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2019EE30577 Grp 30

Problem 5: BONUS: 10 marks

The solution to this problem may be submitted on Moodle by end of today (23:59).

A coffee can contains some black beans and white beans. The following process is to be repeated as long as possible:

- Randomly select two beans from the can. If they are the same colour, throw them out, but put another black bean in (Assume that enough black beans are available to do this). If they are of different colours, place the white one back into the can and throw the black one away.

Execution of this process reduces the number of beans in the can by one. Repetition of this process must terminate with exactly one bean in the can. What can you say about the colour of this last bean depending on the number of black beans and the number of white beans that were there in the can to start with? Give reasons for your answer.

We solve the problem by considering three cases for any arbitrary pull:

~~B, B~~ Let the initial number of black beans be p and white beans be q.

Case I	Case II	Case III
B, B	W, W	B, W

The number of beans changes as:

black $= p-2+1 = p-1$	$p+1$	$p-1$
white $= q$	$q-2$	q

From this, we see that the number of white beans either decreases by 2 or remains the same in all the 3 cases.

As the no. of beans decreases by 1 in each case, we are bound to reach exactly one bean.

Now, if we started with odd no. of white beans in the can, we would end up with a white bean as the bean count of white will decrease only in multiples of 2.

Whereas if we started with an even no. of white beans, all the white beans would get exhausted at the end and we would be left with a black bean.

In short,

Odd no. of white beans ⇒ end with white

even no. of ~~black~~ white beans ⇒ end with black.

We can also write an invariant for the whole process as:

$$(q_0 \bmod 2) = (q \bmod 2)$$

(initial white beans be q_0 and at any stage, q.)

Declaration of originality: No sources were consulted.