ASSIGNMENT 1 - KSHITIJ MITTAL

Q1

$E(X) = 2$ $\quad VAR(X) = 9$ $\quad \sigma_X = \sqrt{9} = 3$

$E(Y) = 0$ $\quad VAR(Y) = 4$ $\quad \sigma_Y = \sqrt{4} = 2$

$CORR(X,Y) = 0.25$

$$COV(X,Y) = CORR(X,Y) * (\sigma_X)(\sigma_Y)$$
$$= 0.25 * 3 * 2$$
$$= 1.5$$

(a) $Var(X+Y) = Var(X) + Var(Y) + 2\,Cov(X,Y)$

$= 9 + 4 + 2*1.5$

$= 16$

(b) $Cov(X, X+Y) = Cov(X,X) + Cov(X,Y)$

$Cov(X,X)$:

$= E[(X-E(X))(X-E(X))]$

$= E(X-E(X))^2$

$= Var(X)$

$= 9$

$Cov(X,Y)$:

$= 1.5$

(from above)

$\therefore Cov(X, X+Y) = 9 + 1.5$

$= 10.5$

(c) $CORR(X+Y, X-Y) = CORR(X,X) - CORR(X,Y) + CORR(X,Y) - CORR(Y,Y)$

$$CORR(X+Y, X-Y) = \frac{COV(X+Y, X-Y)}{\sqrt{Var(X+Y)}\sqrt{Var(X-Y)}}$$

$COV(X+Y, X-Y) = COV(X,X) - \cancel{COV(X,Y)} + \cancel{COV(Y,X)} - COV(Y,Y)$

$$= VAR(X) - VAR(Y)$$
$$= 9-4 \quad = 5$$

$$VAR(X+Y) = VAR(X) + VAR(Y) + 2COV(X,Y) = 9+4+2*1.5 = 16$$
$$VAR(X-Y) = VAR(X) + VAR(Y) - 2COV(X,Y) = 9+4-3 = 10$$

$$\therefore CORR(X+Y, X-Y) = \frac{5}{\sqrt{16}\sqrt{10}} = \frac{5}{4\cdot\sqrt{2}\cdot\sqrt{5}} = \frac{\sqrt{5}}{4\sqrt{2}} = 0.395$$

Q2 X, Y dependent

$Var(X) = Var(Y)$

$$COV(X+Y, X-Y) = COV(X, X-Y) + COV(Y, X-Y)$$
$$= COV(X,X) - \cancel{COV(X,Y)} + \cancel{COV(Y,X)} - COV(Y,Y)$$
$$= VAR(X) - VAR(Y)$$
$$= 0$$

Q3 $Y_t = \underbrace{5 + 2t}_{u_t} + X_t$

non-constant function

$X_t \Rightarrow$ zero mean stationary series

$\gamma_k \Rightarrow$ AUTO COVARIANCE

linear relationship b/w lagged values of a time series y

1) $E(Y_t) = E(5+2t) + E(X_t)$
$$= 5+2t + 0$$
$$= 5+2t$$

2) Auto-covariance function for $\{Y_t\}$

$= Cov(Y_t, \underbrace{Y_{t-k}}_{\text{lag period}})$ 		let $5+2t = u_t$

$= Cov(u_t + X_t, u_{t-k} + X_{t-k})$

$= Cov(u_t, u_{t-k}) + \underbrace{Cov(\cancel{u_t, X_{t-k}})}_{0} + \underbrace{Cov(\cancel{X_t, u_{t-k}})}_{0} + Cov(X_t, X_{t-k})$

$= \underbrace{Cov(u_t, u_{t-k})}_{\hookrightarrow \text{constant}} + Cov(X_t, X_{t-k})$] Auto covariance for X_t at lag k $\therefore \gamma_k$

$= \gamma_k$

3) As mean of $Y_t = E(Y_t) = 5+2t$, we can see that the mean does not stay constant

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Q4

(1) Monthly Accidental Deaths → A : Strong auto-correlation at every 12 months lag

(2) Monthly Air Passengers → C : Overall upward trend, with spikes ~~at~~ 12 months

