1. If $a \geq 0$ and $b > 0$ are two integers, then there exists $q \geq 0$ and $0 \leq r < b$ such that $a = q * b + r$. The *mod* function is defined as $mod(a, b) = r$. Develop an `ML` program for computing $mod(a, b)$ using addition and subtraction. Prove the correctness of your algorithm using mathematical induction.

2. Complete the following iterative algorithm for reversing a given positive integer in base 10. That is, if n is the input integer such that $n = \sum_{i=0}^{m} 10^i a_i = 10^m * a_m + 10^{m-1} * a_{m-1} + ... + 10^0 * a_0$, then your function $rev : \mathbb{N} \to \mathbb{N}$ should return the number x such that $x = \sum_{i=0}^{m} 10^{m-i} a_i = 10^m * a_0 + 10^{m-1} * a_1 + ... + 10^0 * a_m$. Indicate the number of steps required by the process. Use the invariant: after k iterations (note that x_0 is the intial value of x)
$INV = (n = \sum_{i=0}^{m-k} 10^i a_{i+k} = a_k * 10^0 + a_{k+1} * 10^1 + ... + a_m * 10^{m-k} \geq 0)$ and $(x = x_0 10^k + \sum_{i=0}^{k-1} a_{k-i-1} 10^i = x_0 10^k + a_0 * 10^{k-1} + a_1 * 10^{k-2} + ... + a_{k-1} * 10^0 \geq 0)$ and $(0 \leq k \leq m + 1)$

3. (a) Two frogs are sitting at bottom of a flight of 10 steps and debating in how many ways can they jump up the stairs. They can jump one, two or three steps at a time. For example they can cover the 10 steps by jumping (3, 3, 3 , 1) or (2 ,3, 3, 2) or other suitable combinations. Their mathematics is not very strong (being frogs) and they approach you for help. Complete the following `ML` function that provides a general solution (not only for 10 but for general n steps). Note that (3,3,3,1) is distinct from (1,3,3,3) and likewise and that we only want to count the number of solutions and not report the solutions.

```
fun frog(n) =
  if n =1 then 1 else if n =2 then 2 else if n = 3 then 4

  else ________________________________________________
```

   (b) Encouraged by your solution, one of the frogs becomes more ambitious and tries to find out the answer for $n = 100$ and runs out of patience. Write an alternate (faster) `ML` program (analysis not necessary).