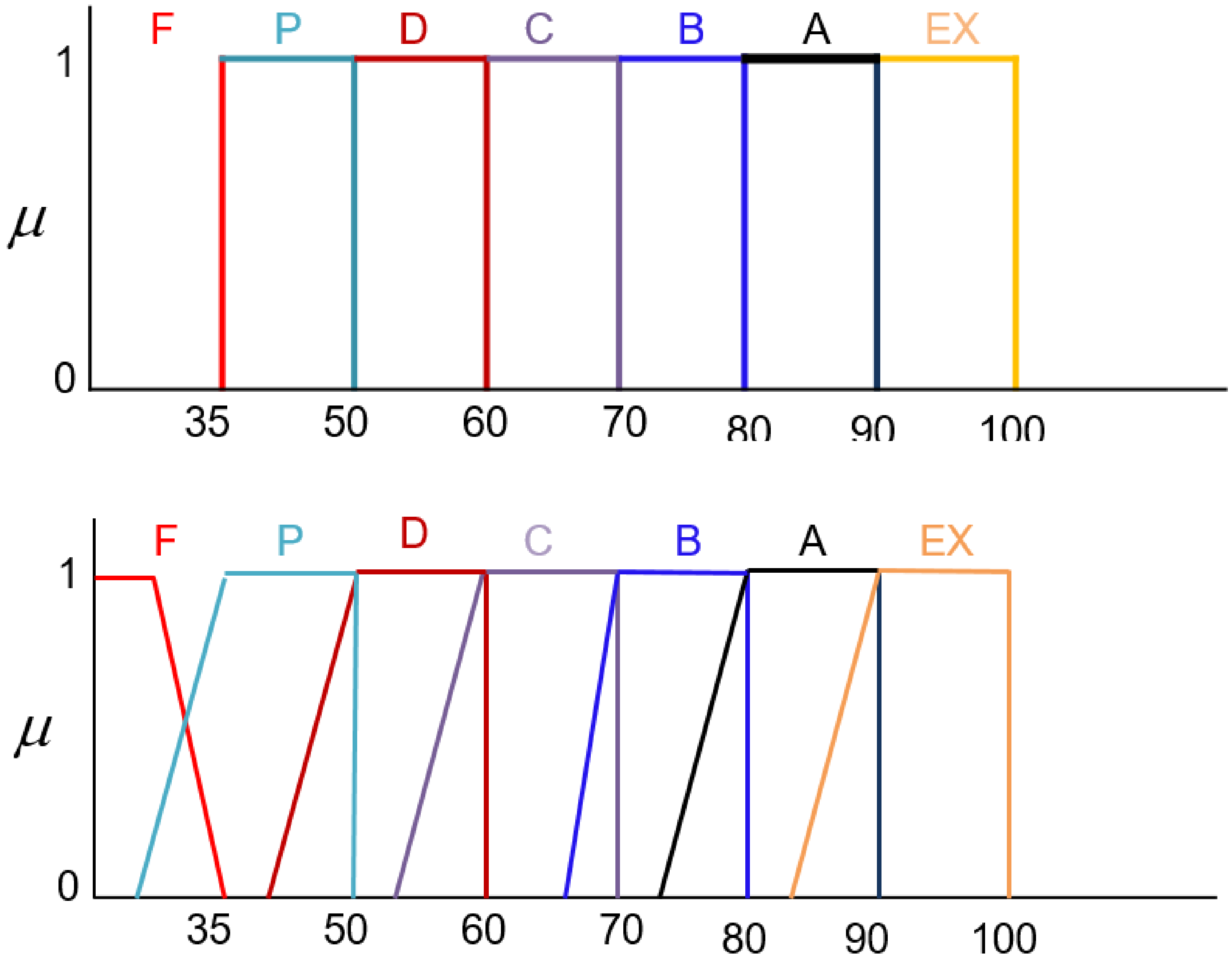
Fuzzy set vs. Crisp set

Crisp Set	Fuzzy Set
1. S = {s | s $\in$ X }	1. F = (s, μ) | s $\in$ X and μ(s) is the degree of s.
2. It is a collection of el-ements.	2. It is collection of or-dered pairs.
3. Inclusion of an el-ement s $\in$ X into S is crisp, that is, has strict boundary **yes** or **no**.	3. Inclusion of an el-ement s $\in$ X into F is fuzzy, that is, if present, then with a degree of **membership**.

Note: A crisp set is a fuzzy set, but, a fuzzy set is not necessarily a crisp set.

Example: Course evaluation in a crisp way vs fuzzy way

1. EX = Marks ≥90
2. A = 80 ≤ Marks < 90
3. B = 70 ≤ Marks < 80
4. C = 60 ≤ Marks < 70
5. D = 50 ≤ Marks < 60
6. P = 35 ≤ Marks < 50
7. F = Marks < 35



Membership Functions

- A characteristic function: the values assigned to the elements of the universal set fall within a specified range and indicate the membership grade of these elements in the set.
- Larger values denote higher degrees of set membership.
- A set defined by membership functions is a fuzzy set.
- The most commonly used range of values of membership functions is the unit interval [0,1].
- We think the universal set X is always a crisp set.
- Notation:
  - The membership function of a fuzzy set A is denoted by :

