Unit-2

Finite Differences

Suppose a table of values (x_i, y_i), $i = 0$ to n of any function $y = f(x)$, the values of x being equally spaced
i.e $x_i = x_0 + ih$.

we are required to obtain the values of $f(x)$ for some intermediate values of x or to obtain the derivative of $f(x)$ for some value of $f(x)$
Then the following 3 type of the differences are useful.

1) Forward Difference
2) Backward Difference
3) Central Difference

Forward Difference

If a function $f(x)$ is tabulated for equally spaced arguments
$x_0, x_0 + h, x_0 + 2h, \cdots \cdots x_0 + nh$.

The values of y are

$$y_0 = f(x_0), \quad y_1 = f(x_0 + h)$$
$$y_2 = f(x_0 + 2h), \cdots \cdots y_n = f(x_0 + nh)$$

then the forward difference operator Δ is defined as

$$\Delta y_0 = y_1 - y_0, \quad \Delta y_1 = y_2 - y_1, \quad \ldots$$

$$\Delta y_{n-1} = y_n - y_{n-1}$$

and called first forward difference

The second forward differences are

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

$$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$$

$$\vdots$$

$$\Delta^2 y_{n-2} = \Delta y_{n-1} - \Delta y_{n-2}.$$

The forward difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0	$\Delta y_0 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$
x_1	y_1	$\Delta y_1 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$
x_2	y_2	$\Delta y_2 = y_3 - y_2$	$\Delta^2 y_2 = \Delta y_3 - \Delta y_1$	$\Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2$.
x_3	y_3	$\Delta y_3 = y_4 - y_3$	$\Delta^2 y_3 = \Delta y_4 - \Delta y_3$	
x_4	y_4			

Backward Difference

If a function $y=f(x)$ is tabulated for equally spaced arguments

$x_0, x_0+h, x_0+2h, --- x_0+nh$

the values of y are

$y_0=f(x_0)$, $y_1=f(x_0+h)$, $y_2=f(x_0+2h)$,

$y_n=f(x_0+nh)$

then the backward difference operator ∇ is defined as

$\nabla y_1 = y_1 - y_0$

$\nabla y_2 = y_2 - y_1$

$\nabla y_3 = y_3 - y_2$

$\nabla y_n = y_n - y_{n-1}$

Scanned by CamScanner

$y_1 - y_0 = \Delta y_0$
$y_1 - y_0 = \nabla y_1$
$y_1 - y_0 = \delta y_{1/2}$

Central Difference

Sometime it is very useful to use another system of differences known as central difference. The central (δ) difference operator is defined as.

$\cancel{y_1 - y_0 = \Delta y_0}$

$$y_1 - y_0 = \delta y_{1/2}.$$

$$y_2 - y_1 = \delta y_{3/2}$$

$$y_3 - y_2 = \delta y_{5/2}.$$

etc.

Scanned by CamScanner

Q Construct a forward diff table for

$f(1) = 1$, $f(2) = 3$, $f(3) = 8$, $f(4) = 15$, $f(5) = 25$ and find $\Delta^4(1)$.

Sol)

x	$y = f(x)$	$\Delta y = \Delta f(x)$	$\Delta^2 y = \Delta^2 f(x)$	$\Delta^3 y = \Delta^3 f(x)$	$\Delta^4 y = \Delta^4 f(x)$
1	1	2	3	-1	2
2	3	5	2	1	
3	8	7	3		
4	15	10			
5	25				

$\Delta^4 f(1) = 2$

$\Delta^3 f(2) = 1$

The Shift Operator

The shift operator $\mathcal{E}$ is defined as

$$\mathcal{E} f(x) = f(x+h)$$

$$\mathcal{E}^2 f(x) = f(x+2h)$$

$$\boxed{\mathcal{E}^n f(x) = f(x+nh)}$$

Q Show That

1) $\mathcal{E} = 1 + \Delta$
2) $\mathcal{E} \cdot \Delta = \Delta \cdot \mathcal{E}$
3) $\Delta \cdot \nabla = \Delta - \nabla$
4) $\nabla = 1 - \mathcal{E}^{-1}$
5) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

