1. Given the following matrix:

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

a. Find the transpose of matrix A i.e. A^T

b. Find the resultant matrix B, when matrix A is multiplied by its transpose i.e. $B = AA^T$.

c. Find the eigenvalues and eigenvectors of matrix $B = AA^T$.

d. Now find the resultant matrix C, when matrix A^T is multiplied by A i.e. $C = A^T A$.

e. Find the eigenvalues and eigenvectors of matrix $C = A^T A$.

f. What do you conclude about the eigenvalues of matrices B and C?

2. Solve the system of equations to determine the values of x, y and z as given below:

$$2x + 3y - z = 15$$
$$4x - 3y - z = 19$$
$$x - 3y + 3z = -5$$

Hint: Use the equation $AX = B$ to solve the system of equation above, where A and B are the matrices for values of LHS and RHS of the equations and X consist of x, y and z variables.

So $X = A^{-1}B$, which means you need to find out the inverse of A and compute the dot product of A^{-1} and B (or by multiplying the inverse of the A matrix by the B matrix) to get the solution.

3. What is the inverse of the following matrix?

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

4. Given the following matrix:

$$A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

Is the given matrix above an orthogonal matrix?

5. Given:

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

and

$$v = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

a. Compute $u^T u$

b. Compute $u^T v$