$$\mu_A : X \rightarrow [0,1]$$

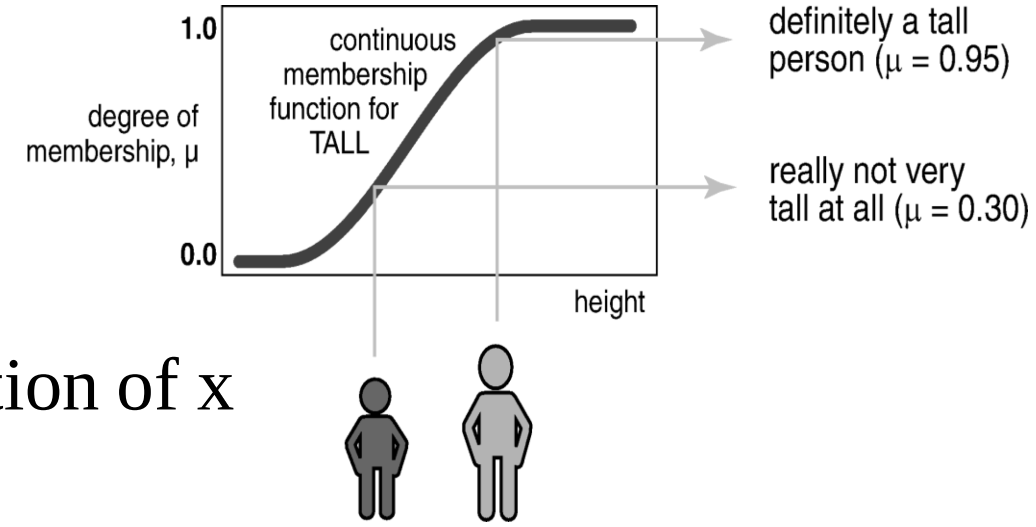
  - In the other one, the function is denoted by A and has the same form

$$A : X \rightarrow [0,1]$$

Membership Functions and Fuzzy Set

- A classical set is defined by a crisp membership for example $A = \{x|x > 6\}$
- A fuzzy set is an extension of a classical set where the membership function describes a degree of belonging.
- If X is the **universe of discourse** and its elements are denoted by x, then a fuzzy set A in X is defined as a set of ordered pairs.

$$\boxed{A = \{x, \mu_A(x)|x \in X\}}$$



- $\mu_A(x)$ is called the membership function of x in A.
- The membership function maps each element of X to a membership value between 0 and 1.

Exercise:

- Develop a reasonable membership function for the following fuzzy sets based on moving vehicles on a freeway for the speed range of 0–75 mph.
    - a. Fast
    - b. Moderate
    - c. Slow

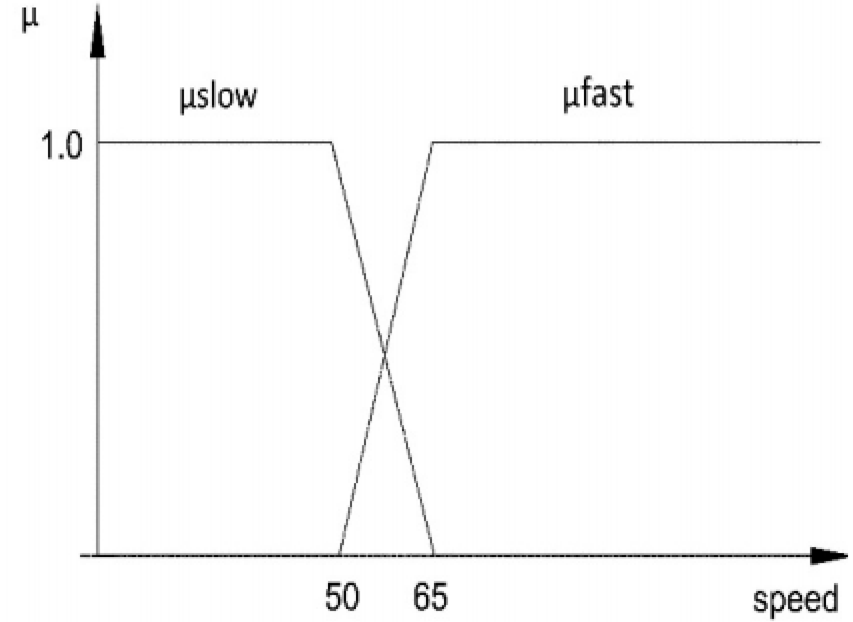
Let us take 0-50 slow

50-65 moderate

65-75 fast

$\mu_A = \{1/0 + 1/50 + 0/65 + 0/75\}$

$\mu_B = \{0/0 + 0/50 + 1/65 + 1/75\}$



Types of Fuzzy Set

$$A = \left\{ (x, \mu_A(x)) \middle| x \in U \right\}$$

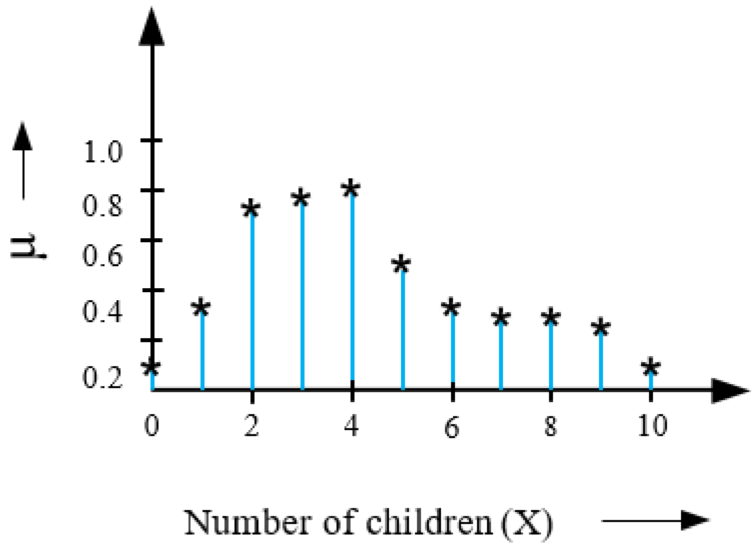
U : discrete universe ⇒ $$A = \sum_{x_i \in U} \mu_A(x_i) / x_i$$

U : continuous universe ⇒ $$A = \int_U \mu_A(x) / x$$

Note that $\sum$ and integral signs stand for the union of membership grades; " / " stands for a marker and does not imply division.

