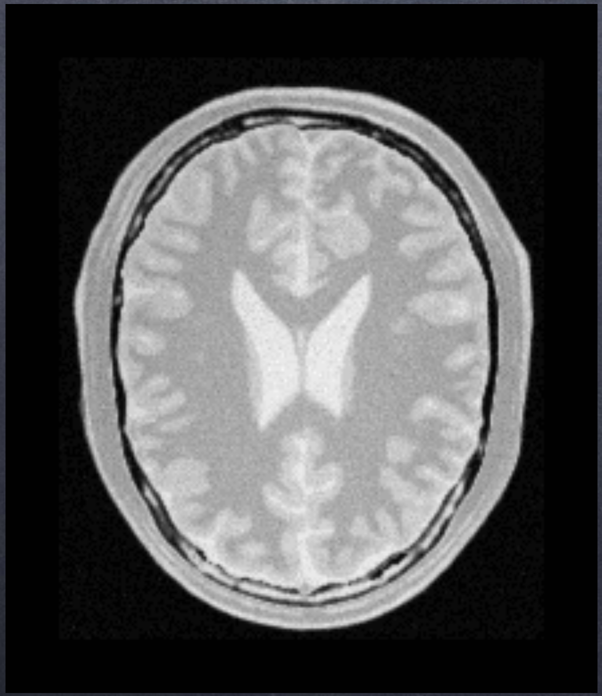
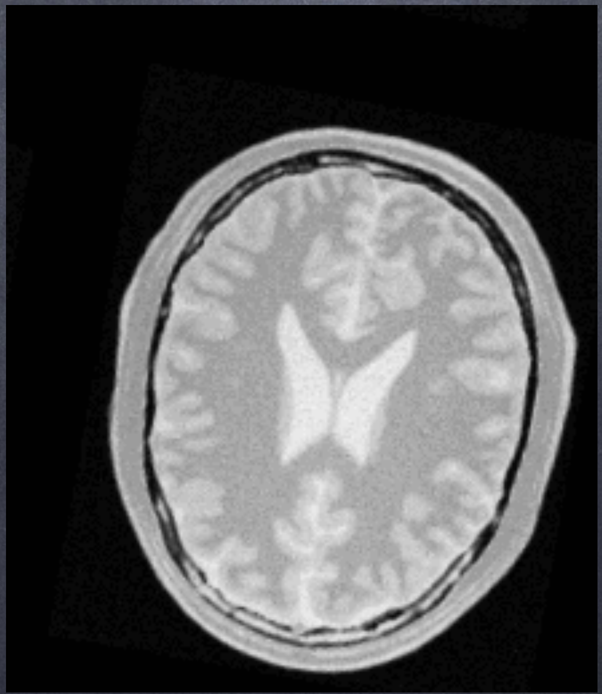
Computer Vision

Image Registration

- How is it similar to camera calibration?
- How we can register two images with respect to each other?