Sol

1) We Know That

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta f(x) = \mathcal{E} f(x) - f(x)$$

$$\Delta f(x) = (\mathcal{E} - 1) f(x)$$

$$\Delta = \mathcal{E} - 1$$

$$\boxed{\mathcal{E} = 1 + \Delta}$$

$E' f(x) = f(x+h)$
$E^{-1} f(x) = f(x-h)$

2)

$$E \cdot \Delta f(x) = E\left[f(x+h) - f(x)\right]$$
$$= E f(x+h) - E f(x)$$
$$= f(x+2h) - f(x+h)$$
$$= \Delta f(x+h)$$
$$E \cdot \Delta f(x) = \Delta \cdot E f(x)$$

$$\boxed{E \cdot \Delta = \Delta \cdot E}$$

4)

$$\nabla = 1 - E^{-1}$$

$$\nabla f(x) = f(x) - f(x-h)$$
$$= f(x) - E^{-1} f(x)$$
$$\nabla f(x) = f(x)\left[1 - E^{-1}\right)$$

$$\boxed{\nabla = 1 - E^{-1}}$$

3.)

$$\Delta \cdot \nabla f(x)$$
$$= (E-1)(1 - E^{-1}) f(x)$$
$$= (E-1)\left[f(x) - E^{-1} f(x)\right]$$
$$= (E-1)\left[f(x) - f(x-h)\right]$$

$= Ef(x) - Ef(x-h) - f(x) + f(x-h)$

$= f(x+h) - f(x) - f(x) + f(x-h)$

$= [f(x+h) - f(x)] - [f(x) - f(x-h)]$.

$\Delta \cdot \nabla f(x) = \Delta f(x) - \nabla f(x)$

$\Delta \cdot \nabla f(x) = (\Delta - \nabla) f(x)$

$$\boxed{\Delta \cdot \nabla = \Delta - \nabla}$$

5.)

Theorem (Differences of a Polynomial)

The n^{th} difference of polynomial of n^{th} degree are constant and all higher order differences are zero when the values of independent variables are at equal interval

Q Evaluate $\Delta^3(1-x)(1-2x)(1-3x)$ if $h=1$.

Sol

Let

$$f(x) = (1-x)(1-2x)(1-3x)$$
$$f(x) = -6x^3+11x^2-6x+1$$

$$\Delta^3 f(x) = \Delta^3(-6x^3+11x^2-6x+1)$$

$$= -6\Delta^3(x^3)+11\Delta^3(x^2)-6\Delta^3(x)+\Delta^3(1)$$

$$= -6.3!+0-0+0$$

~~Explanation~~

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta f(x) = f(x+1) - f(x)$$

$$= (x+1)^3 - (x)^3$$
$$= x^3+3x^2+3x+1-x^3$$
$$\Delta f(x) = 3x^2+3x+1$$

Scanned by CamScanner

$$\Delta^2 f(x) = [3(x+1)^2 + 3(x+1) + 1] - [3x^2 + 3x + 1]$$

$$= \cancel{3x^2} + 6x + 3 + \cancel{3x} + 3 + \cancel{1} - \cancel{3x^2} - \cancel{3x} - \cancel{1}$$

$$\Delta^2 f(x) = 6x + 6$$

$$\Delta^3 f(x) = [6(x+1) + 6] - [6x + 6]$$

$$= 6x + 6 + 6 - 6x - 6$$

$$= 6$$

$$= 3!$$

Q. Evaluate

$$\Delta^4 (1-ax)(1-bx)(1-cx)(1-dx)$$

if $h = 1$

Factorial Notation

$x^{(n)}$ is defined as

$$x^{(n)} = x(x-h)\cdot(x-2h)\cdot\ ---\ (x-(n-1)h)$$

In particular

$x^{(0)} = 1$, $x^{(1)} = x$, $x^{(2)} = x\cdot(x-h)$, etc

Expression of any function in factorial notation

Let $f(x)$ be a f^n of degree n

$$f(x) = a_0 x^n + a_1 x^{n-1} + --- + a_n$$

Thus the polynomial $f(x)$ can be expressed in the factorial notation as

$$f(x) = A_0 x^{(n)} + A_1 x^{(n-1)} + --- + A_n$$

Now, we find the values of $A_0, A_1, --- A_n$

Scanned by CamScanner

Q Find factorial form of $2x^3+3x^2-5x+4$

Sol

Let factorial form of the given f^n is

$f(x) = Ax^{(3)} + Bx^{(2)} + Cx^{(1)} + D$ — (1)