Example : Discrete

- Either elements or their membership values (or both) also may be of discrete values



A = "Happy family"

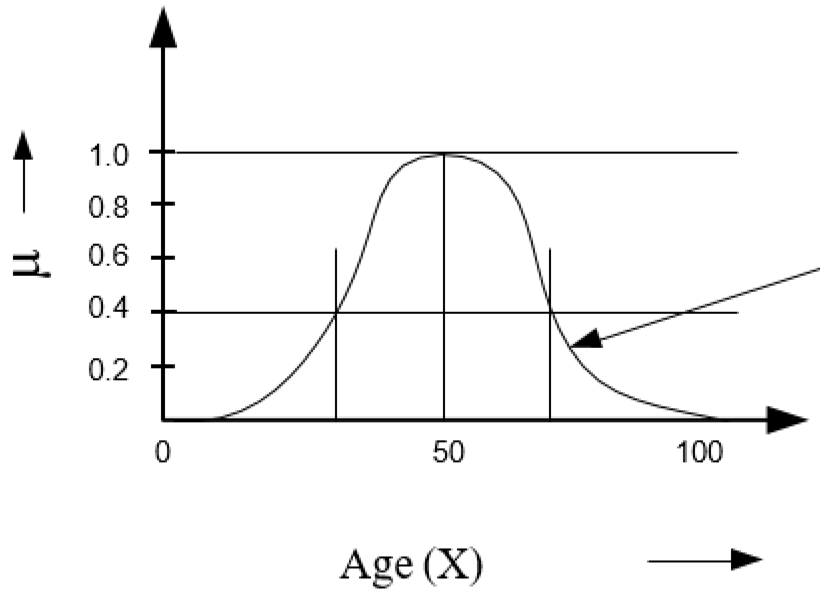
A ={(0,0.1),(1,0.30),(2,0.78)……(10,0.1)}

Note : X = discrete value

How you measure happiness ??

$A = 0.1/1 + 0.3/2 + 0.8/3 + 1.0/4 + 0.9/5 + 0.5/6 + 0.2/7 + 0.1/8$

Example : Continuous



B = "Middle aged"

$$\mu_B(x) = \frac{1}{1+\left(\frac{x-50}{5}\right)^4}$$

$$B = \left\{(x, \mu_B(x)) \middle| x \in U\right\}$$

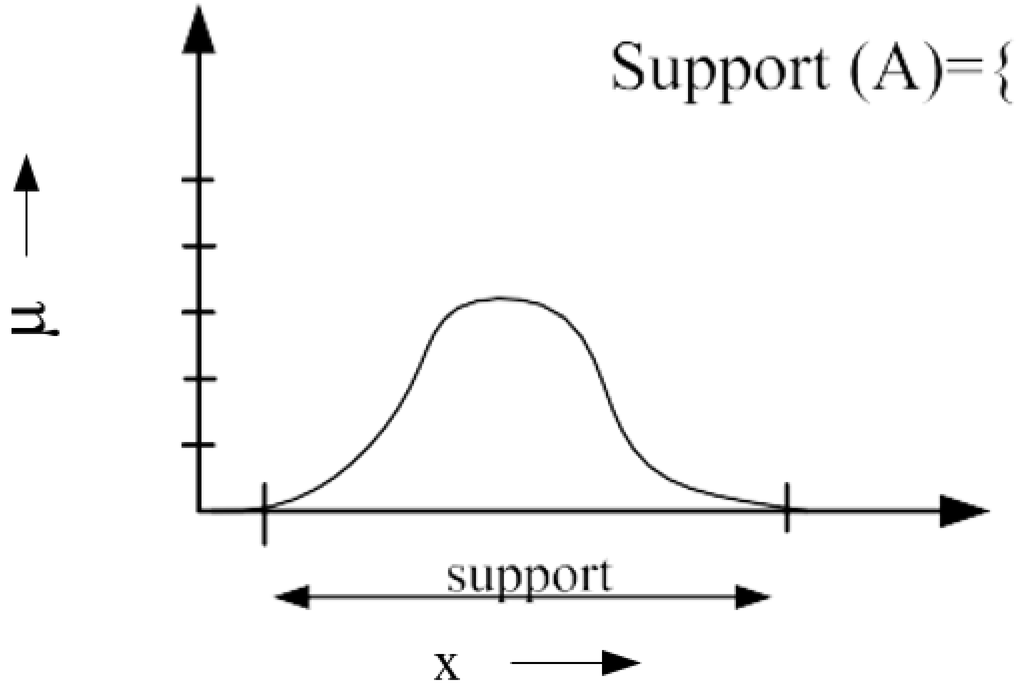
Note : x = real value
= R^+

Alternative Representation:

$$B = \int_{R+} \frac{1}{1+\left(\frac{x-50}{5}\right)^4} \Big/ x$$

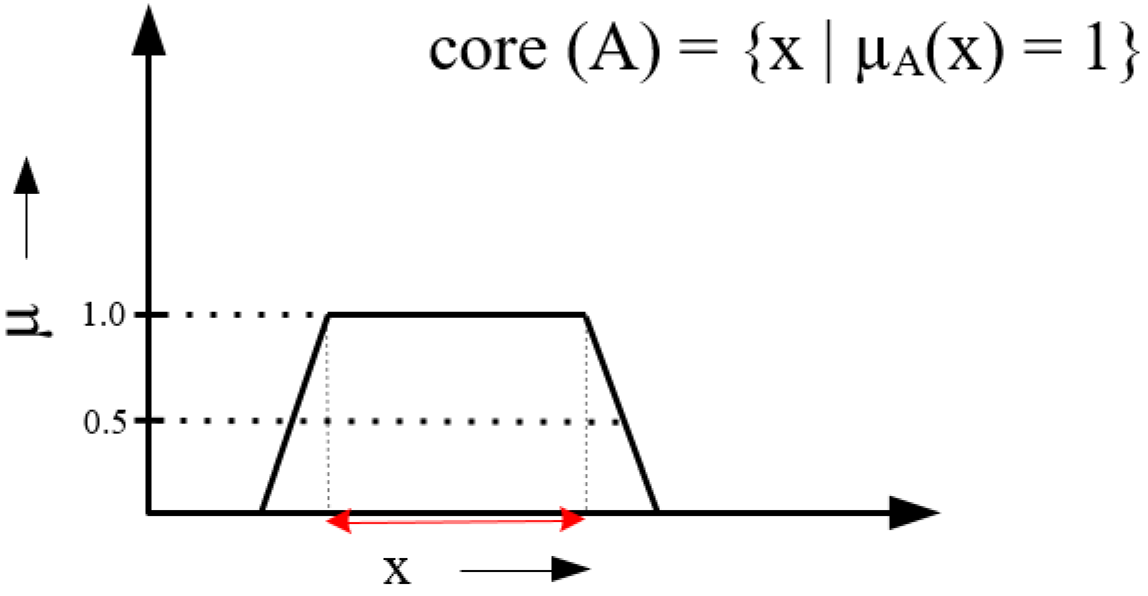
Fuzzy terminologies: Support

- Support: The support of a fuzzy set A is the set of all points x ∈ X such that μA(x) > 0



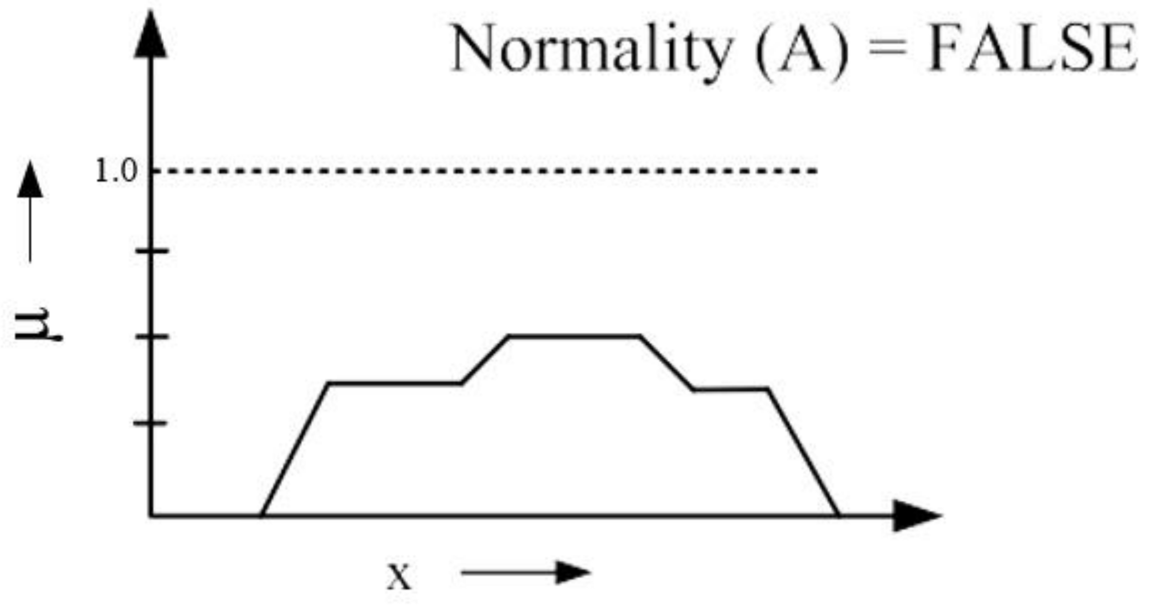
Fuzzy terminologies: Core

- Core: The core of a fuzzy set A is the set of all points x in X such that μA(x) = 1



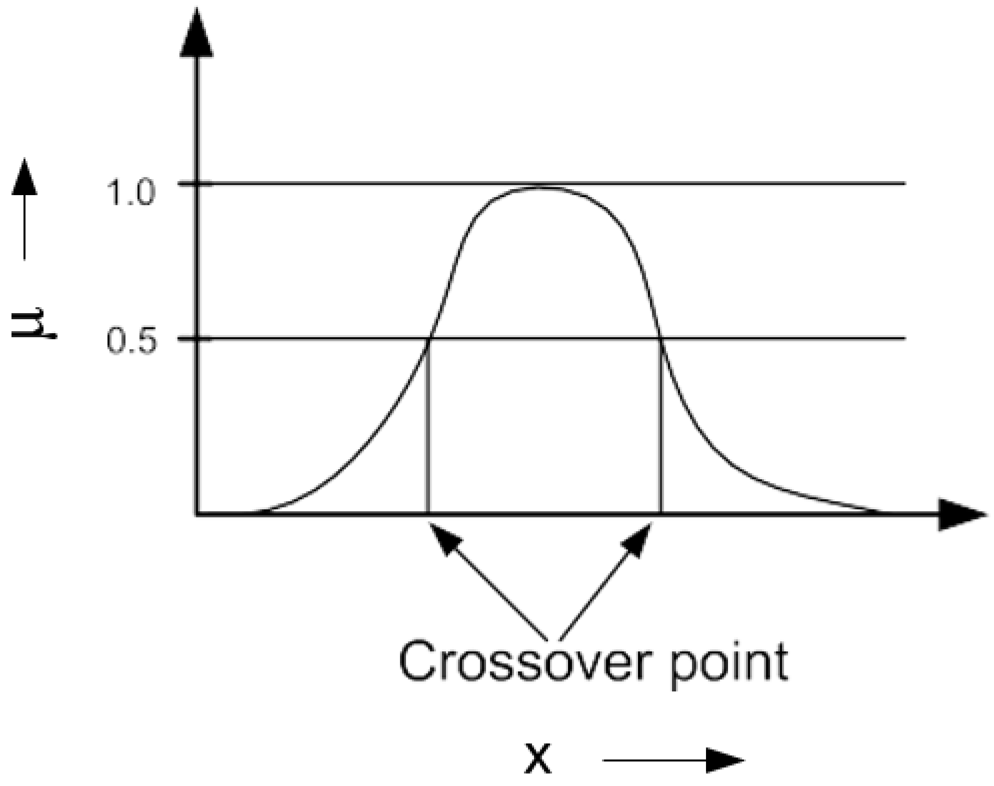
Fuzzy terminologies: Normality

- A fuzzy set A is a normal if its core is non-empty. In other words, we can always find a point x ∈ X such that μA(x) = 1.