I_m



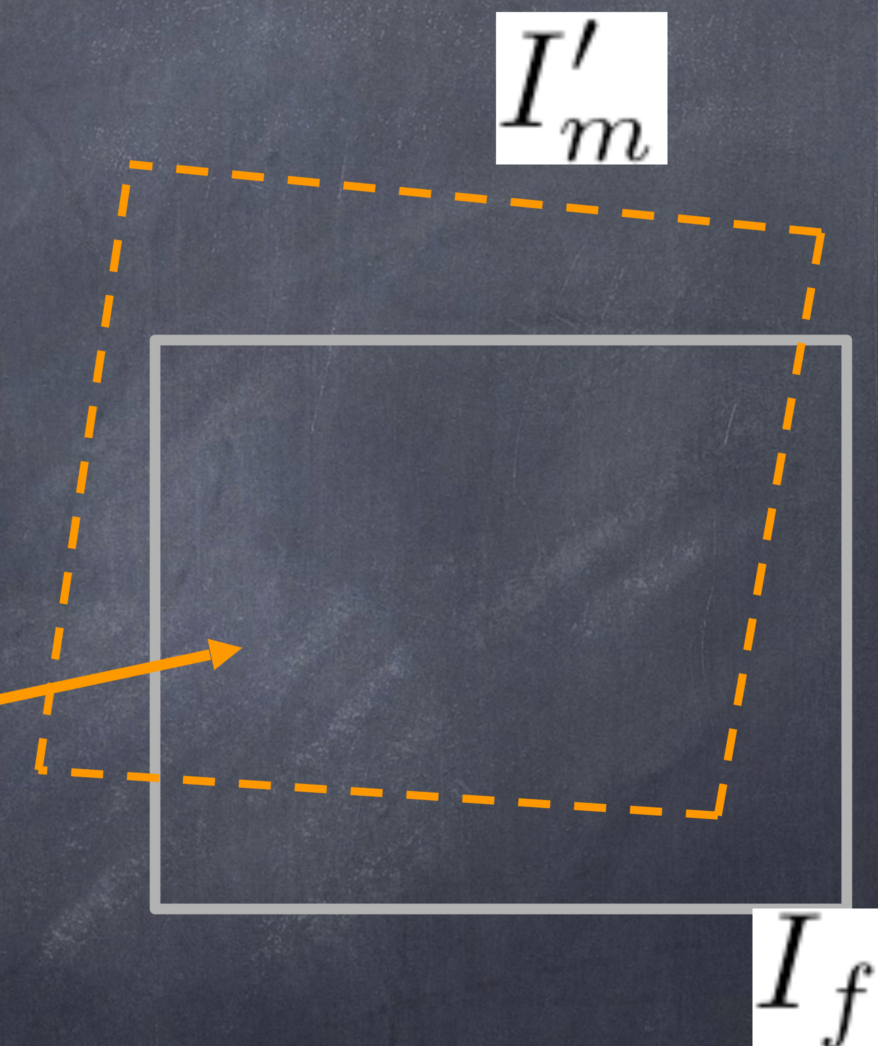
I_f

Intensity based Registration

- How we can do that?