Using the method of synthetic division we divide by

$x, (x-1), (x-2)$ successively as

1	2	3	-5	4.	= D.
	0	2	5		
2	2	5	0		= C
	0	4			
3	2	9			= B
	0				
	2				
	= A				

$f(x) = 2x^{(3)} + 9x^{(2)} + 4$

Q. Find factorial form of

$x^4 - 12x^3 + 24x^2 - 30x + 9$

Sol)

Let the factorial form is

$f(x) = Ax^{(4)} + Bx^{(3)} + Cx^{(2)} + Dx^{(1)} + E$

Now

By using synthetic division, we divide by

$x, (x-1), (x-2), (x-3)$ successively as

1	1	-12	24	-30	9	$\Rightarrow E$
	0	1	-11	13		
2	1	-11	13	-17		$\Rightarrow D$
	0	2	-18			
3	1	-9	-5			$\Rightarrow C$
	0	3				
4	1	-6				$\Rightarrow B$
	0					
	1					$\Rightarrow A$

$f(x) = x^{(4)} - 6x^{(3)} - 5x^{(2)} - 17x^{(1)} + 9$

Q. Assuming that the following values of y being to be a polynomial of degree 4 Compute rest of values.

x	0	1	2	3	4	5	6
y	1	-1	1	-1	1	-	-

Sol

Construct a forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	~~$\Delta^5 y$~~
0	1	-2	4	-8	16	~~$y_5 - 31$~~
1	-1	2	-4	8	$y_5 - 15$	~~$y_6 - 5y_5 + 26$~~
2	1	-2	4	$y_5 - 7$	$y_6 - 4y_5 + 11$	
3	-1	2	$y_5 - 3$	$y_6 - 3y_5 + 4$		
4	1	$y_5 - 1$	$y_6 - 2y_5 + 1$			
5	y_5	$y_6 - y_5$				
6	y_6					

Since the degree of polynomial is 4

$\therefore$ 4th order difference must be equal to constant and higher order ~~diff~~ will be equal to 0 which is possible if

$$y_5 - 15 = 16 \quad \text{--- (1)}$$

$$y_6 - 4y_5 + 11 = 16 \quad \text{--- (2)}$$

from (1) & (2)

$$\boxed{y_5 = 31} \qquad \boxed{y_6 = 129}$$

Q If the values of the f^n $y = \log x$ are given

x	1	2	3	4	5
y	1	1.3010	1.4771	1.6021	1.6990

Then find. $\nabla^2 \log 2$ $\cancel{, \nabla^3 \log 1}$ and $\Delta^4 \log 1$

Sol

Construct a forward diff table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1	0.3010	−0.1249	0.0738	−0.0588
2	1.3010	0.1761	−0.0511	0.0230	
3	1.4771	0.1250	−0.0281		
4	1.6021	0.0969			
5	1.6990				

$$\Delta^4 \log 1 = -0.0588$$

$$\Delta^2 \log 1 = -0.1249$$

Q Given That

x	1	2	3	4
y	1	1.3010	1.4771	1.6021

Where $y = \log x$, Then find the values of $\nabla^2 \log 4$, $\nabla^2 \log 3$ and $\nabla^3 \log 4$

Sol Construct a backward difference table

	x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
x_0	1	1	0.3010	-0.1249	0.0738
x_1	2	1.3010	0.1761	-0.0511	
x_2	3	1.4771	0.1250		
x_3	4	1.6021			