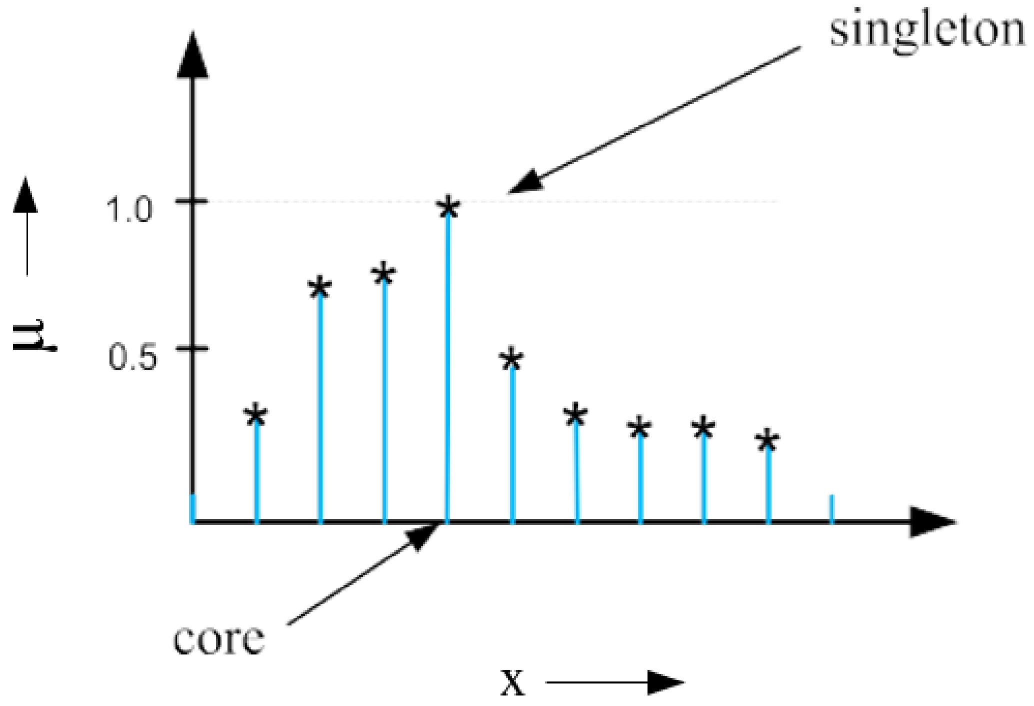
Fuzzy terminologies: Crossover points

- A crossover point of a fuzzy set A is a point $x \in X$ at which $\mu_A(x) = 0.5$. That is Crossover (A) = $\{x | \mu_A(x) = 0.5\}$.



Fuzzy terminologies: Fuzzy Singleton

- A fuzzy set whose support is a single point in X with $\mu_A(x) = 1$ is called a fuzzy singleton.
- That is $|A| = |\{ x \mid \mu A(x) = 1\}| = 1$. Following fuzzy set is not a fuzzy singleton.



Fuzzy terminologies: α-cut and strong α-cut

- α-cut and strong α-cut :
  - The α-cut of a fuzzy set A is a crisp set defined by
  - $A_\alpha = \{x \mid \mu_A(x) \geq \alpha \}$
- Strong α-cut is defined similarly :
  - $A_{\alpha'} = \{x \mid \mu_A(x) > \alpha \}$

- Note : Support(A) = A_0’ and Core(A) = A_1.

Fuzzy terminologies: Convexity

- A fuzzy set A is convex if and only if for any x_1 and $x_2 \in X$ and any $\lambda \in [0, 1]$

$$\mu_A (\lambda x_1 + (1 - \lambda) x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

- Note :
  - A is convex if all its α- level sets are convex.
  - Convexity (A_α) =⇒ A_α is composed of a single line segment only

