1. Given the following matrix:

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

Compute the Singular Value Decomposition of matrix A.

2. Compute the pseudoinverse of the following matrix:

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

3a. Explain what is meant by rank of a matrix.

3b. For the following matrix, determine the rank.

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

Hint: Try and reduce the given matrix in its reduced row-echelon form.

4. Calculate the 1-norm of the following matrix:

$$B = \begin{bmatrix} 5 & -4 & 2 \\ -1 & 2 & 3 \\ -2 & 1 & 0 \end{bmatrix}$$

5. Calculate the infinity-norm of the following matrix:

$$C = \begin{bmatrix} 3 & 6 & -1 \\ 3 & 1 & 0 \\ 2 & 4 & -7 \end{bmatrix}$$

6. Calculate the norm of the vector $\vec{u} = (2, -2, 3, -4)$

7. Compute the Jacobian matrix of the function $F : \mathbf{R}^3 \to \mathbf{R}^4$ with components:

$y_1 = x_1$
$y_2 = 5x_3$
$y_3 = 4x_2^2 - 2x_3$
$y_4 = x_3 \sin x_1$

8. Find the gradient of the radial coordinate r.

Hint:
$r = \sqrt{x^2 + y^2 + z^2}$ is the magnitude of a radial vector, $r = x\hat{i} + y\hat{j} + z\hat{k}$.

9. Find the derivative of the following Scalar-by-Scalar with respect to a scalar x

$f(x) = cosx + sinx + x^2$

10. Find the derivative of the following matrix with respect to a scalar x

$$A = \begin{bmatrix} x^2 + 2cosx & sinx \\ cosx & x^3 + x^2 + x + 1 \end{bmatrix}$$

11. Let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^{3 \times 1}$, let $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, and finally, let $B = x^T A x$. Then what is the value of $\frac{\partial B}{\partial x}$?