Example Error Measure: SSD

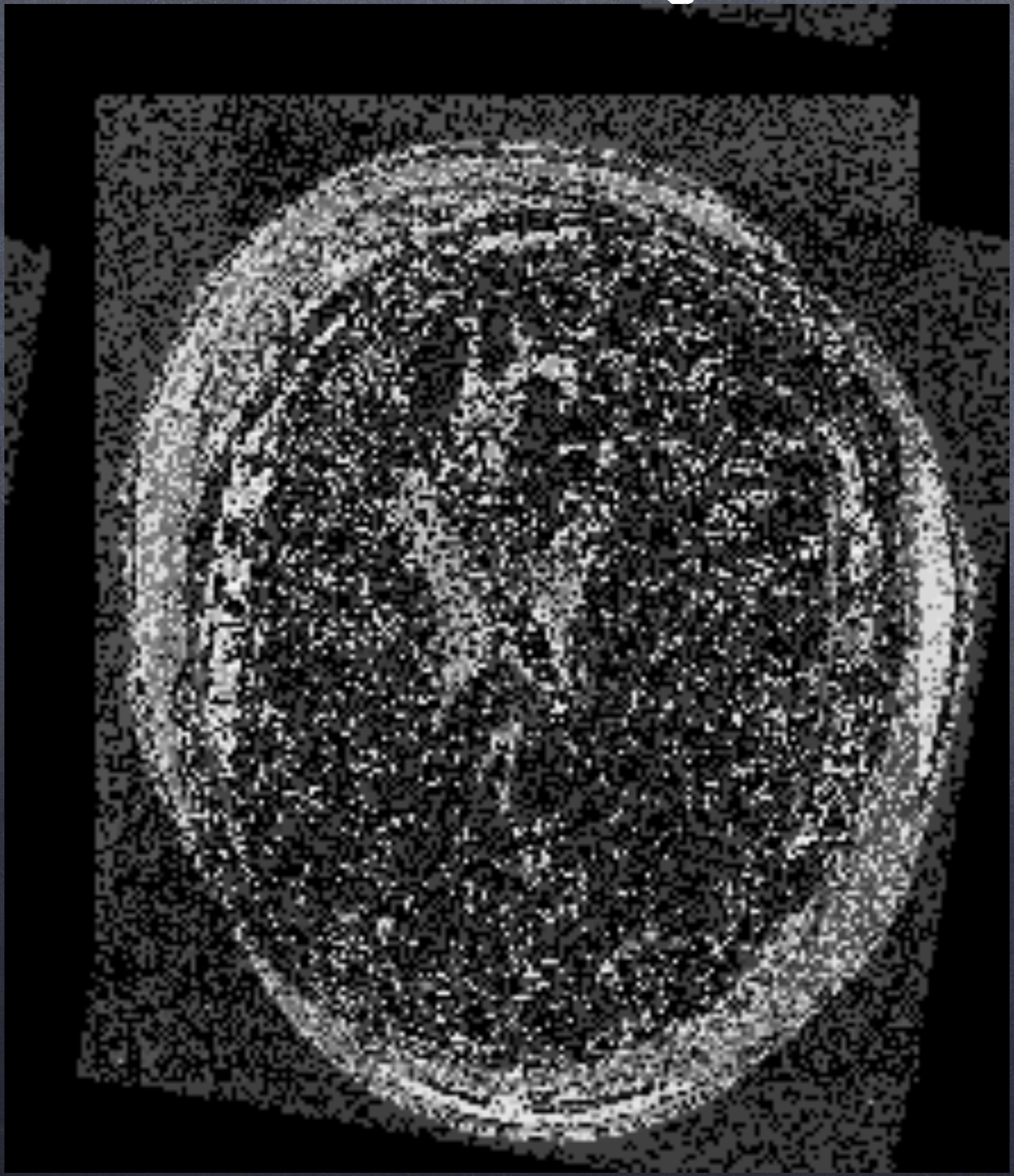
$$\sum_{\mathbf{p}\in\Omega}\left[I_f(\mathbf{p}) - I'_m(\mathbf{p})\right]^2$$



Ω Region of intersection between images

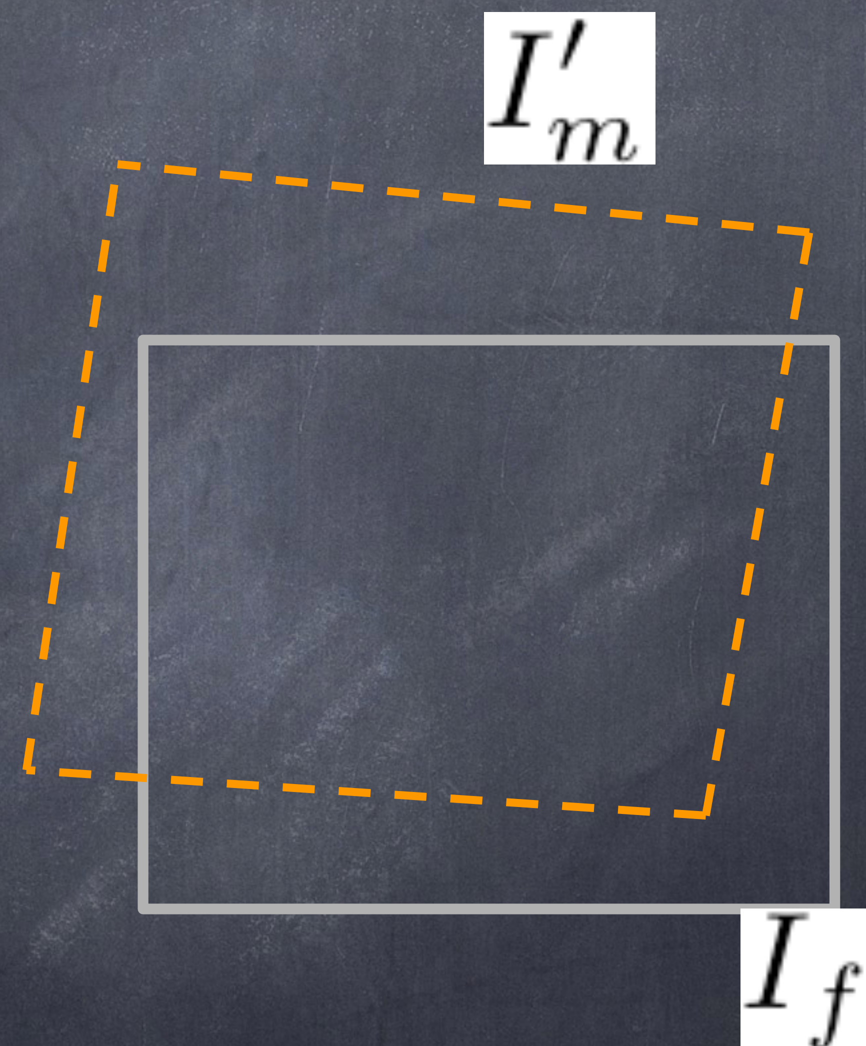
$\mathbf{p}$ Pixel location within region