$\nabla^2 \log 4 = -0.0511$

$\nabla^2 \log 3 = -0.1249$

$\nabla^3 \log 4 = 0.0738$

Q If $f(1)=1$, $f(2)=3$
$f(3)=8$, $f(4)=15$
$f(5)=25$
then find $\Delta^4 f(1)$ and $\Delta^2 f(2)$

Sol

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1	2	3	-1	2
2	3	5	2	1	
3	8	7	3		
4	15	10			
5	25				

$\Delta^4 f(1) = 2.$

$\Delta^2 f(2) = 2.$

Newton's Gregory forward interpolation formula

Let $y_i = f(x_i)$, $i = 0, 1, 2, \ldots n$ be function for equally spaced arguments, then

$$f(x) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0)$$

$$+ \frac{u(u-1)(u-2)}{3!}\Delta^3 f(x_0) + \ldots$$

where $\boxed{u = \frac{x - x_0}{h}}$

and h is interval size i.e $\boxed{h = x_{i+1} - x_i}$

Newton's Gregory Backward Interpolation formula

$$f(x) = f(x_n) + u\nabla f(x_n) + \frac{u(u+1)}{2!}\nabla^2 f(x_n)$$

$$+ \frac{u(u+1)(u+2)}{3!}\nabla^3 f(x_n) + \ldots$$

where $\boxed{u = \frac{x - x_n}{h}}$

Scanned by CamScanner

Q Find a cubic polynomial f(x) which takes the following

x	0	1	2	3
$f(x)$	1	2	1	10

Also find $f(0.5)$, $f'(1.5)$

Sol

Since the value of x are equally spaced construct a forward diff table

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1	1	-2	12.
1	2	-1	10	
2	1	9		
3	10.			

$$u = \frac{x - x_0}{h} = \frac{x-0}{1}$$

$$\boxed{u = x}$$

By Newton's forward formula

$$f(x) = f(x_0) + \frac{u}{1!}\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(x_0)$$

$$f(x) = 1 + x + \frac{x(x-1)}{2}(-2) +$$

$$\frac{x(x-1)(x-2)}{6} \times 12$$

$$f(x) = 2x^3 - 7x^2 + 6x + 1$$

$$f(0.5) = 2(0.5)^3 - 7(0.5)^2 + 6(0.5) + 1$$

$$=$$

$$f'(x) = 6x^2 - 14x + 6$$

$$f'(1.5)$$

$$= 6(1.5)^2 - 14(1.5) + 6$$

Scanned by CamScanner

Q Find $y(3.4)$ from the following table

x	1	2	3	4
y	2.105	2.808	3.614	4.604

Sol

Since the value of x are equally spaced. So we can use Newton's forward or Backward formula

Construct a Backward diff table

	x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
x_0	1	2.105	0.703	0.103	0.081
x_1	2	2.808	0.806	0.184	
x_2	3	2.614	0.990		
x_3	4	4.604			

Here $n = 3$ & $h = 1$

$$u = \frac{x - x_3}{h} = \frac{x-4}{1}$$

By Newton's Backward formula

$$f(x) = f(x_3) + u \cdot \nabla f(x_3) + \frac{u(u+1)}{2!} \nabla^2 f(x_3) +$$

$\frac{u(u+1)(u+2)}{3!} \nabla^3 f(x_3)$

$f(x) = 4.604 + (x-4)\, 0.990 +$

$\frac{(x-4)(x-3)}{2}\, 0.184 +$

$\frac{(x-4)(x-3)(x-2)}{3!} \times 0.081$

$f(3.4) = 4.604 + (-0.6) \times 0.990 +$

$\frac{(-0.6)\, 0.4}{2} \times 0.184 +$

$\frac{(-0.6)(0.4)(1.4)}{6} \times 0.081$

$= 3.9833$

$\nabla^2 f(x_2)$

Q From the following table of half yearly premium for policies maturing at different ages, estimate the premium for maturing at the age 63.