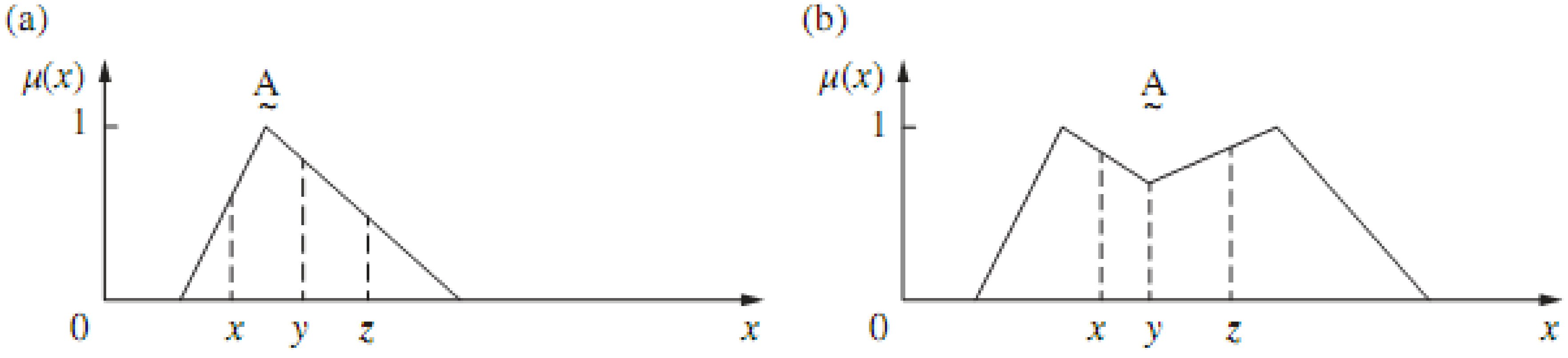


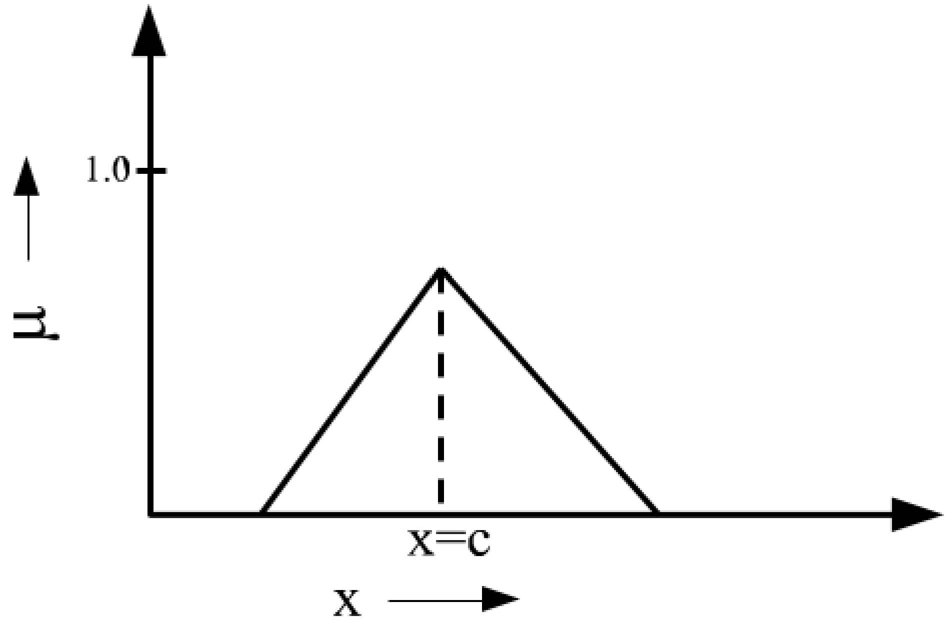
Figure 4.3 Convex, normal fuzzy set (a) and nonconvex, normal fuzzy set (b).

Fuzzy terminologies: Bandwidth

- For a normal and convex fuzzy set, the bandwidth (or width) is defined as the distance the two unique crossover points:
- Bandwidth(A) = | x1 - x2 |
- where μA(x1) = μA(x2) = 0.5

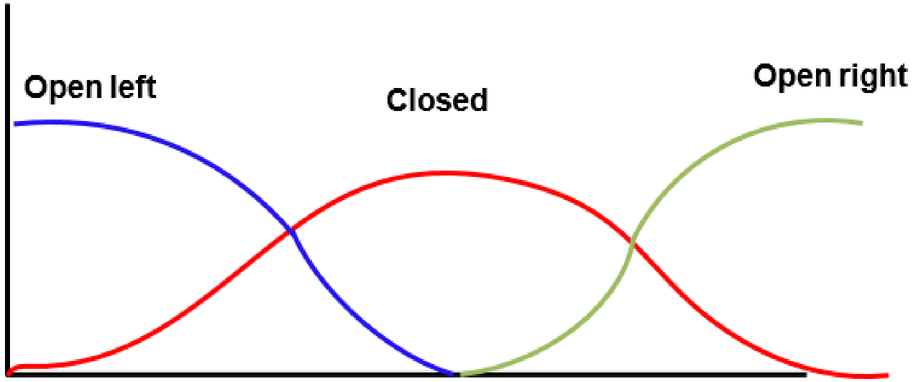
Fuzzy terminologies: Symmetry

- A fuzzy set A is symmetric if its membership function around a certain point x = c, namely $\mu_A(x + c) = \mu_A(x - c)$ for all $x \in X$.



Fuzzy terminologies: Open and Closed

- A fuzzy set A is
- Open left
  - If $\lim_{x \to -\infty} \mu_A(x) = 1$ and $\lim_{x \to -\infty} \mu_A(x) = 0$
- Open right:
  - If $\lim_{x \to -\infty} \mu_A(x) = 0$ and $\lim_{x \to -\infty} \mu_A(x) = 1$
- Closed
  - If : $\lim_{x \to -\infty} \mu_A(x) = \lim_{x \to -\infty} \mu_A(x) = 0$



Fuzzy vs. Probability

- Fuzzy : When we say about certainty of a thing
  - Example: A patient come to the doctor and he has to diagnose so that medicine can be prescribed.
  - Doctor prescribed a medicine with certainty 60% that the patient is suffering from flue. So, the disease will be cured with certainty of 60% and uncertainty 40%. Here, instead of flue, other diseases with some other certainties may be.
- Probability: When we say about the chance of an event to occur
  - Example: SL will win the T20 tournament with a chance 60% means that out of 100 matches, India own 60 matches.

Prediction vs. Forecasting

- The Fuzzy vs. Probability is analogical to Prediction vs. Forecasting
- Prediction : When you start guessing about things.
- Forecasting : When you take the information from the past job and apply it to new job.
- The main difference:
  - Prediction is based on the best guess from experiences.
  - Forecasting is based on data you have actually recorded and packed from previous job.