SSD Example: Initial Alignment



How we can improve it further?

- Mapped (transformed) version of the moving image I_m (based on an estimated transformation) and Fixed image I_f



$$\sum_{\mathbf{p}\in\Omega}\left[I_f(\mathbf{p}) - I'_m(\mathbf{p})\right]^2$$

How we can improve it further?

$$I'_m(\mathbf{p}) = I_m(\mathbf{T}^{-1}(\mathbf{p}; \mathbf{\Theta}))$$

$$\sum_{\mathbf{p} \in \Omega} \left[I_f(\mathbf{p}) - I_m(\mathbf{T}^{-1}(\mathbf{p}; \mathbf{\Theta})) \right]^2$$

Think Abstractly

$$E(\mathbf{\Theta}) = \sum_{\mathbf{p}\in\Omega} \left[I_f(\mathbf{p}) - I_m(\mathbf{T}(\mathbf{p};\mathbf{\Theta})) \right]^2$$

- Can we minimize this function?

Think Abstractly

$$E(\mathbf{\Theta}) = \sum_{\mathbf{p}\in\Omega} \left[I_f(\mathbf{p}) - I_m(\mathbf{T}(\mathbf{p};\mathbf{\Theta}))\right]^2$$

- Can we minimize this function?

$$\nabla E(\mathbf{\Theta}_t) = \frac{\partial E}{\partial \mathbf{\Theta}}(\mathbf{\Theta}_t)$$

$$\mathbf{\Theta}_{t+1} = \mathbf{\Theta}_t - \eta \nabla E(\mathbf{\Theta}_t)$$

Continuous Computation of Derivative

$$\frac{\partial E}{\partial \boldsymbol{\Theta}} = \sum_{\mathbf{p}\in\Omega} -2\left[I_f(\mathbf{p}) - I_m(\mathbf{T}(\mathbf{p};\boldsymbol{\Theta}))\right] \frac{\partial I_m}{\partial \mathbf{T}} \frac{\partial \mathbf{T}}{\partial \boldsymbol{\Theta}}$$

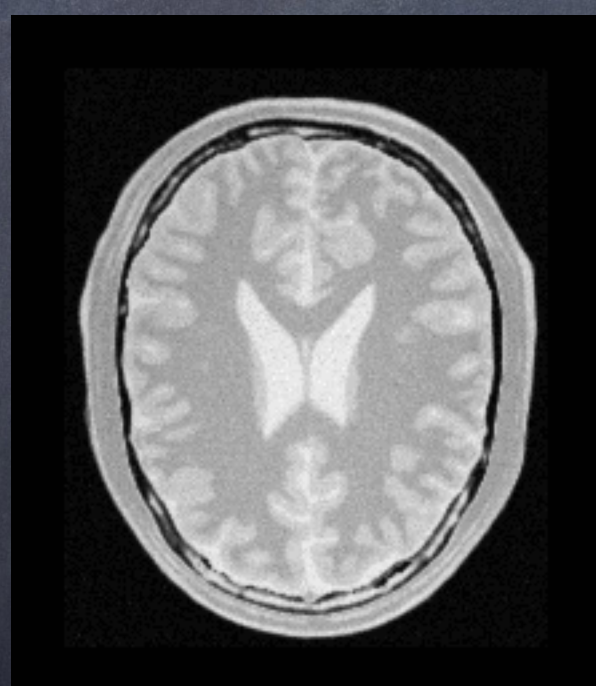
Current error at pixel location $\Delta I(\mathbf{p})$

