ELL409 Assignment 3 Report

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Part 1A.

In this part of the assignment, we experiment with our own implementation of Neural Networks, compare its performance to a standard library (Tensorflow) as well as try to understand the representations learnt by the neural network.

First we implement our own class of Neural Network using Numpy.

The following is the initialization portion of the neural network where we define the various parameters we will be using and also initialize the weights and the biases randomly.

```
class NN:
    def __init__(self,epochs=100,layers=2,nodes=[50,75],alpha=0.0003,bs=16,eps=0.
↪001,activation=1,reg=1,lam =0):
        np.random.seed(42)
        self.epochs = epochs
```

```
        self.layers = layers
        self.nodes = nodes
        self.alpha = alpha
        self.bs = bs
        self.eps = eps
        self.activation = activation
        self.reg = reg
        self.lam = lam
        self.w = [np.random.randn(NUM_FEATURES,self.nodes[0])]
        self.b = [np.random.randn(self.nodes[0])]
        self.z = [0] * (self.layers +1)
        self.a = [0] * (self.layers)
        self.dw = [0]*(self.layers+1)
        self.db = [0]*(self.layers+1)

        for i in range(1,self.layers):
            self.w.append(np.random.randn(self.nodes[i-1],self.nodes[i]))
            self.b.append(np.random.randn(self.nodes[i]))

        self.w.append(0.1*np.random.randn(self.nodes[self.layers-1],10))
        self.b.append(np.random.randn(10))
```

Next, we define our activation functions which we will be using in the hidden layers as well as the softmax layer for the final output.

```
def relu(x): # 1
    return np.maximum(0,x)

def relu_der(x):
    x[x>0] = 1
    x[x<=0] = 0
    return np.array(x)

def sigmoid(x): # 2
    return 1/(1+np.exp(-1.*x))

def sigmoid_der(x):
    return sigmoid(x)*(1-sigmoid(x))

def tanh(x): # 3
    return np.tanh(x)

def tanh_der(x):
    return 1-np.square(np.tanh(x))

def softmax(x):
    return np.exp(x) / np.sum(np.exp(x), axis=1, keepdims=True)
```

These functions are called using the following helper functions defined in our Neural Network class:

```
    def activation_fn(self,x):
        if self.activation == 1:
            return relu(x)
        elif self.activation == 2:
            return sigmoid(x)
        elif self.activation == 3:
            return tanh(x)

    def der_activation_fn(self,x):
        if self.activation == 1:
            return relu_der(x)
        elif self.activation == 2:
            return sigmoid_der(x)
        elif self.activation == 3:
            return tanh_der(x)
```

Next we move to the forward propagation step of our neural network:

```
    def f_prop(self,X):
        self.z[0] = np.dot(X,self.w[0]) + self.b[0]
        self.a[0] = self.activation_fn(self.z[0])
        for i in range(1,self.layers):
            self.z[i] = np.dot(self.a[i-1],self.w[i]) + self.b[i]
            self.a[i] = self.activation_fn(self.z[i])
        self.z[self.layers] = np.dot(self.a[self.layers-1],self.w[self.layers])␣
↪+ self.b[self.layers]
```

For computing the gradients we perform the following backpropagation:

```
    def back_prop(self,X,t,y):
        temp = y-t
        self.dw[self.layers] = (np.dot(self.a[self.layers-1].T,temp)+
                                self.lam*self.w[self.layers])/self.bs
        self.db[self.layers] = (np.sum(temp,axis=0))/self.bs

        for i in range(self.layers-1,-1,-1):
            aux = np.dot(temp,self.w[i+1].T)
            der = self.der_activation_fn(self.a[i])
            temp = aux*der
            if i>0:
                self.dw[i] = (np.dot(self.a[i-1].T,temp) + self.lam*self.w[i])/
↪self.bs
            else:
                self.dw[0] = (np.dot(X.T,temp) + self.lam*self.w[0])/self.bs
            self.db[i] = np.sum(temp,axis=0)/self.bs
```

Now for updating our weights, we perform the following steps in our train function of the Neural Network class:

```
        for epoch in tqdm(range(self.epochs)):
            t1 = time.time()
            for chunk in range(len(Xchunks)):
                Xcon = Xchunks[chunk]
                tcon = tchunks[chunk]

                self.f_prop(Xcon)
                y = softmax(self.z[self.layers])

                # call backprop here
                self.back_prop(Xcon,tcon,y)

                for i in range(self.layers+1):
                    self.w[i] -= self.alpha * self.dw[i]
                    self.b[i] -= self.alpha * self.db[i]
```

Here, the chunks are used for implementing the batch gradient descent.

The following are the tuneable parameters in the Neural Network class:

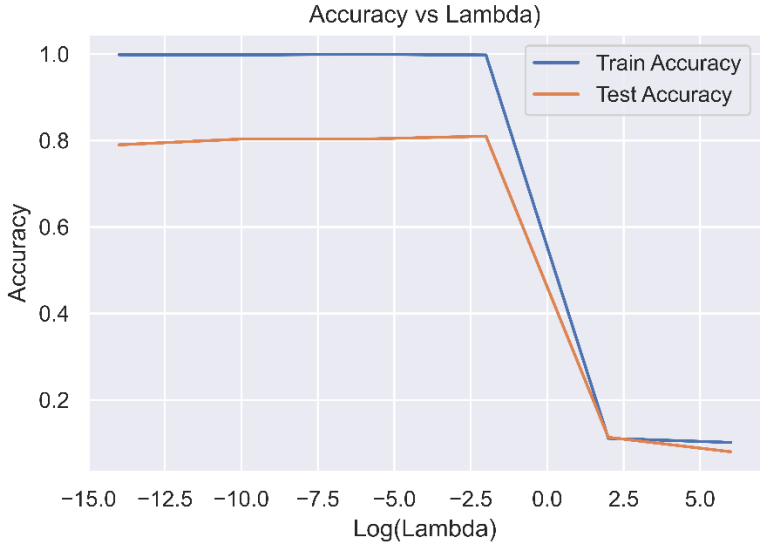
- Number of hidden layers
- Number of nodes (neurons) in each hidden layers
- Learning rate
- Batch Size
- Number of Epochs
- Regularization parameters
- Type of activation function to be used in the hidden layers
- Epsilon for early stopping

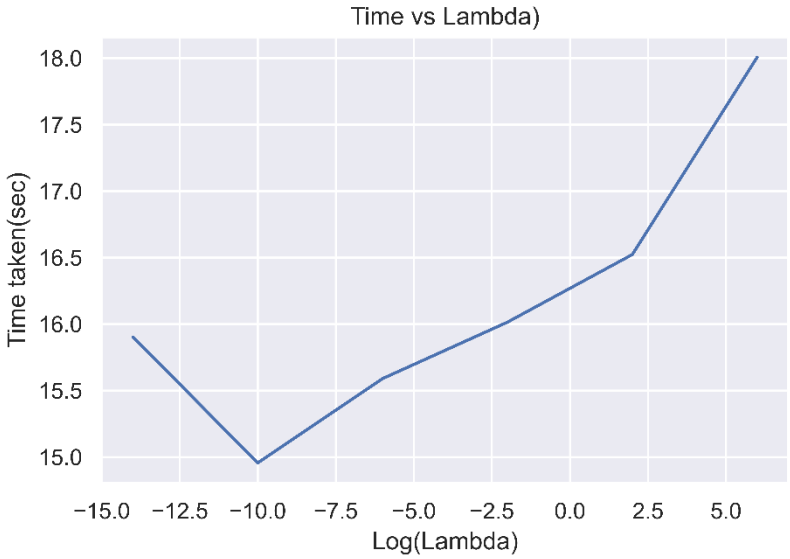
We also use early stopping in which we see if the train accuracy has been within a threshold for 3 iterations, we break the loop there.

In the next section, we see how the change in hyperparameters affect our train/test accuracies.