Age	45	50	55	60	65
Premium	114.84	96.16	83.32	74.48	68.48

Ans = 70.585

Q By using Newton's forward formula, find a function $f(x)$ which takes the following values.

x	0	1	2	3	4	5
$f(x)$	2	4	-3	54	21	-6

and hence evaluate $f(0.6)$ &

$f''(0.6)$

Gauss's Central difference formulae

Let $y = f(x)$ be a f^n which takes the value $\ldots, y_{-2}, y_{-1}, y_0, y_1, y_2 \ldots$ for equally spaced values of argument

$x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots$

with unit interval of u

i.e where $u = \frac{x - x_0}{h}$.

Then Gauss forward formulae is.

$$y = y_0 + u \cdot \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_{-1} +$$

$$\frac{(u+1)u(u-1)}{3!} \Delta^3 y_{-1} +$$

$$\frac{(u+1)u(u-1)(u-2)}{4!} \Delta^4 y_{-2} +$$

$$\frac{(u+2)(u+1)u(u-1)(u-2)}{5!} \Delta^5 y_{-2} + -$$

$$- - - -$$

2nd gauss Backward formula is.

$$y = y_0 + u \cdot \Delta y_{-1} + \frac{(u+1)u}{2!} \Delta^2 y_{-1} +$$

$$\frac{(u+1)u(u-1)}{3!} \Delta^3 y_{-2} + \frac{(u+2)(u+1)u(u-1)}{4!} \Delta^4 y_{-2}$$

$$+ \frac{(u+2)(u+1)u(u-1)(u-2)}{5!} \Delta^5 y_{-3} + \ldots$$

Note - The gauss formula gives more accurate result if u lies b/w 0 & 1

The gauss backward give more accurate result if u lies b/w -1 & 0

Q Using gauss forward & backward interpolation formula, find the value of y when $x = 3.75$ for the following data

x	2.5	3.0	3.5	4.0	4.5	5.0
y	24.145	22.043	20.225	18.644	17.262	16.047

Sol Construct a difference table

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-2	2.5	24.145	-2.102	0.284	-0.047	0.009
-1	3.0	22.043	-1.818	0.237	-0.038	0.006
0	3.5	20.225	-1.581	0.199	-0.032	
1	4.0	18.644	-1.382	0.167		
2	4.5	17.262	-1.215			
3	5.0	16.047				

$\Delta^5 y$
-0.003

i By gauss forward formulae

$$y = y_0 + u \cdot \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_{-1} +$$

$$\frac{(u+1)u(u-1)}{3!} \Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{4!} \Delta^4 y_{-2} +$$

$$\frac{(u+2)(u+1)u(u-1)(u-2)}{5!} \Delta^5 y_{-2} \quad \text{--- (1).}$$

$x = 3.75$ lies b/w 3.5 & 4.0.

we take $x_0 = 3.5$ & $h = 0.5$

K.2?

$$u = \frac{x - x_0}{h} = \frac{3.75 - 3.5}{0.5} = 0.5$$

from (1)

$$y(3.75) = 20.225 + 0.5 \times (-1.581) +$$

$$\frac{0.5 \times (-0.5)}{2} (0.232) +$$

$$\frac{1.5 \times 0.5 \times (-0.5)}{6} \times (-0.038) +$$

$$\frac{1.5 \times 0.5 \times (-0.5)(-1.5)}{24} \times 0.009 +$$

$$\frac{2.5 \times 1.5 \times 0.5 \times (-0.5) \times (-1.5)}{120} \times (-0.003)$$

$$= 19.407.$$

$$\boxed{y(3.75) = 19.407}$$

BLACK

K=3

ii By gauss Backward formula

Pink

$$y = y_0 + u \cdot \Delta y_{-1} + \frac{(u+1)u}{2!} \Delta^2 y_{-1} +$$

$$\frac{(u+1)u(u-1)}{3!} \Delta^3 y_{-2} + \frac{(u+2)(u+1)u(u-1)(u-2)}{4!}$$

$$\cdot \Delta^4 y_{-2} + \frac{(u+2)(u+1)u(u-1)(u-2)}{5!} \Delta^5 y_{-3}$$

(1)

$x = 3.75$ lies b/w 3.5 & 4.0, so we take $x_0 = 4.0$.