Operations on Fuzzy Sets

- A fuzzy set operations are the operations on fuzzy sets.
- The fuzzy set operations are generalization of crisp set operations.
    - Union
    - Intersection
    - Complement

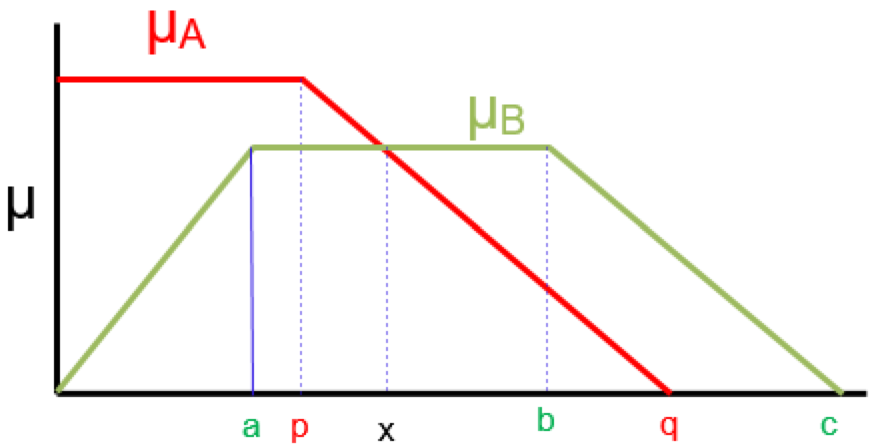
Basic fuzzy set operations: Union

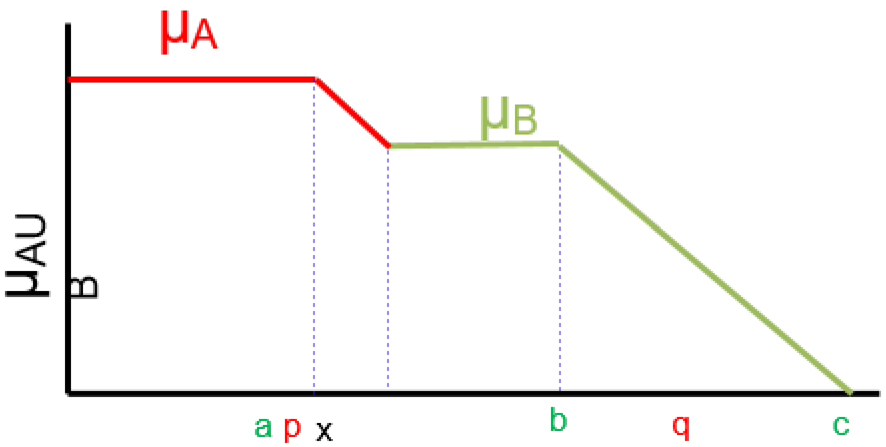
- Union (A ∪ B):

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$

- Example:
- A = {(x1, 0.5), (x2, 0.1), (x3, 0.4)} and
- B = {(x1, 0.2), (x2, 0.3), (x3, 0.5)};
- C = A ∪ B = {(x1, 0.5), (x2, 0.3), (x3, 0.5)}

A OR B (B →, A ↓)	0	0.25	0.5	0.75	1.0
0	0	0.25	0.5	0.75	1.0
0.25	0.25	0.25	0.5	0.75	1.0
0.5	0.5	0.5	0.5	0.75	1.0
0.75	0.75	0.75	0.75	0.75	1.0
1	1.0	1.0	1.0	1.0	1.0





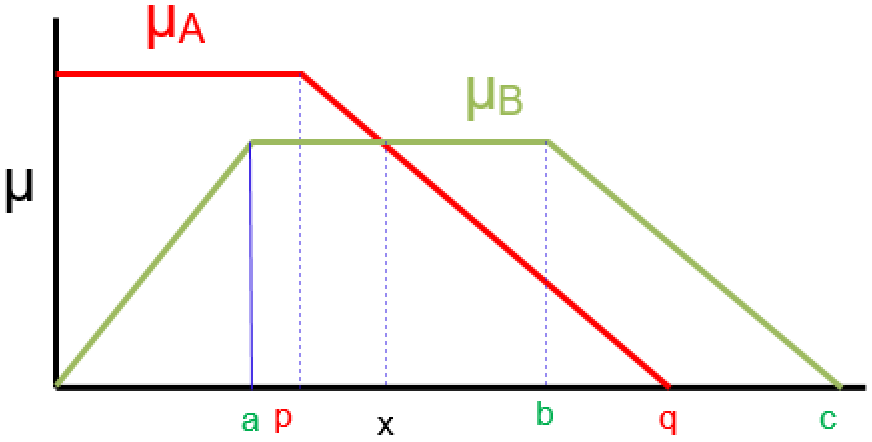
Basic fuzzy set operations: Intersection

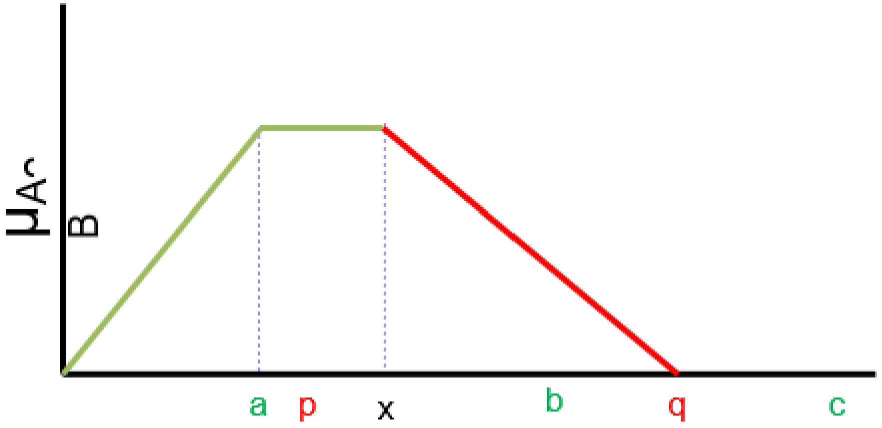
- Intersection (A ∩ B):

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$$

- Example:
- A = {(x1, 0.5), (x2, 0.1), (x3, 0.4)} and
- B = {(x1, 0.2), (x2, 0.3), (x3, 0.5)};
- C = A ∩ B = {(x1, 0.2), (x2, 0.1), (x3, 0.4)}