(3) Annual Milk Trappings → B : Strong auto-correlation at every 10 years lag

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Q5

Time Series Assignment 1

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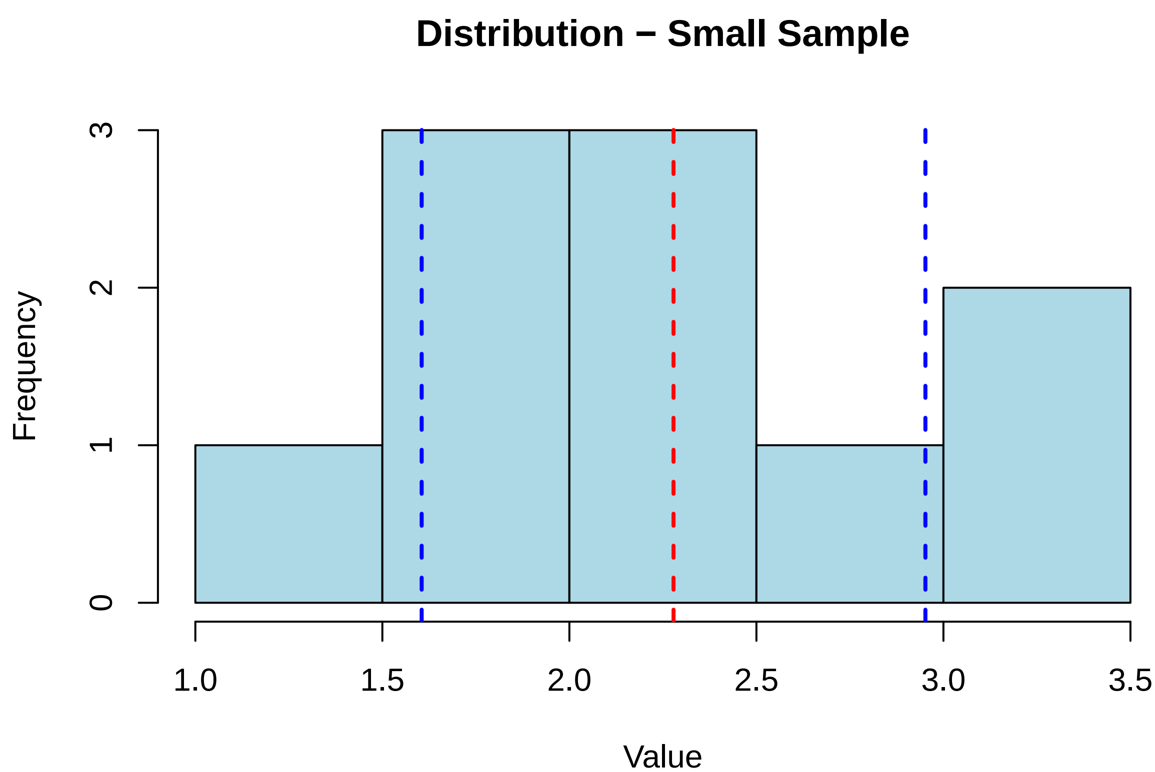
2023-06-26

Assignment 1

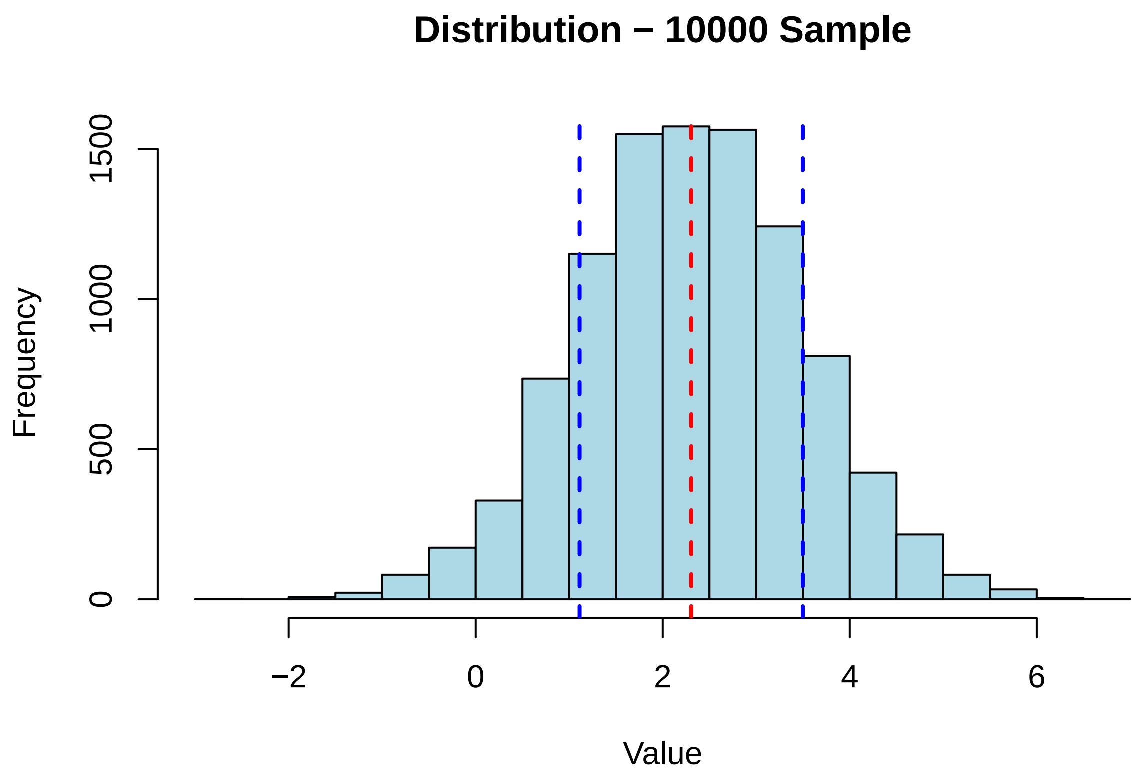
Making the small sample

```r
set.seed(100)

sample_q5_small=rnorm(10, mean = 2.3, sd=1.2)
small_sample_mean=mean(sample_q5_small)
small_sample_sd=sd(sample_q5_small)
```



Expanding Sample Size to 10_000

```r
set.seed(100)

sample_q5_big=rnorm(10000, mean = 2.3, sd=1.2)
big_sample_mean=mean(sample_q5_big)
big_sample_sd=sd(sample_q5_big)
```



Larger samples tend to provide more accurate estimates of population parameters. Increasing the sample size from 10 to 10,000 can improve the precision of the estimated sample mean and sample standard deviation. The larger sample size allows for a better representation of the underlying distribution and reduces the impact of random fluctuations

Sample Mean and SD from sample size of 10000 would be more reliable estimates