Variation with lambda (# of layers = 2)

First we fix the value of learning rate to be 0.004, number of layers to be 2, batch size of 16 and run for 100 epochs. Now by running a sweep over the values of lambda we get the following results:

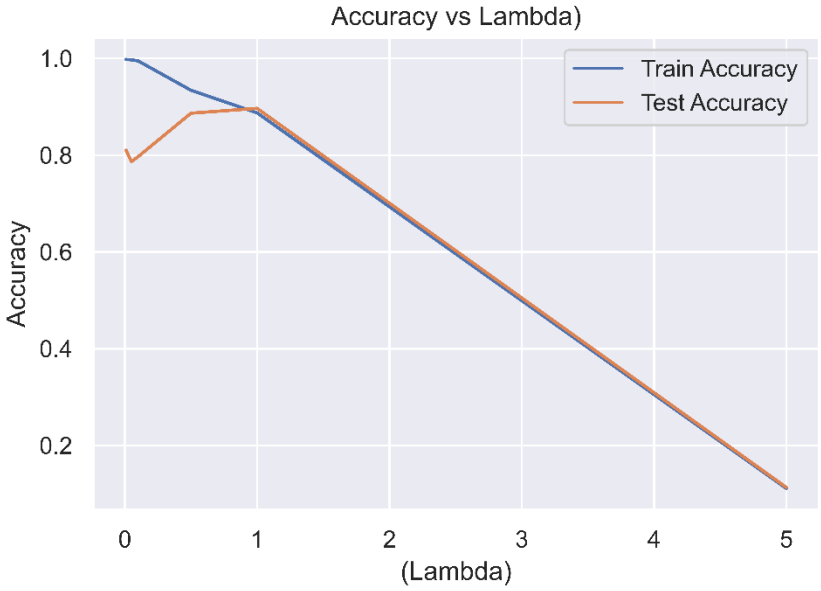


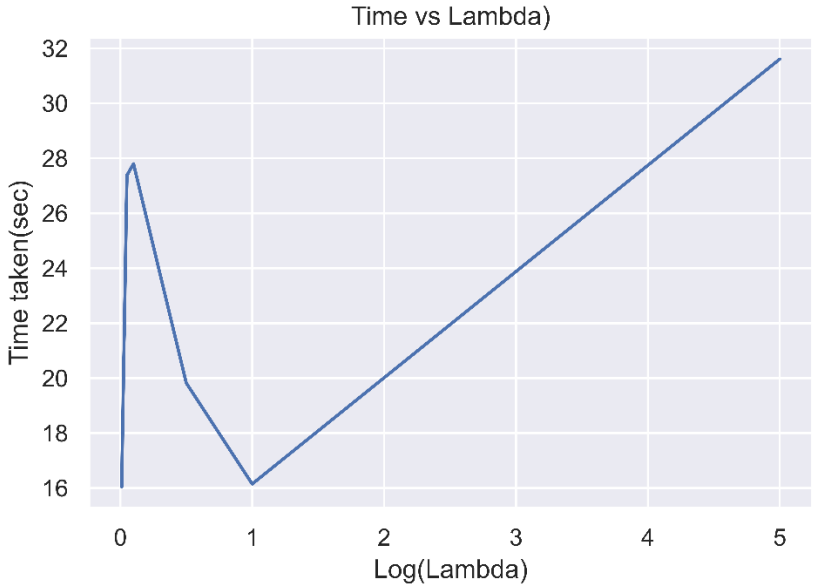


	lam	train_acc	test_acc	time_taken
0	-14	0.998519	0.790000	15.902141
1	-10	0.998148	0.803333	14.956310
2	-6	0.999259	0.803333	15.590128
3	-2	0.997778	0.810000	16.014454
4	2	0.111111	0.113333	16.522977
5	6	0.101481	0.080000	18.006861

We can clearly see **overfitting** in the initial values of lambda and **underfitting** when the lambda becomes larger.

Tuning this further for values between 0 and 5 we see the following curve:

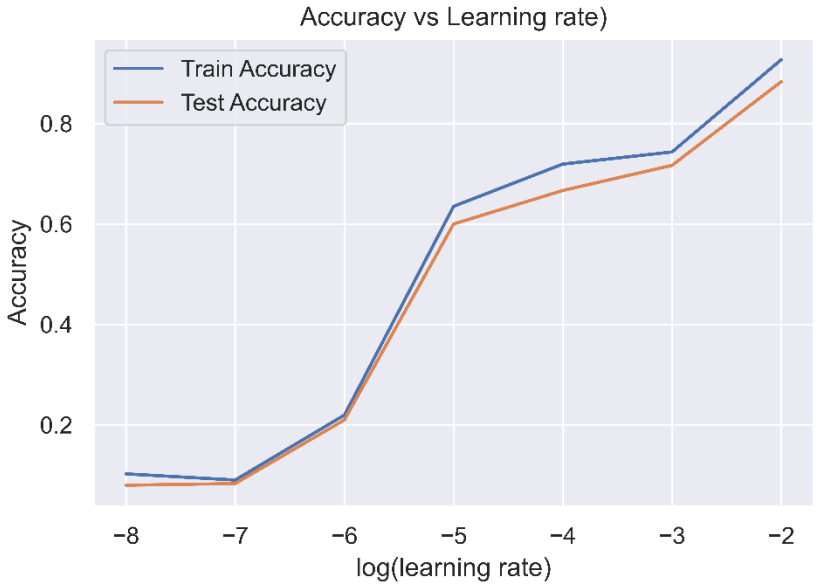


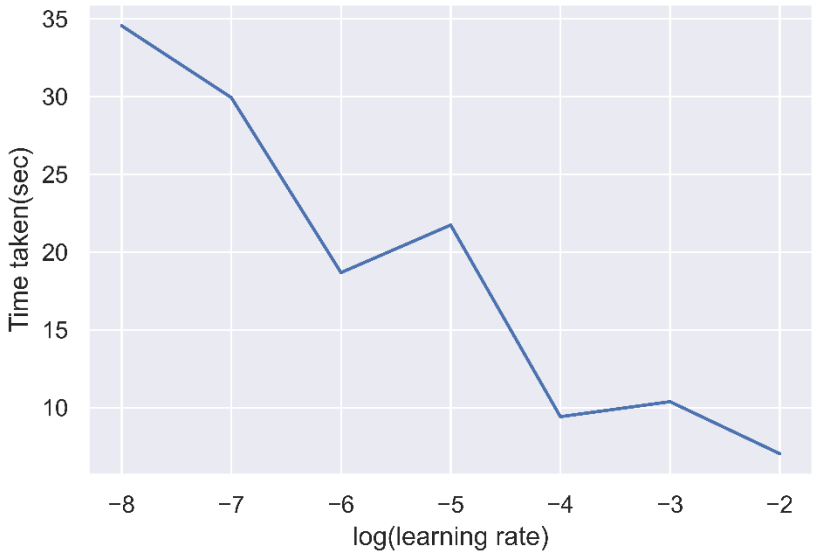


	lam	train_acc	test_acc	time_taken
0	0.01	0.997778	0.810000	16.045384
1	0.05	0.997037	0.786667	27.378482
2	0.10	0.994815	0.796667	27.799001
3	0.50	0.934074	0.886667	19.820700
4	1.00	0.887407	0.896667	16.145968
5	5.00	0.111111	0.113333	31.607286

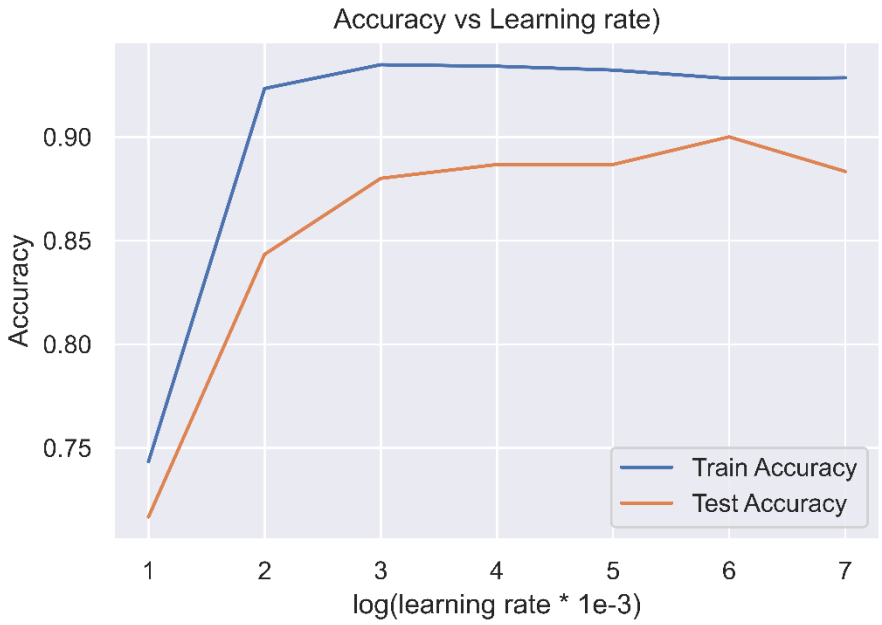
Variation with learning rate

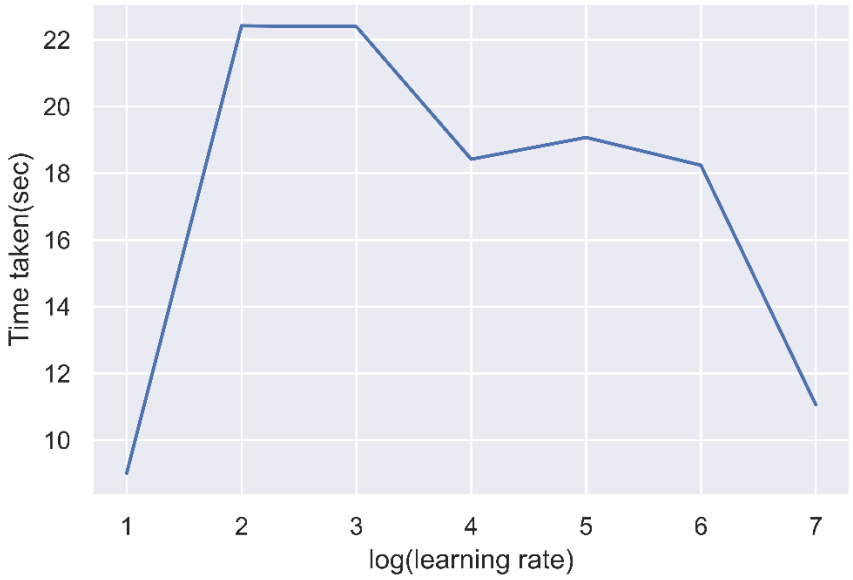
Here we fix the value of regularization lambda to be 0.5 and vary the learning rate. We get the following results:





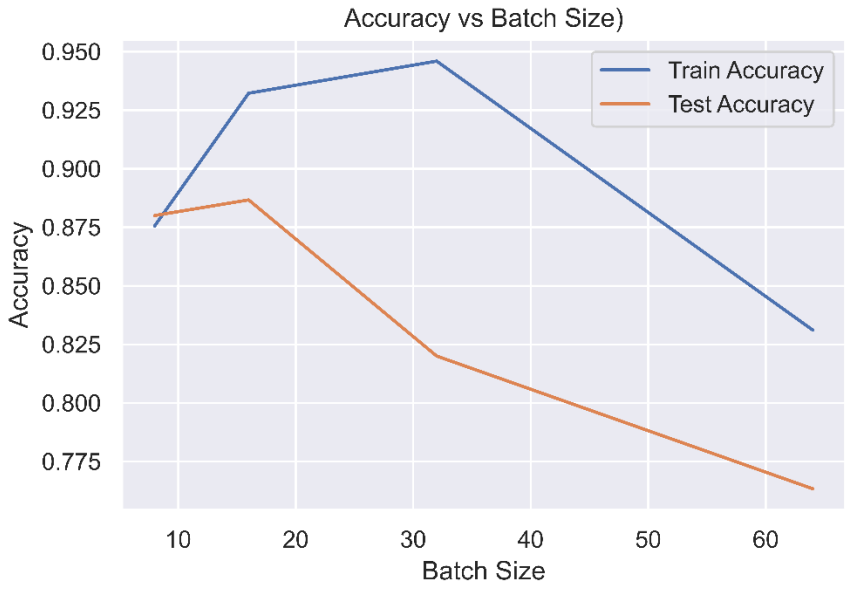
	LR	train_acc	test_acc	time_taken
0	1.000000e-08	0.102593	0.080000	34.551819
1	1.000000e-07	0.090370	0.083333	29.931592
2	1.000000e-06	0.219630	0.210000	18.679262
3	1.000000e-05	0.635185	0.600000	21.738911
4	1.000000e-04	0.719259	0.666667	9.415470
5	1.000000e-03	0.743333	0.716667	10.381021
6	1.000000e-02	0.927037	0.883333	7.037894

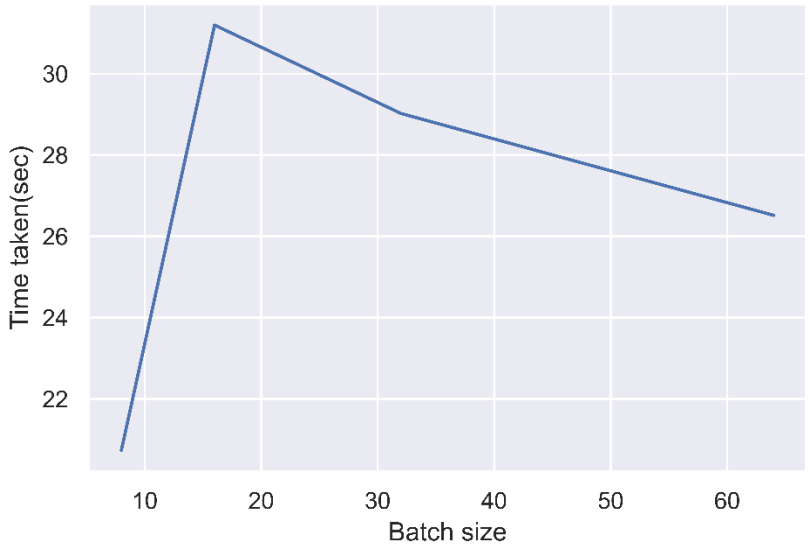




	LR	train_acc	test_acc	time_taken
0	0.001	0.743333	0.716667	9.007805
1	0.002	0.923333	0.843333	22.419587
2	0.003	0.934815	0.880000	22.394939
3	0.004	0.934074	0.886667	18.418861
4	0.005	0.932222	0.886667	19.071711
5	0.006	0.928148	0.900000	18.239354
6	0.007	0.928519	0.883333	11.061211

Variation of Batch size

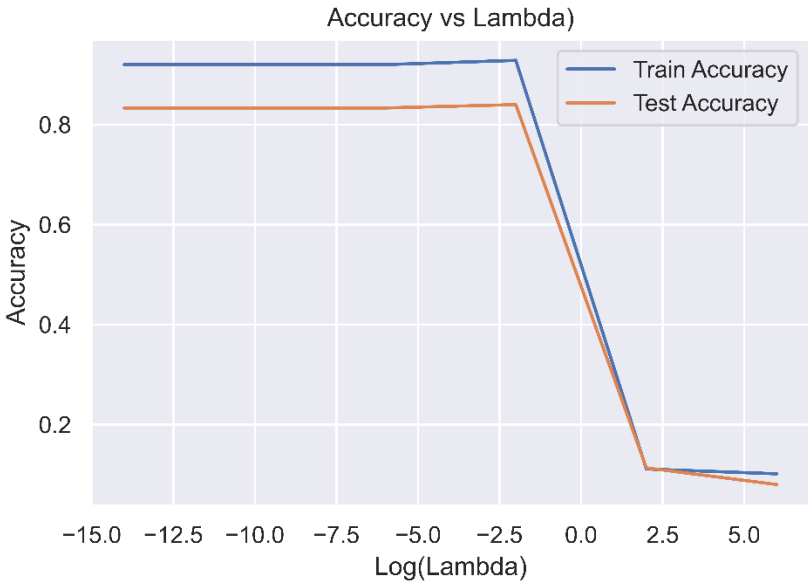


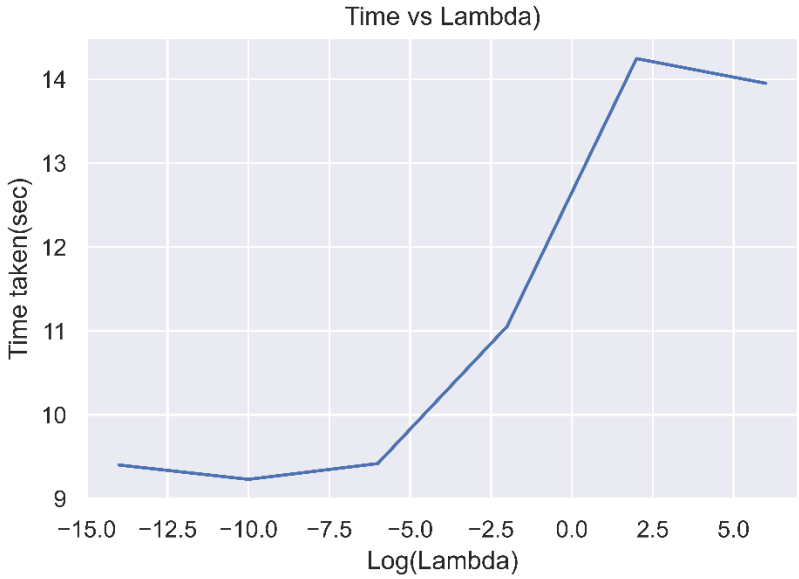


	BS	train_acc	test_acc	time_taken
0	8	0.875556	0.880000	20.737143
1	16	0.932222	0.886667	31.194967
2	32	0.945926	0.820000	29.021666
3	64	0.831111	0.763333	26.515240

Variation with lambda (# of layers = 1)

First we fix the value of learning rate to be 0.004, number of layers to be 1, batch size of 16 and run for 100 epochs. Now by running a sweep over the values of lambda we get the following results:

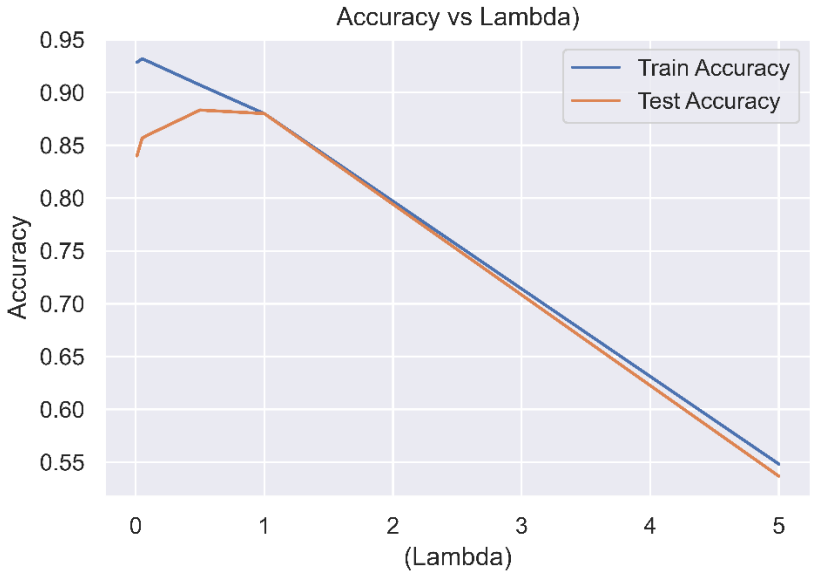


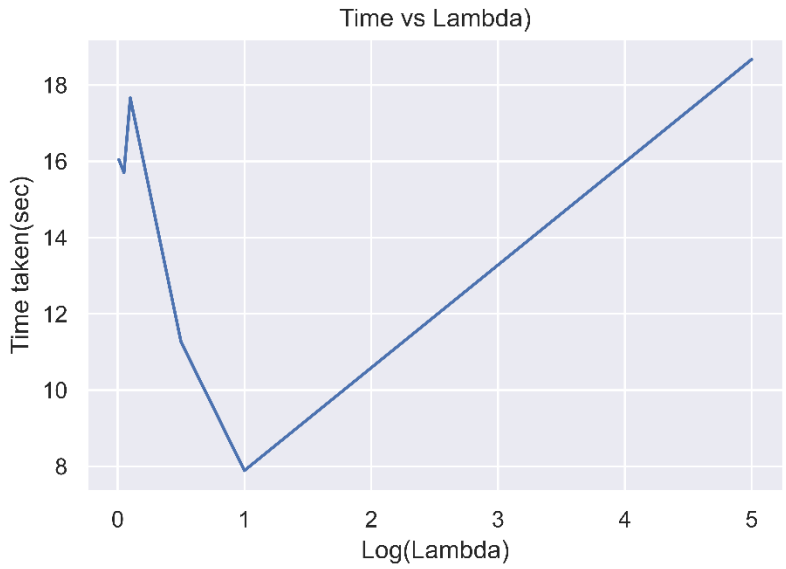


	lam	train_acc	test_acc	time_taken
0	-14	0.920000	0.833333	9.401528
1	-10	0.920000	0.833333	9.230801
2	-6	0.920000	0.833333	9.419060
3	-2	0.928519	0.840000	11.055267
4	2	0.111111	0.113333	14.246521
5	6	0.101481	0.080000	13.951764

We can clearly see **overfitting** in the initial values of lambda and **underfitting** when the lambda becomes larger.

Tuning this further for values between 0 and 5 we see the following curve:

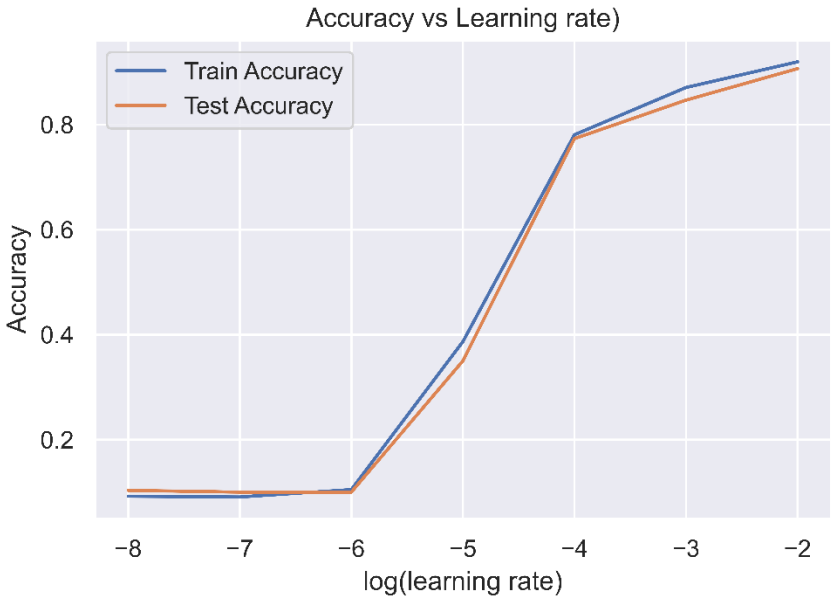


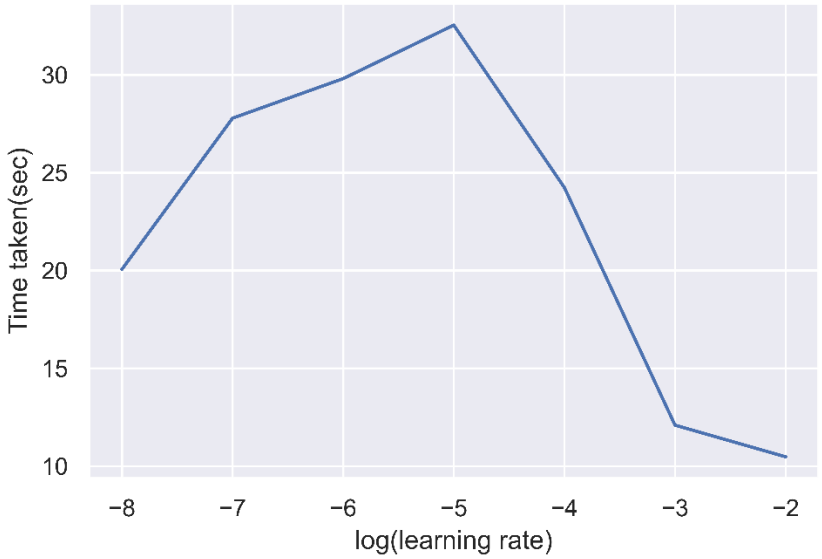


	lam	train_acc	test_acc	time_taken
0	0.01	0.928519	0.840000	16.044469
1	0.05	0.931852	0.856667	15.705497
2	0.10	0.929259	0.860000	17.658879
3	0.50	0.907037	0.883333	11.265135
4	1.00	0.880000	0.880000	7.890082
5	5.00	0.548148	0.536667	18.670657

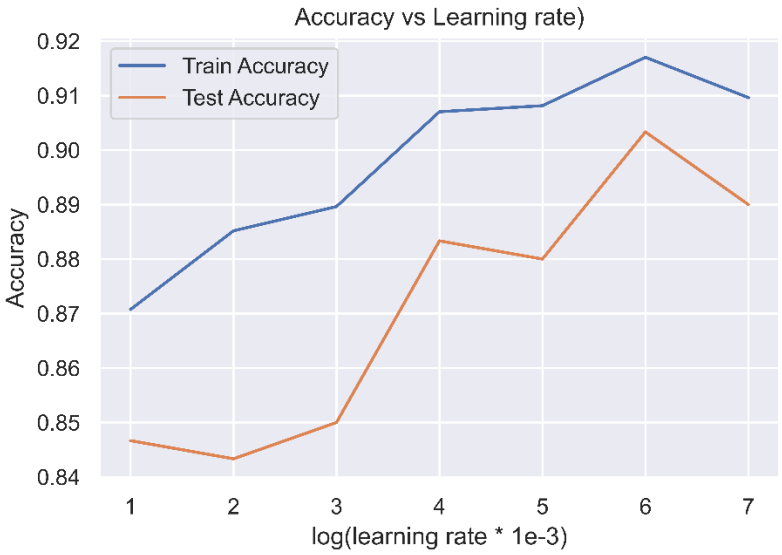
Variation with learning rate

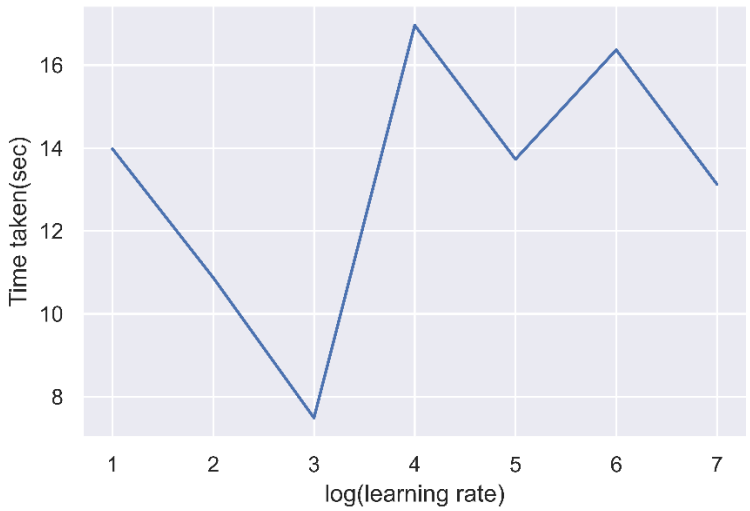
Here we fix the value of regularization lambda to be 0.5 and vary the learning rate. We get the following results:





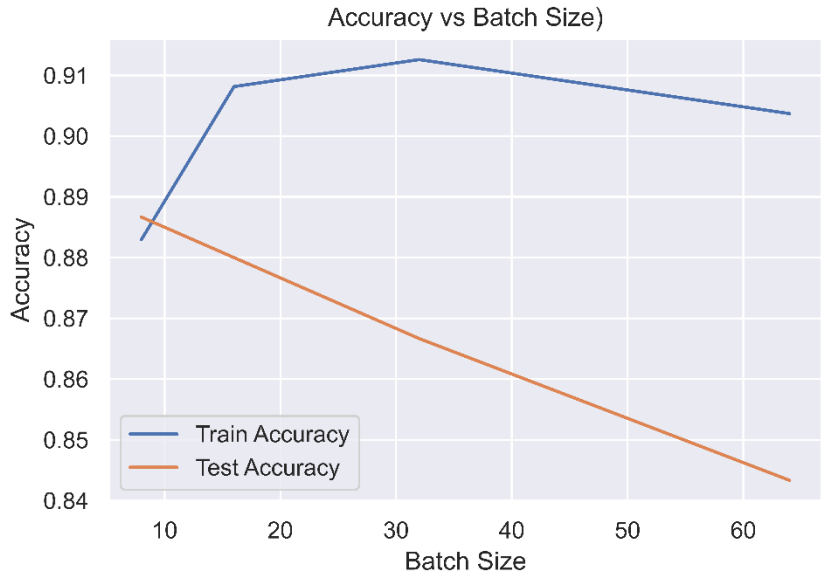
	LR	train_acc	test_acc	time_taken
0	1.000000e-08	0.092222	0.103333	20.061172
1	1.000000e-07	0.090741	0.100000	27.785561
2	1.000000e-06	0.104815	0.100000	29.807645
3	1.000000e-05	0.386667	0.350000	32.541521
4	1.000000e-04	0.780741	0.773333	24.240064
5	1.000000e-03	0.870741	0.846667	12.096484
6	1.000000e-02	0.919630	0.906667	10.478140

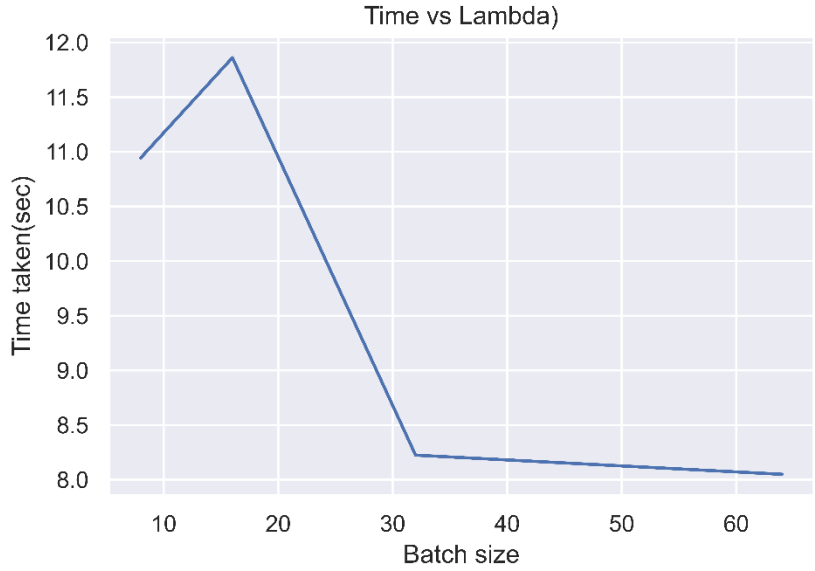




	LR	train_acc	test_acc	time_taken
0	0.001	0.870741	0.846667	13.978901
1	0.002	0.885185	0.843333	10.872702
2	0.003	0.889630	0.850000	7.490244
3	0.004	0.907037	0.883333	16.960423
4	0.005	0.908148	0.880000	13.727301
5	0.006	0.917037	0.903333	16.368423
6	0.007	0.909630	0.890000	13.126486

Variation with Batch size





	BS	train_acc	test_acc	time_taken
0	8	0.882963	0.886667	10.944870
1	16	0.908148	0.880000	11.861279
2	32	0.912593	0.866667	8.223766
3	64	0.903704	0.843333	8.048591

Grid Search:

We fix the value of learning rate to be 0.004 and the batch size to be 16 and then vary the other hyperparameters based on the ranges derived from previous plots.

```
df = pd.DataFrame()
bestLayer, bestLd, bestact = 0,0,0
bestAcc = 0
for layer in [1,2]:
    for activation in [1,2,3]:
        for lbd in tqdm([1e-8,1e-4,1e-2,5e-2,1e-1,5e-1]):
            try:
                nn = NN(epochs = 100,alpha=0.
↪004,layers=layer,nodes=[100,64],lam=lbd,activation=activation,early=False)
                train_acc_list,test_acc_list,train_error_list,test_error_list =␣
↪nn.train(Xorig,Yorig,log=False)
                test_acc = test_acc_list[-1]
                train_acc = train_acc_list[-1]
                if test_acc>bestAcc:
                    bestLayer = layer
                    bestLd = lbd
                    bestact = activation
                    bestAcc = test_acc
                it = {'Layer':layer,'Activation':activation,'Lambd':
↪lbd,'trainAcc':train_acc, 'testAcc':test_acc}
                df = df.append(it,ignore_index=True)
            except:
                continue
        df.to_csv('grid_pt1.csv',index=False)
```

The following are the top results that we get from this:

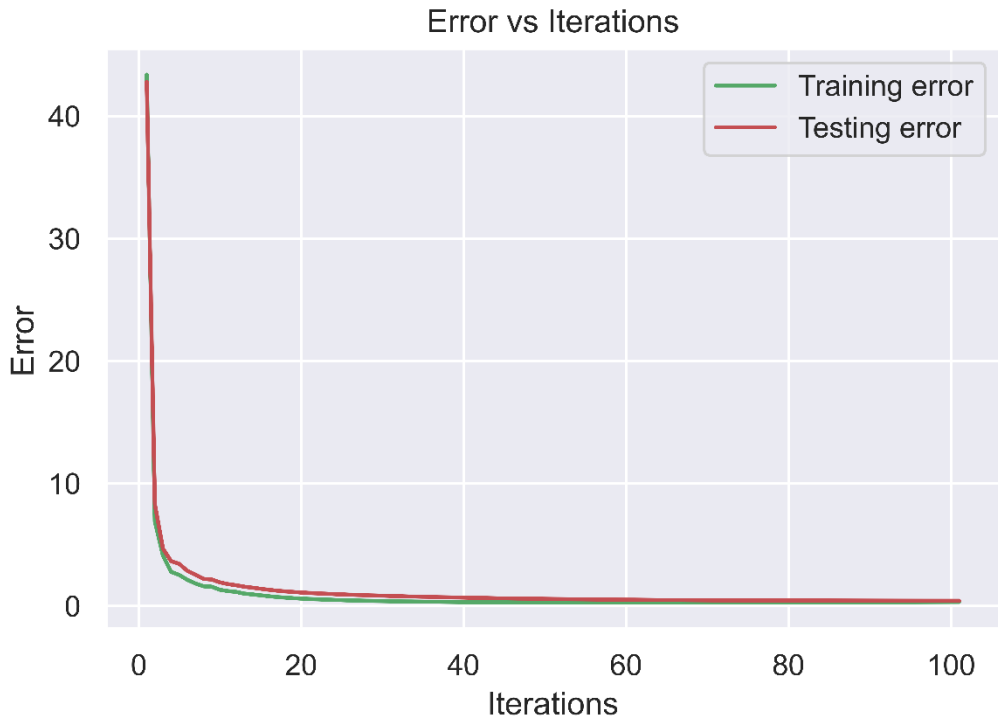
Activation	Lambda	# Layer	Test Acc.	Train Acc.
Relu	0.5	1	0.903333	0.918519
Tanh	0.5	1	0.903333	0.902593
Relu	0.5	2	0.903333	0.931481
Tanh	0.5	2	0.893333	0.914815

Finally, we pick the following set of hyperparameters:

Hyperparameter	Value
Learning rate	0.004
Batch Size	16
Activation Function	Relu
# of layers	2
Nodes in the layers	(100,64)
Regularization Lambda	0.5

Using **5-fold cross validation**, we get an average test Accuracy of **0.9033.**

The following are the training curves that we get for this hyperparameter setting:



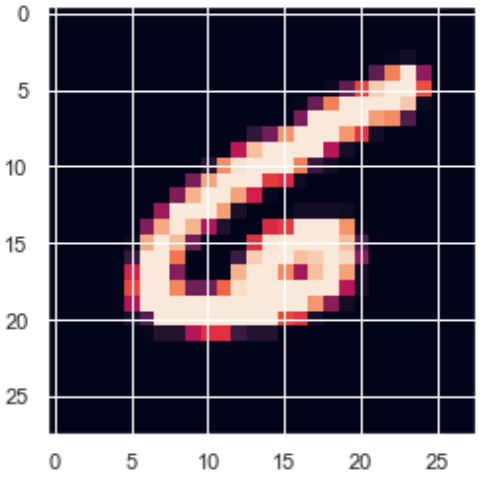
Error Analysis

We get the following confusion matrix for this set of hyperparameters:

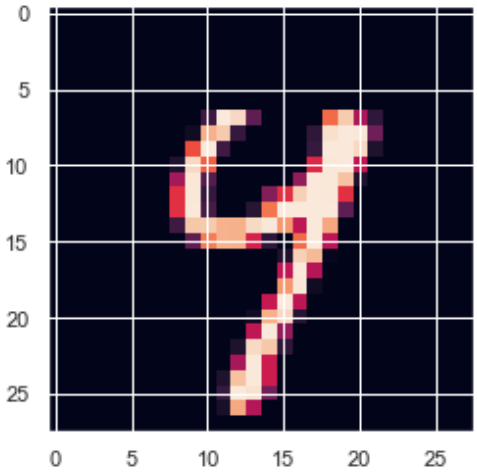
Predicted	0	1	2	3	4	5	6	7	8	9	All
Actual											
0	26	0	0	0	0	0	0	0	0	0	26
1	0	24	1	0	0	0	0	1	1	0	27
2	0	0	26	0	0	0	1	1	0	0	28
3	2	0	1	38	0	0	0	0	0	0	41
4	0	0	0	0	18	0	0	0	0	3	21
5	1	0	0	1	0	26	0	0	2	0	30
6	1	0	0	0	1	2	24	0	0	0	28
7	0	0	0	0	0	0	0	32	0	1	33
8	0	1	0	1	0	1	1	0	32	0	36
9	0	0	0	0	2	0	0	1	0	27	30
All	30	25	28	40	21	29	26	35	35	31	300

Following are some of the data-points where the model is going wrong:

Original number: 6, Prediction: 4;



Original number: 4, Prediction: 9



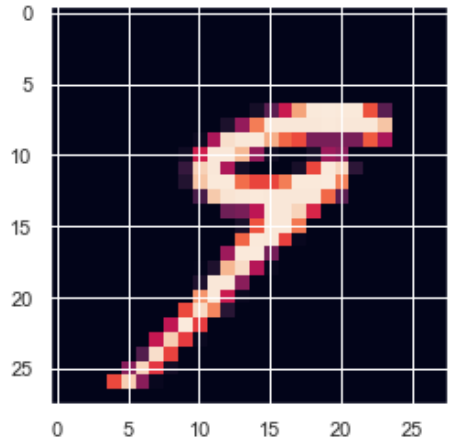
We can see that these images which the model is classifying incorrectly are indeed difficult to categorize. In the first picture, it got confused between 6 and 4 since the circular portion of 6 is not drawn perfectly and indeed would have looked more like a 4 had it been extended downwards. The second picture also can be easily confused with a 4 and a 9 as most of the features are similar to a 9 except the top being connected.

The model is able to extract the features but sometimes struggles when some features are very common.

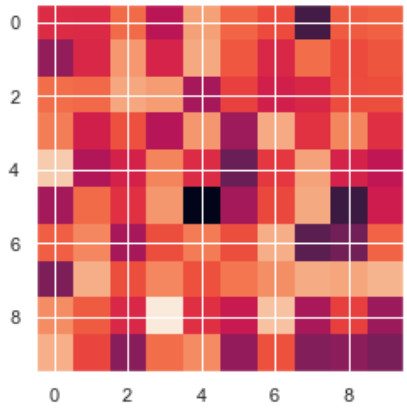
Representations learned by the layers

By plotting the various representations learned by the intermediate layers, we get the following:

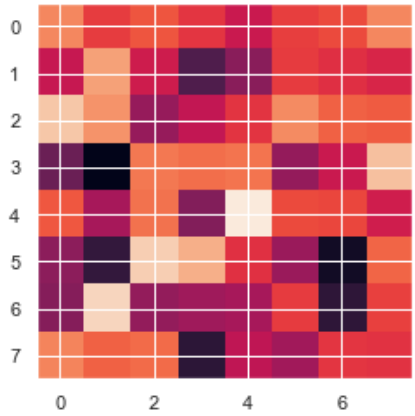
Input image:



Representation learned by layer 1 (100 nodes):



Representation learned by layer 2 (64 nodes):



Using a standard library

If we use a standard library with the same hyperparameters and the architecture, we get the following results:

```
Model: "sequential_1"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_3 (Dense)              (None, 100)               78500
_________________________________________________________________
dense_4 (Dense)              (None, 64)                6464
_________________________________________________________________
dense_5 (Dense)              (None, 10)                650
=================================================================
Total params: 85,614
Trainable params: 85,614
Non-trainable params: 0
_________________________________________________________________
```

After running for 100 epochs and early stopping with best val_accuracy:

```
Epoch 37/100
2700/2700 [==============================] - 0s 123us/sample - loss: 0.0629
- accuracy: 0.9537 - val_loss: 0.3278 - val_accuracy: 0.9167
```

We get a **validation accuracy of 0.9167** and a training accuracy of 0.9537.

The validation accuracy is very close to what we got using our own implementation. The few differences that occur might be due to the following:

1) Better initialization of the random weight matrices leading to better fitting.
2) Training time is also faster (33.25 second vs 41 second) because of better implementation using tensors in Tensorflow.

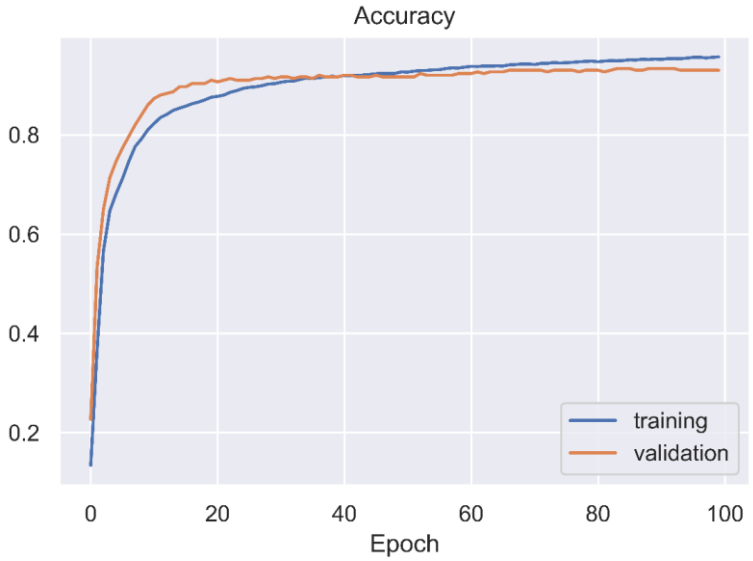
Part 1B

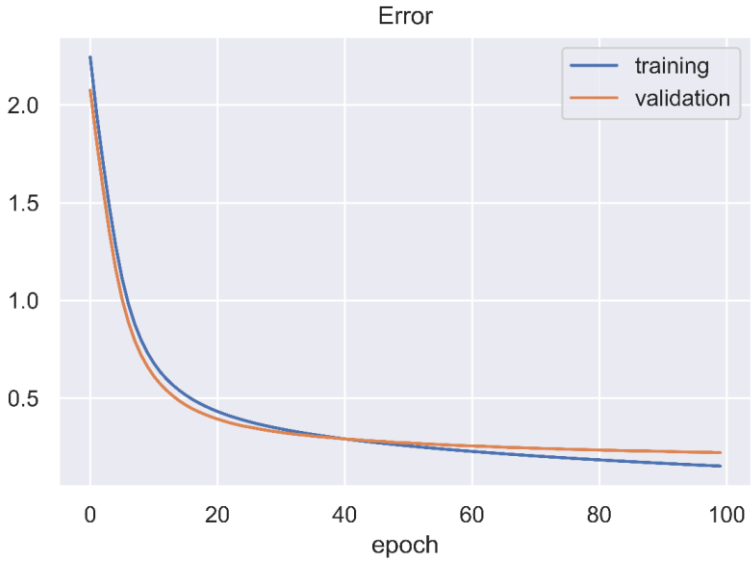
Here we use the PCA representation given to us in the second assignment.

We first tune the parameters using the same approach as followed in part 1A. After tuning, we compare the performances across various experiments as follows:

Setting	**Training Accuracy**	**Testing Accuracy**
Raw pixels + 2 hidden layers	95.37%	91.67%
PCA + 0 hidden layers	87.15%	88.67%
PCA + 1 hidden layer	94.44%	91.67%
PCA + 2 hidden layers	95.70%	93.00%

We also get the following training plots for the last case:





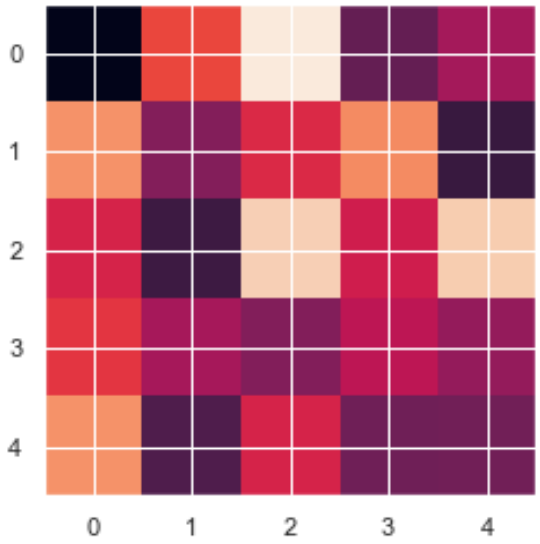
We see that adding hidden layers do help in increasing the test accuracy of the model. Simple logistic regression was not able to perform very well but once we added an additional layer, the accuracy jumped past 90%.

Moreover, we see that using the raw pixels with 2 hidden layers has similar performance as that in the case of PCA + 1 hidden layer. This shows that the PCA representation is very effective and is probably an embedding extracted from an intermediate layer of a dense neural network.

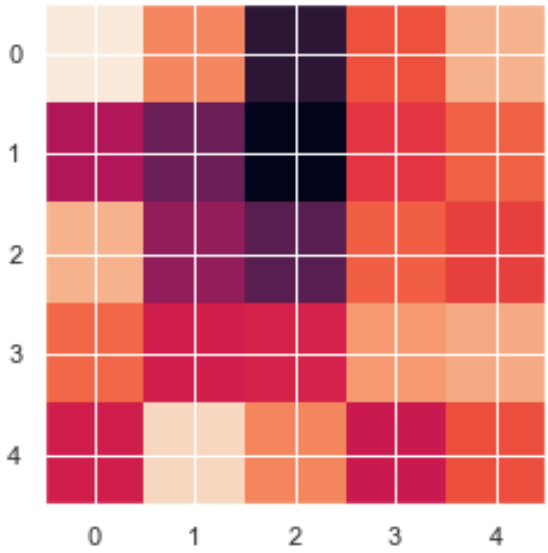
Representations learned:

We also try plotting the representations learned by the neural network with along with the PCA representation given to us:

Input label: 9



Input label: 5



These representations are similar to what we were getting by plotting the intermediate layers of the neural network implemented in problem 1A.

Part 2A (Using CNN)

In this part of the assignment, we use Tensorflow library to implement Convolutional Neural Networks (CNN) for the problem. We use the full MNIST dataset available on the internet. We implement the following architecture:

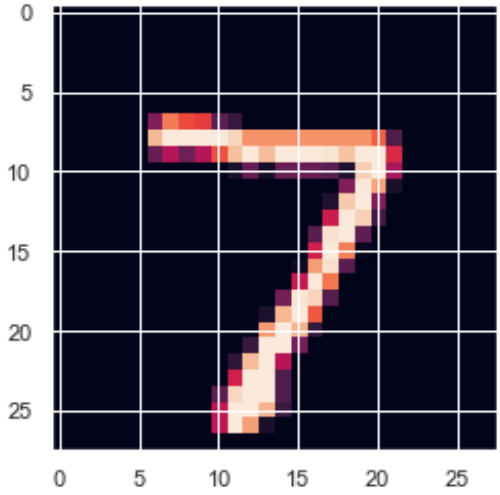
```
Model: "sequential_4"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
conv2d_5 (Conv2D)            (None, 26, 26, 32)        320
_________________________________________________________________
max_pooling2d_5 (MaxPooling2 (None, 13, 13, 32)        0
_________________________________________________________________
flatten_4 (Flatten)          (None, 5408)              0
_________________________________________________________________
dense_7 (Dense)              (None, 25)                135225
_________________________________________________________________
dropout_2 (Dropout)          (None, 25)                0
_________________________________________________________________
dense_8 (Dense)              (None, 10)                260
=================================================================
Total params: 135,805
Trainable params: 135,805
Non-trainable params: 0
_________________________________________________________________
```

After training for 10 epochs, we get the following results:

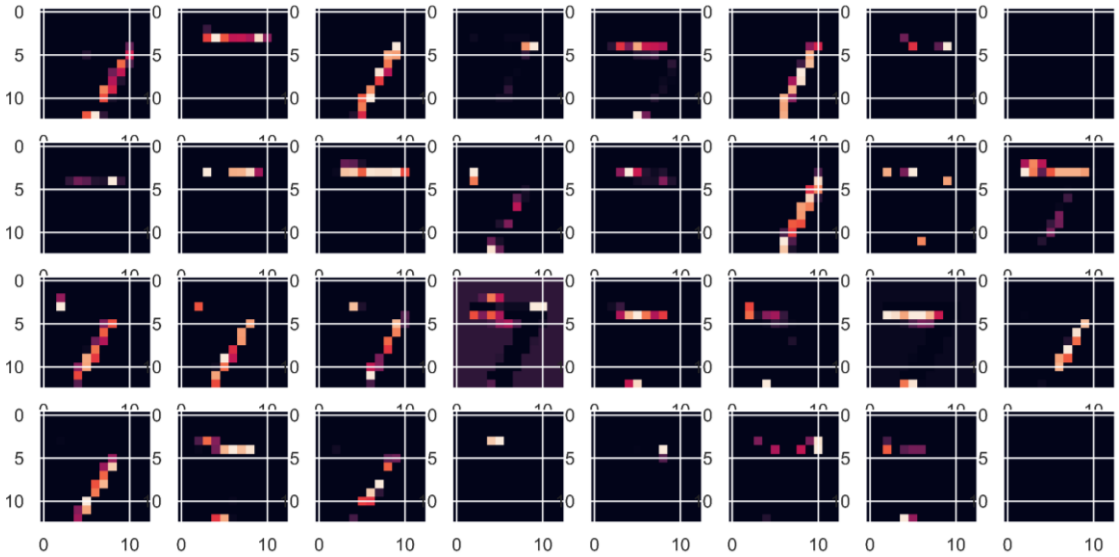
```
Epoch 10/10
54000/54000 [==============================] - 22s 399us/sample - loss:
0.2172 - accuracy: 0.9221 - val_loss: 0.0515 - val_accuracy: 0.9867
```

Here, we get a **validation accuracy of 98.67%** and a **test accuracy of 98.35%** which is way greater than what we were getting without using a convolutional layer.

Next, we try to visualize the representations learned by the layers of the neural net especially the ones learned by the filters of the conv2D layers. We feed in the following image to the CNN model:

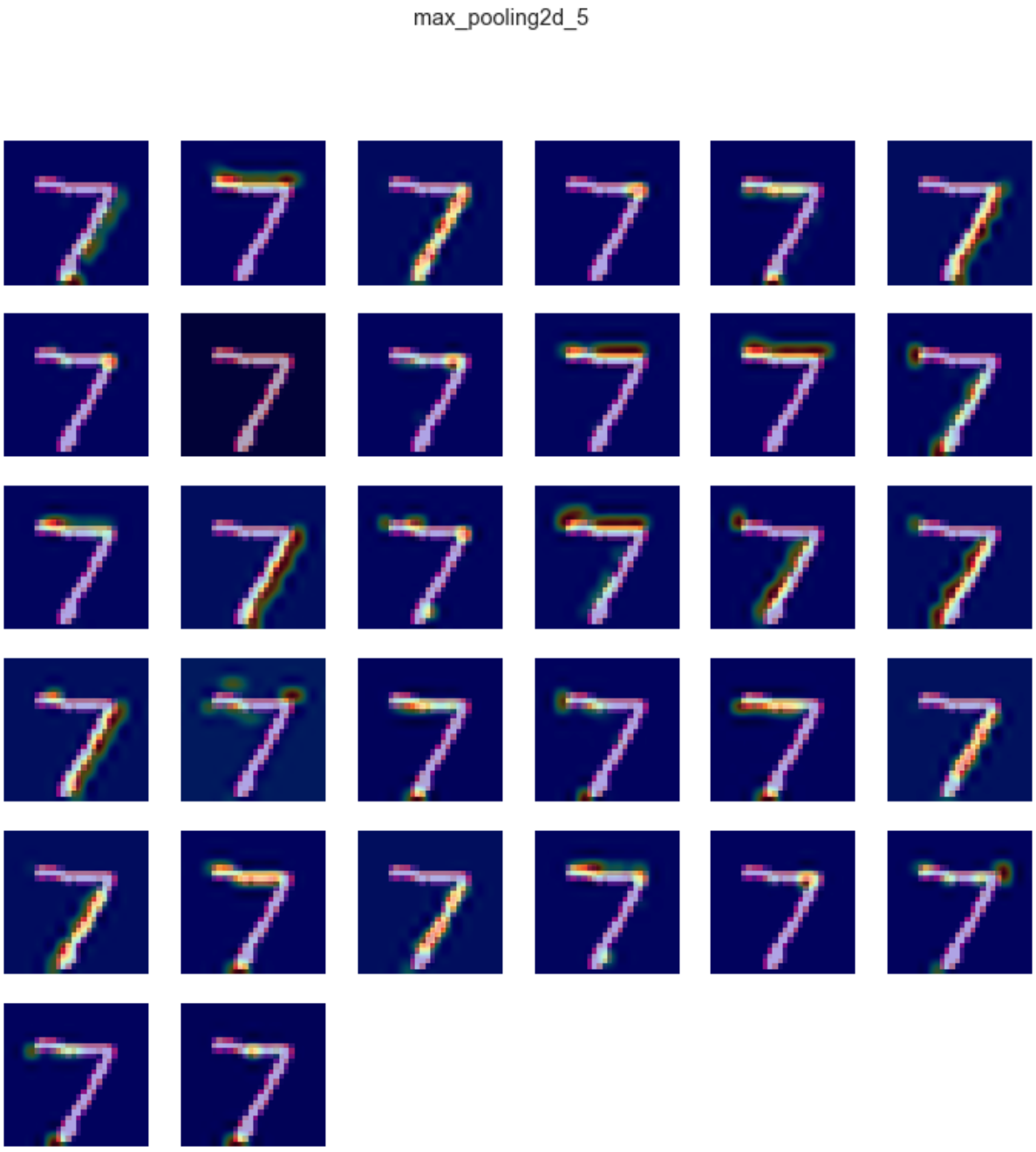


Following are the representations learned by the Max pooling layer:



We observe that the Conv2D layer is able to learn the representation of edges of the digit given in the input. This is possible because we have not flattened out the input image and there's a filter moving on the 2D image which is capable of learning this.

We also try plotting the heat-map (trigger points) using the Keract library (reference given at the end):

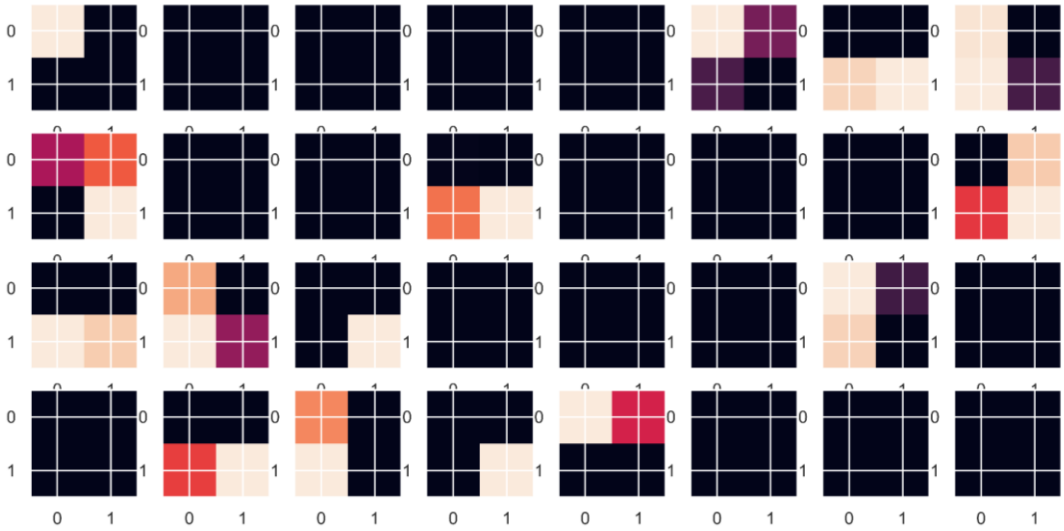


We also try varying the size of the filters (kernel size) of the conv2D layer to see the effect it has on the model performance:

Kernel size	Test Accuracy
(2x2)	97.68%
(3x3)	98.35%
(4x4)	98.48%
(8x8)	98.64%
(24x24)	96.90%

We see that on increasing the kernel size, the accuracy increases but after a certain limit, it starts falling as the filters become too large and are not able to capture the minute information required for categorization. Hence, there is a sweet spot in between.

For the 24x24 case, the following are the representations learned by the max pooling layer:



We observe that as the kernel size is increased beyond a limit, the representations start making way lesser sense as before.

Part 2B (Auto-encoders)

In this part, we experiment using Auto-encoders in Tensorflow. Auto-encoders are used to reconstruct an input image after passing through a neural network.

We implement the following two models:

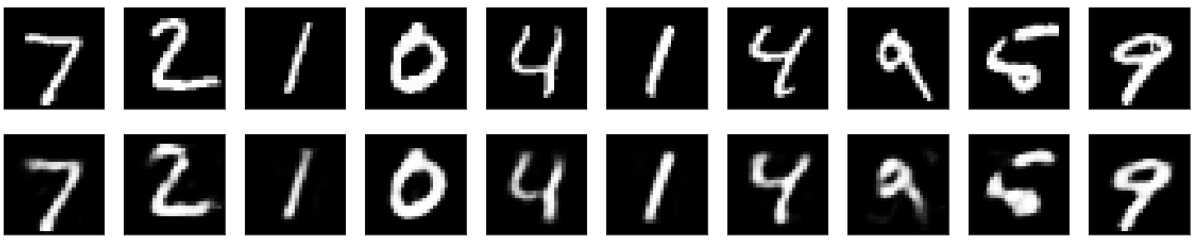
The encoder:

```
Model: "model_1"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_1 (InputLayer)         [(None, 784)]             0
_________________________________________________________________
dense (Dense)                (None, 32)                25120
=================================================================
Total params: 25,120
Trainable params: 25,120
Non-trainable params: 0
_________________________________________________________________
```

The decoder:

```
Model: "model_2"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_2 (InputLayer)         [(None, 32)]              0
_________________________________________________________________
dense_1 (Dense)              (None, 784)               25872
=================================================================
Total params: 25,872
Trainable params: 25,872
Non-trainable params: 0
_________________________________________________________________
```

Here, the model takes in a 784 dimensional input and encodes it (embedding) to a 32 dimensional space. Then the decoder maps it back to a 784 dimensional space (same as the input dimension). Now, we visualize the performance of this model:



On the top are the input images and the bottom row are the reconstructed images after passing through our auto-encoder model.

So what we do now is, after training this auto-encoder model, we feed this model the data we had for part 1A containing 3000 points (784 dimensional) and get the output of the intermediate layer (32 dimensional). This way we have mapped down the 784 dimensional data to 32 dimensional data by using a model trained in an unsupervised way.

Now we feed this 32 dimensional data to the same neural network we created in part 1A, except now we have the input of 32 dimensions.

```
Model: "sequential_1"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
dense_7 (Dense)              (None, 100)               3300
_________________________________________________________________
dense_8 (Dense)              (None, 64)                6464
_________________________________________________________________
dense_9 (Dense)              (None, 10)                650
=================================================================
Total params: 10,414
Trainable params: 10,414
Non-trainable params: 0
_________________________________________________________________
```

After training this model, we get the following:

```
Epoch 65/65
2700/2700 [==============================] - 0s 74us/sample - loss: 0.0142
- accuracy: 0.9996 - val_loss: 0.2483 - val_accuracy: 0.9267
```

The **validation accuracy** we get is **92.67%**, which is even greater than what we got before. Also, it took only 11 seconds to train.

So in a nutshell, we mapped down the 784 dimensional data to 32 dimensions using sparse auto-encoders working in an unsupervised setting. This was trained on the full MNIST data. Next, we feed our n=3000 data given to us for 1A to this auto-encoder to get the 32 dimensional data from the intermediate layer. Finally, we fed this data to our dense neural network to perform the classification task.

The accuracy of this can be further improved if we use CNN in the encoder to model more complex features into this embedding.

References

1) https://blog.keras.io/building-autoencoders-in-keras.html
2) https://towardsdatascience.com/how-to-make-an-autoencoder-2f2d99cd5103
3) https://www.kaggle.com/arpitjain007/guide-to-visualize-filters-and-feature-maps-in-cnn
4) https://keras.io/examples/vision/mnist_convnet/
5) https://peterroelants.github.io/posts/neural-network-implementation-part04/
6) http://cs229.stanford.edu/summer2020/cs229-notes-deep_learning.pdf
7) https://www.machinecurve.com/index.php/2019/12/02/visualize-layer-outputs-of-your-keras-classifier-with-keract/
8) https://pypi.org/project/keract/
9) https://stackoverflow.com/questions/41711190/keras-how-to-get-the-output-of-each-layer