$$u = \frac{x - x_0}{h} = \frac{3.75 - 4.0}{0.5} = -0.5$$

$y_0 = 18.644$

$\Delta y_{-1} = -1.581$

$\Delta^2 y_{-1} = 0.199$

$\Delta^3 y_{-2} = -0.038$

$\Delta^4 y_{-2} = 0.006$

$\Delta^5 y_{-3} = -0.003$

Stirling formula

$$y = y_0 + u\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{u^2}{2!}\Delta^2 y_{-1} +$$

$$\frac{u(u^2-1^2)}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) +$$

$$\frac{u^2(u^2-1)}{4!}\Delta^4 y_{-2} + \cdots$$

Bessel's formula

$$y = \left(\frac{y_0 + y_1}{2}\right) + \left(u - \frac{1}{2}\right)\Delta y_0 +$$

$$\frac{u(u-1)}{2!}\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right) +$$

$$\frac{u(u-1)(u-\frac{1}{2})}{3!}\Delta^3 y_{-1} + \cdots$$

Laplace Everett's formula

$$y = \left[u y_1 + \frac{u(u^2-1^2)}{3!} \Delta^2 y_0 + \frac{u(u^2-1^2)(u^2-2^2)}{5!} \Delta^4 y_{-1} + \cdots \right] +$$

$$\left[v y_0 + \frac{v(v^2-1^2)}{3!} \Delta^2 y_{-1} + \frac{v(u^2-1^2)(v^2-2^2)}{5!} \Delta^4 y_{-2} + \cdots \right]$$

where

$$v = 1-u$$

and

$$u = \frac{x - x_0}{h}.$$

Q Using stirling formula find y at $x=32$ (Also find by using Everett's formula)

if

x	20	30	40	50
y	512	439	346	243

Since $x=32$ lies b/w 30 and 40 we choose $x_0 = 30$

$$u = \frac{x-x_0}{h} = \frac{32-30}{10} = 0.2$$

Construct a diff table

u	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	20	512	$-73\ \Delta y_{-1}$	$-20\ \Delta^2 y_{-1}$	$10\ \Delta^3 y_{-1}$
0	30	$439\ y_0$	$-93\ \Delta y_0$	-10	
1	40	346	-103		
2	50	243			

$$v = 1-u$$
$$= 1-0.2$$
$$= 0.8$$

By stirling formula

$$y = y_0 + u\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{u^2}{2!}\Delta^2 y_{-1} + \frac{u(u^2-1^2)}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right)$$

$$y = 439 + 0.2 \times \left(\frac{-93-73}{2}\right) + \frac{(0.2)^2}{2} \times (-20) +$$

$$0.2 \frac{\left[(0.2)^2 - 1^2\right]}{6} \left(\frac{10+0}{2}\right).$$

$$y = 421.84$$

By Laplace Everette's formula.

$$y = \left[uy_1 + \frac{u(u^2-1^2)}{3!}\Delta^2 y_0 + \frac{u(u^2-1^2)(u^2-2^2)}{5!}\Delta^4 y_{-1} + \right.$$

$$\left. --- \right] + \left[vy_0 + \frac{v(v^2-1^2)}{3!}\Delta^2 y_1 + \right.$$

$$\left. \frac{v(v^2-1^2)(v^2-2^2)}{5!}\Delta^4 y_{-2} + --- \right]$$

$$= \left[0.2 \times 346 + \frac{0.2\left[(0.2)^2 - 1\right]}{6} \times (-10) + 0\right]$$

$$+ \left[0.8 \times 439 + \frac{0.8 \times ((0.8)^2 - 1)}{6} \times (-10 + 0)\right]$$

$$= 421.68 \text{ approx}$$

Q2 Using Bessel's Interpolation formulae find y_{25} if

$y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$,

$y_{32} = 3992$

Since $x = 25$ lies b/w 24 & 28.
we choose $x_0 = 24$.

$$u = \frac{x - x_0}{h} = \frac{25 - 24}{4} = 1/4$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	2854	308	74	-8
24	3162	382	66	
28	3544	448		
32	3992			