A AND B (B → / A ↓)	0	0.25	0.5	0.75	1.0
0	0	0	0	0	0
0.25	0	0.25	0.25	0.25	0.25
0.5	0	0.25	0.5	0.5	0.5
0.75	0	0.25	0.5	0.75	0.75
1	0	0.25	0.5	0.75	1





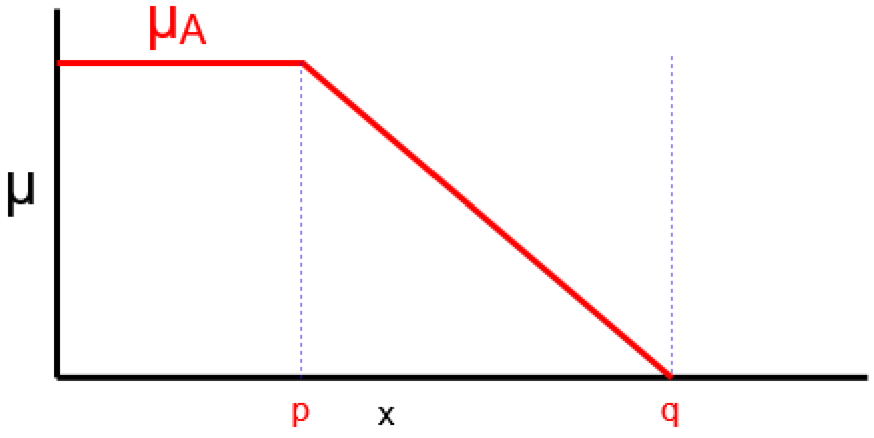
Basic fuzzy set operations: Complement

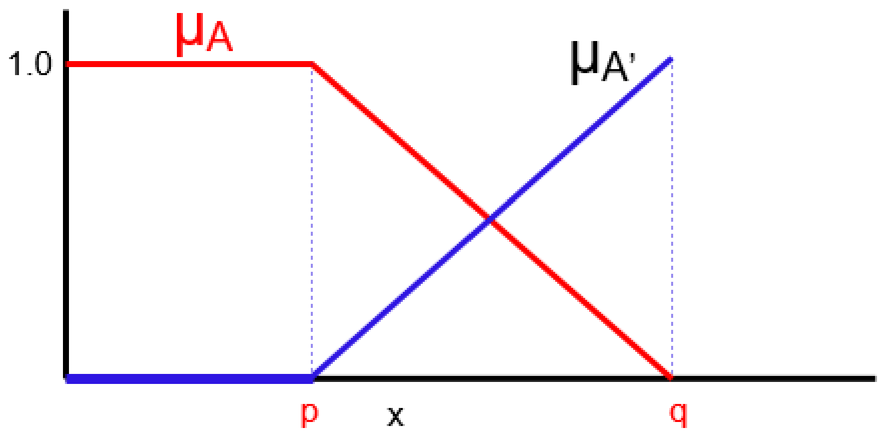
- Complement (A^C):

$$\mu_A{}^{AC}(x) = 1-\mu_A(x)$$

- Example:
- A = {(x1, 0.5), (x2, 0.1), (x3, 0.4)}
- C = AC = {(x1, 0.5), (x2, 0.9), (x3, 0.6)}

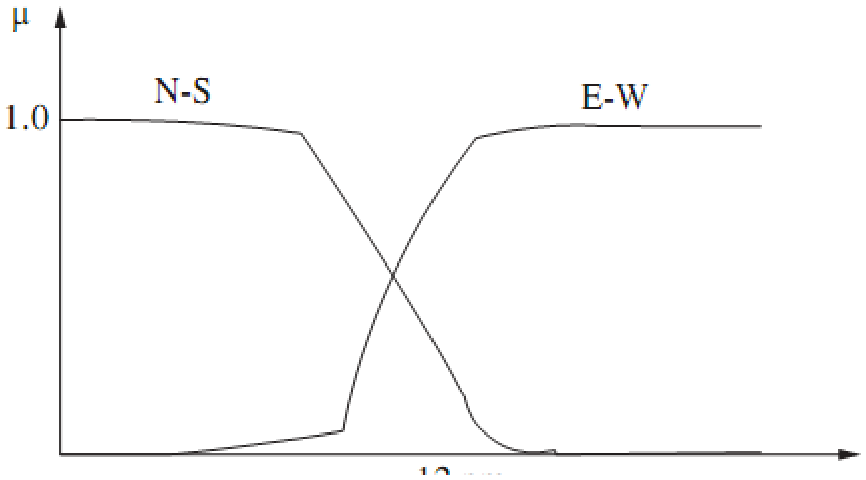
A	NOT A
0	1
0.25	0.75
0.5	0.5
0.75	0.25
1	0





Exercise:

- The direction of the wind was in a North–South direction from morning until noon. Then it changed direction to East–West. The time noon is considered a transition period for changing the wind direction from North–South to East–West as seen in Figure. Find the intersection, union, and difference for the two directions of wind flow.



NS = $\{\frac{1}{morning} + \frac{0.5}{midnoon} + \frac{0}{evening}\}$

EW = $\{\frac{0}{morning} + \frac{0.5}{midnoon} + \frac{1}{evening}\}$

NS U EW = $\{\frac{1}{morning} + \frac{0.5}{midnoon} + \frac{1}{evening}\}$

NS ∩ EW = $\{\frac{0}{morning} + \frac{0.5}{midnoon} + \frac{0}{evening}\}$

NS|EW = NS ∩ $\overline{EW}$ =

$\{\frac{1}{morning} + \frac{0.5}{midnoon} + \frac{0}{evening}\}$

EW|NS = EW ∩ $\overline{NS}$ =

$\{\frac{0}{morning} + \frac{0.5}{midnoon} + \frac{1}{evening}\}$

Thank You!

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Comments, Questions, Suggestions