ASSIGNMENT 1 - KSHITIJ MITTAL

Q1

$E(X) = 2$ $\qquad VAR(X) = 9 \qquad \sigma_X = \sqrt{9} = 3$

$E(Y) = 0$ $\qquad VAR(Y) = 4 \qquad \sigma_Y = \sqrt{4} = 2$

$CORR(X,Y) = 0.25$

$$COV(X,Y) = CORR(X,Y) * (\sigma_X)(\sigma_Y)$$
$$= 0.25 * 3 * 2$$
$$= 1.5$$

(a) $Var(X+Y) = Var(X) + Var(Y) + 2\,Cov(X,Y)$
$$= 9 + 4 + 2*1.5$$
$$= 16$$

(b) $Cov(X, X+Y) = Cov(X,X) + Cov(X,Y)$

$= E[(X-E(X))(X-E(X))]$

$= E(X-E(X))^2$

$= Var(X)$

$= 9$

$= 1.5$ (from above)

$\therefore Cov(X, X+Y) = 9 + 1.5$
$$= 10.5$$

(c) $CORR(X+Y, X-Y) = CORR(X,X) - CORR(X,Y) + CORR(X,Y) - CORR(Y,Y)$

$$CORR(X+Y, X-Y) = \frac{COV(X+Y, X-Y)}{\sqrt{Var(X+Y)}\sqrt{Var(X-Y)}}$$

$COV(X+Y, X-Y) = COV(X,X) - COV(\cancel{X,Y}) + COV(\cancel{Y,X}) - COV(Y,Y)$

$$= VAR(X) - VAR(Y)$$
$$= 9-4 \quad = 5$$

$$VAR(X+Y) = VAR(X) + VAR(Y) + 2COV(X,Y) = 9+4+2*1.5 = 16$$
$$VAR(X-Y) = VAR(X) + VAR(Y) - 2COV(X,Y) = 9+4-3 = 10$$

$$\therefore CORR(X+Y, X-Y) = \frac{5}{\sqrt{16}\sqrt{10}} = \frac{5}{4 \cdot \sqrt{2} \cdot \sqrt{5}} = \frac{\sqrt{5}}{4\sqrt{2}} = 0.395$$

Q2 X, Y dependent

$Var(X) = Var(Y)$

$$COV(X+Y, X-Y) = COV(X, X-Y) + COV(Y, X-Y)$$
$$= COV(X,X) - \cancel{COV(X,Y)} + \cancel{COV(Y,X)} - COV(Y,Y)$$
$$= VAR(X) - VAR(Y)$$
$$= 0$$

Q3 $Y_t = \underbrace{5 + 2t}_{u_t} + X_t$

non-constant function

$X_t \to$ zero mean stationary series

$\gamma_k \to$ AUTO COVARIANCE

linear relationship b/w lagged values of a time series y

1) $E(Y_t) = E(5+2t) + E(X_t)$
$$= 5+2t + 0$$
$$= 5+2t$$

2) Auto-covariance function for $\{Y_t\}$

$= Cov(Y_t, Y_{t-k})$ lag period $\qquad$ let $5 + 2t = u_t$

$= Cov(u_t + X_t, u_{t-k} + X_{t-k})$

$= Cov(u_t, u_{t-k}) + \cancel{Cov(u_t, X_{t-k})}^{0} + \cancel{Cov(X_t, u_{t-k})}^{0} + Cov(X_t, X_{t-k})$

$= \underbrace{Cov(u_t, u_{t-k})}_{\text{constant}} + Cov(X_t, X_{t-k})$] Auto covariance for X_t at lag k $\therefore \gamma_k$

$= \gamma_k$

3) As mean of $Y_t = E(Y_t) = 5 + 2t$, we can see that the mean does not stay constant

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Q4

(1) Monthly Accidental Deaths → A : Strong auto-corelation at every 12 months lag

(2) Monthly Air Passengers → C : Overall upward trend, with spikes at 12 months

(3) Annual Milk Trappings → B : Strong auto-correlation at every 10 years lag

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Q5