Lab Assignment 4b: Optimization for Machine Learning
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Write python codes of descent methods with inexact line search technique for the following function:

(1) $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x) = (x_1 - r)^4 + (x_1 - 2x_2)^2$ with and $x^0 = (r-1, r+1)$ where r is the last digit of your roll number. If last digit of your roll number is 0 then choose $r = 1.75$. Use $\beta_1 = 10^{-4}$, $\beta_2 = 0.9$, $r = 0.5$, and stopping criteria $\|\nabla f(x^k)\| < 10^{-4}$ or maximum 500 iterations. Find number of iterations, function evaluations and gradient evaluations.

(2) Solve the above problem use Bregman function $h(x) = \frac{1}{2}x^T Bx$ where $B = \begin{bmatrix} 2r & \sqrt{r} \\ \sqrt{r} & r \end{bmatrix}$. Find number of iterations, function evaluations and gradient evaluations. Does this method take less number of iterations?

(3) Solve the following system of equation using Newton method

$$F(x) = \begin{bmatrix} x_1^2 + x_2^2 - 2 \\ e^{x_1 - 1} + x_2^3 - 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{1}$$

Use $x^0 = (2.r, 2.r)^T$, stopping criteria $\|F(x^k)\| < 10^{-4}$ or maximum 500 iterations.

(4) Solve the following problem

$$\min_x (x_1 - r)^4 + (x_1 - 2x_2)^2$$

using Newton method. Use $x^0 = (r+1, r-1)$ as initial approximation. Use stopping criteria $\|\nabla f(x^k)\| < 10^{-4}$ or maximum 500 iterations.

(5) Suppose $f_1, f_2 : \mathbb{R}^n \to \mathbb{R}$, for $n = 30$ are defined as

$$\begin{aligned} I_1 &= \{i \in \{2, 3, ..., n\} \mid i \mod 2 = 1\} \\ I_2 &= \{i \in \{2, 3, ..., n\} \mid i \mod 2 = 0\}; \\ f_1(x) &= x_1 + \frac{2}{|I_1|}\sum_{i \in I_1}(x_i - sin(6\pi x_1 + i\pi/n))^2 \\ f_2(x) &= 1 - \sqrt{x_1} + \frac{2}{|I_2|}\sum_{i \in I_2}(x_i - sin(6\pi x_1 + i\pi/n))^2 \end{aligned}$$

Using modified Newton method solve

$$\begin{aligned} &\min 0.r f_1(x) + (1 - 0.r) f_2(x) \\ &\qquad s.t. \quad 0.001 \leq x_1 \leq 1, \quad -1 \leq x_i \leq 1, \quad i = 2, 3, ..., n. \end{aligned}$$

where r is last digit of your roll no. If last digit of your roll no is use then use $r = 15$.

(6) Given the dataset construct unconstrained optimization for logistic regression and solve using (i) gradient descent (ii) mirror descent method by using a symmetric positive definite matrix with diagonal elements in $[5, 10]$ and off diagonal elements in $[0, 1]$ (iii) Newton method