By Bessel's Formula

u	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	20	2854	308	74 $\Delta^2 y_{-1}$	-8 $\Delta^3 y_{-1}$
0	24	3162 y_0	382 Δy_0	66 $\Delta^2 y_0$	
1	28	3544 y_1	448		
2	32	3992			

By Bessel's formula

$$y = \left(\frac{y_0 + y_1}{2}\right) + \left(\frac{u - 1}{2}\right)\Delta y_0 + \frac{u(u-1)}{2!}\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right)$$

$$+ \frac{(u - \frac{1}{2})\, u(u-1)}{3!}\Delta^3 y_{-1} + - - .$$

$y = 3250.8750$ approx

Q Using Stirling formula find y_{28} if $y_{20} = 49225$, $y_{25} = 48316$, $y_{30} = 47236$, $y_{35} = 45926.8$, $y_{40} = 44306$.

Ans = 47692

formula for unequal interval

If $y = f(x)$ be function and some of the values $y_0, y_1, y_2 - - y_n$ are given at $x_0, x_1, x_2 - - x_n$ where the interval in arguments is unequal

Lagrange's Interpolation formula

$$f(x) = \frac{(x-x_1)(x-x_2) - - (x-x_n)}{(x_0-x_1)(x_0-x_2) - - - (x_0-x_n)} y_0 +$$

$$\frac{(x-x_0)(x-x_2) - - (x-x_n)}{(x_1-x_0)(x_1-x_2) - - - (x_1-x_n)} y_1 + -$$

$$- - - \frac{(x-x_0)(x-x_1) - - (x-x_{n-1})}{(x_n-x_0)(x_n-x_1) - - (x_n-x_{n-1})} y_n$$

Q Find a polynomial $P(x)$ satisfying the values $P(1) = 1$, $P(3) = 27$, $P(4) = 64$ by using Lagrange's interpolation formula and hence find $P(2)$

Sol

$x_0 = 1$ $x_1 = 3$ $x_2 = 4$

$y_0 = 1$ $y_1 = 27$ $y_2 = 64$

Scanned by CamScanner

By Lagrange's formula

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(x-3)(x-4)}{(1-3)(1-4)} \times 1 + \frac{(x-1)(x-4)}{(3-1)(3-4)} \times 27 +$$

$$\frac{(x-1)(x-3)}{(4-1)(4-3)} \times 64$$

y_n $f(x) = 8x^2 - 89x + 12$

$= 6$

Q find the cubic interpolation polynomial for the following data

x	0	1	2	5
$f(x)$	2	3	12	147

Also find $f(1.5)$

Divided difference

Let $y_0, y_1, y_2 --- y_n$ be the values of function $y = f(x)$ at $x_0, x_1, x_2 --- x_n$.

Then first divided diff. of x_0 & x_1 is

$$\underset{x_1}{\Delta} y_0 = \frac{y_1 - y_0}{x_1 - x_0}$$

The first divided diff of x_1 & x_2 is.

$$\underset{x_2}{\Delta} y_1 = \frac{y_2 - y_1}{x_2 - x_1}.$$

and so on

Similarly the second divided diff of x_0, x_1, x_2

$$\underset{x_1, x_2}{\Delta^2} y_0 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0},$$

The divided table is,

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0	$\frac{y_1 - y_0}{x_1 - x_0} = \Delta y_0$	$\frac{\Delta y_1 - \Delta y_0}{x_2 - x_0} = \Delta^2 y_0$	$\frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0} = \Delta^3 y_0$	$\frac{\Delta^3 y_1 - \Delta^3 y_0}{x_4 - x_0}$.
x_1	y_1	$\frac{y_2 - y_1}{x_2 - x_1} = \Delta y_1$	$\frac{\Delta y_2 - \Delta y_1}{x_3 - x_1} = \Delta^2 y_1$	$\frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1} = \Delta^3 y_1$	
x_2	y_2	$\frac{y_3 - y_2}{x_3 - x_2} = \Delta y_2$	$\frac{\Delta y_3 - \Delta y_2}{x_4 - x_2} = \Delta^2 y_2$		
x_3	y_3	$\frac{y_4 - y_3}{x_4 - x_3} = \Delta y_3$			
x_4	y_4				

Q Construct a divided diff table for

$y = x^2 - 4x + 1$ at $x = 0, 1, 3, 4, 8$. find $\Delta^2 y_0, \Delta y_2$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1	$\frac{-2-1}{1-0} = -3$	$\frac{0+3}{3-0} = 1$	0
1	-2	$\frac{-2+2}{3-1} = 0$	$\frac{3-0}{4-1} = 1$	0.
3	-2	$\frac{1+2}{4-3} = 3$	$\frac{8-3}{8-3} = 1$	
4	1	$\frac{33-1}{8-4} = 8.$		
8	33.			

$\Delta^2 y_0 = 1$ $\Delta y_2 = 3$

Q Show that

$$\Delta^3_{bcd}\left(\frac{1}{a}\right) = \frac{-1}{abcd}$$

Sol

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
a	$1/a$	$\frac{\frac{1}{b}-\frac{1}{a}}{b-a} = \frac{-1}{ab}$	$\frac{-\frac{1}{bc}+\frac{1}{ab}}{c-a} = \frac{1}{abc}$	$\frac{\frac{1}{bcd}-\frac{1}{abc}}{d-a} = \frac{-1}{abcd}.$
b	$1/b$	$= \frac{-1}{bc}$	$\frac{-\frac{1}{cd}+\frac{1}{bc}}{d-b} = \frac{+1}{bcd}$	
c	$1/c$	$= \frac{-1}{cd}$		
d	$1/d$			

Newton's divided diff formula.

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + - - - + (x-x_0)(x-x_1) - - (x-x_{n-1})\Delta^n y_0$$

Q find the value of function $y = f(x)$ at the pt where the points $x = 2.5$ if the following value are given

x	1	2	4	7	12
$y=f(x)$	22	30	82	106	216

Sol

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	22	$\frac{30-22}{2-1} = 8$	$\frac{26-8}{4-1} = \frac{18}{3} = 6$	$\frac{-3.6-6}{7-1} = \frac{-9.6}{6} = -1.6$	$\frac{0.535+1.6}{12-1} = 0.194$
2	30	$\frac{82-30}{4-2} = \frac{52}{2} = 26$	$\frac{8-26}{7-2} = \frac{-18}{5} = -3.6$	$\frac{1.75+3.6}{12-2} = \frac{5.35}{10} = 0.535$	
4	82	$\frac{106-82}{7-4} = \frac{24}{3} = 8$	$\frac{22-8}{12-4} = \frac{18}{8} = 1.75$		
7	106	$\frac{216-106}{12-7} = \frac{110}{5} = 22$			
12	216				

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 y_4.$$

$$f(x) = 22 + (2.5-1)8 + (2.5-1)(2.5-2)6 + (2.5-1)(2.5-2)(2.5-4)(-1.6) + (2.5-1)(2.5-2)(2.5-4)(2.5-7)(0.194)$$

$$= 41.281.$$

Q Find a polynomial f(x) for the following data

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

by using Newton

Q Find the missing value if the following values are given

x	1	2	3	4	5
y = f(x)	2	-	5	8	12

Since 4 values are known ∴ the 4th diff must be equal to 0.

Sol

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	2	$y_1 - 2$	$7 - 2y_1$	$3y_1 - 9$	$12 - 4y_1$
2	y_1	$5 - y_1$	$y_1 - 2$	$3 - y_1$	
3	5	3	1		
4	8	4			
5	12				

$$12 - 4y_1 = 0$$

$$y_1 = 3$$

Scanned by CamScanner

OR.

x	1	3	4	5
y	2	5	8	12

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1	2	$\frac{5-2}{3-1} = 1.5$	$\frac{3-1.5}{4-1} = 0.5$	0.
3	5	$\frac{8-5}{4-3} = 3$	$\frac{4-3}{5-3} = 0.5$	
4	8	$\frac{12-8}{5-4} = 4$		
5	12			

By Newton divided

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0$$

$$f(x) = 2 + (2-1) \times 1.5 + (2-1)(2-3) \times 0.5$$

$$= 3.0$$