

Question 1

```
import numpy as np
```

```
# function for calculating factorial
```

```
def factorial(n):
```

```
    val = 1;
```

```
    for i in range(2, n + 1):
```

```
        val = val * i;
```

```
    return val;
```

```
# function for calculating u values
```

```
def u_values(u, n):
```

```
    val = u;
```

```
    for i in range(1, n):
```

```
        val = val * (u - i);
```

```
    return val;
```

```
# forward newton raphson function
```

```
def forward_newton_method(X, Y, input_x):
```

```
    n = X.shape[0]
```

```
    # initialising the value of interpolated y to 0
```

```
    calc_y = Y[0][0]
```

```
    # finding the forward differences and storing them in Y matrix
```

```
    for i in range(1, n):
```

```
        for j in range(n - i):
```

```
            Y[j][i] = Y[j + 1][i - 1] - Y[j][i - 1];
```

```
    # initial value of u
```

```
    u = (input_x - X[0]) / (X[1] - X[0])
```

```
    for i in range(1, n):
```

```
        calc_y = calc_y + (u_values(u, i) * Y[0][i]) / factorial(i);
```

```
    return calc_y
```

```
# taking the actual function as  $y = x^3$  and giving 2 inputs initially
```

```
X = np.array([10, 30])
```

```
# array Y will store the y values and forward difference values
```

```
Y = np.zeros((2,2))
```

```
# initialising the first entries of every row in Y to the y values they take
Y[0][0], Y[1][0] = 1000, 27000
```

```
input_x = 25
actual_y = 15625
calc_y = forward_newton_method(X, Y, input_x)
```

```
# printing output
print("f(x) at x = {} is: {}".format(input_x, calc_y))
print("error percentage is:", 100*abs(actual_y - calc_y)/actual_y)
# output below
f(x) at x = 25 is: 20500.0
error percentage is: 31.2
```

```
# now checking if error percentage decreases by adding more terms
```

```
X = np.array([10, 20, 30])
Y = np.zeros((3,3))
Y[0][0], Y[1][0], Y[2][0] = 1000, 8000, 27000
```

```
input_x = 25
actual_y = 15625
calc_y = forward_newton_method(X, Y, input_x)
```

```
# printing output
print("f(x) at x = {} is: {}".format(input_x, calc_y))
print("error percentage is:", 100*abs(actual_y - calc_y)/actual_y)
# output below
f(x) at x = 25 is: 16000.0
error percentage is: 2.4
```

```
# increasing 2 more terms to check if error percentage goes down even more
```

```
n = 5
X = np.array([10, 20, 30, 40, 50])
Y = np.zeros((5,5))
Y[0][0], Y[1][0], Y[2][0], Y[3][0], Y[4][0] = 1000, 8000, 27000, 64000, 125000
```

```
input_x = 25
actual_y = 15625
calc_y = forward_newton_method(X, Y, input_x)
```

```

# printing output
print("f(x) at x = {} is: {}".format(input_x, calc_y))
print("error percentage is:", 100*abs(actual_y - calc_y)/actual_y)
# output below
f(x) at x = 25 is: 15625.0
error percentage is: 0.0

```

Question 2

```

# function to calculate polynomial p2(x)
def lagrange_interpolation(X, Y, input_x):
    n = X.shape[0]
    p2 = 0

    for i in range(n):
        L = 1
        for j in range(n):
            if i != j:
                L = L * (input_x - X[j]) / (X[i] - X[j])
        p2 = p2 + L * Y[i]

    return p2

# known values of (x,y)
X = np.array([0.25, 0.5, 1])
Y = np.array([0.27633, 0.52050, 0.84270])

# value of x at which we have to interpolate
input_x = 0.75
calc_y = lagrange_interpolation(X, Y, input_x)

# Displaying output
print("Value of interpolated f(x) at x = 0.75 is:", calc_y)
# Output below
Value of interpolated f(x) at x = 0.75 is: 0.70929

```