Intensity gradient in moving image

Change in transformation wrt change in parameters

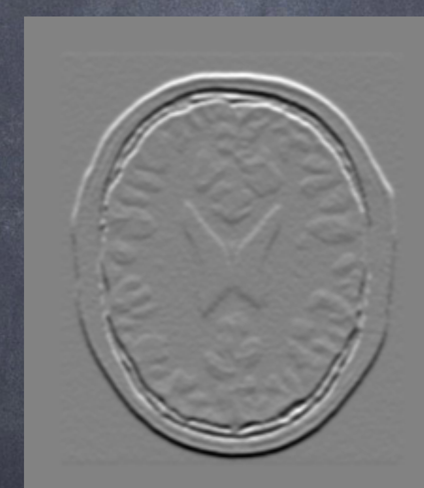
How to compute the derivatives

$$\frac{\partial I_m}{\partial \mathbf{T}}(\mathbf{p}) = \begin{pmatrix} I_{mx}(\mathbf{T}(\mathbf{p};\boldsymbol{\Theta})) & I_{my}(\mathbf{T}(\mathbf{p};\boldsymbol{\Theta})) \end{pmatrix}$$



I_m

I_{mx}



I_{my}

dT/dθ

- Similarity transform:

$$\mathbf{T}(\mathbf{p};\boldsymbol{\Theta}) = \begin{pmatrix} ax - by + t_x \\ bx + ay + t_y \end{pmatrix}$$

  - Where $\boldsymbol{\Theta} = \begin{pmatrix} a & b & t_x & t_y \end{pmatrix}^T$ $\mathbf{p} = (x, y)^T$

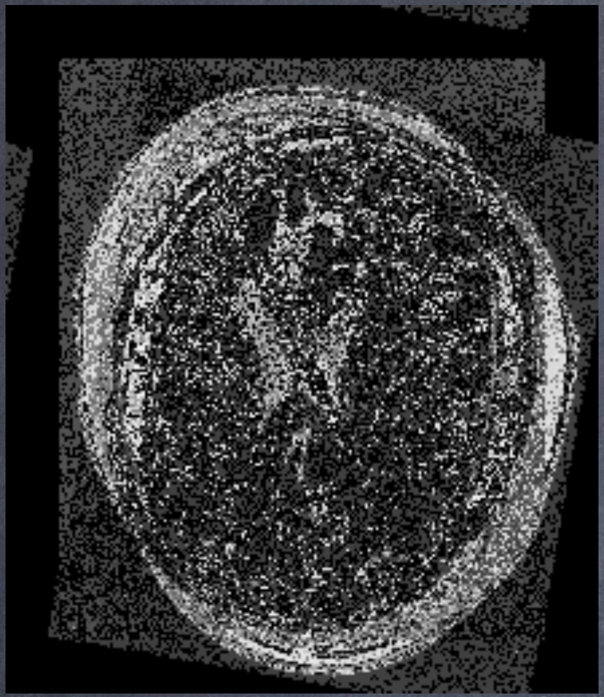
- So derivative is 2x4 matrix (Jacobian):

$$\frac{\partial \mathbf{T}}{\partial \boldsymbol{\Theta}} = \begin{pmatrix} x & -y & 1 & 0 \\ y & x & 0 & 1 \end{pmatrix}$$

Algorithm Outline

- Initialize transformation
- Repeat
  - Compute gradient
  - Make step in gradient direction
  - Update mapping equation
  - