Contents

Artin 2.1.5 Consequences of an equation

We are given that $xyz = 1$ in some group G. So

$$yz = x^{-1}$$

and therefore

$$yzx = 1.$$

But it is not necessarily the case that $yxz = 1$. We know $xy = z^{-1}$ but we don't know that this group is abelian.

Artin 2.1.7 Proving that a given binary op is associative

Given the law of composition

$$ab = a,$$

we must prove that

$$(ab)c = a(bc).$$

The LHS becomes

$$(a)c = ac = a$$

while the RHS becomes

$$a(b) = ab = a$$

so it is proved.

Artin 2.2.1 A cyclic matrix group

The elements of the cyclic group generated by

$$\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

are its powers:

$$\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

So it's a cyclic group of order 6, isomorphic to the integers modulo 6.

Note that $A^3 = -I$, so $A^6 = I$ is immediate and it's easy to get $A^4 = -A$ and $A^5 = -A^2$ from previously computed powers.

Artin 2.2.15 Uniqueness of identity and inverses in subgroups

(a)

Let e_H be the idenity in H. Then

$$e_H e_H = e_H$$

but since the operation is inherited from G we may multiply by e_H^{-1} in G:

$$e_H^{-1} e_H e_H = e_H^{-1} e_H$$

to get

$$e_G e_H = e_G,$$

that is,

$$e_H = e_G.$$

(b)

Let a^{-H} be the inverse in H and a^{-G} be the inverse in G. Then in H we have

$$a a^{-H} = 1$$

but this equation holds in G as well we may multiply on the left by a^{-G} in G to get

$$a^{-G} a a^{-H} = a^{-G}$$

where the first two factors on the left multiply in G to give the identity, so

$$a^{-H} = a^{-G}.$$

Artin 2.2.20(a) The order of products in abelian groups

Let a, b be elements of an abelian group of orders m, n respectively. Then

$$a^m = e$$

and

$$b^n = e.$$

If

$$(ab)^l = e$$

then

$$a^l b^l = e$$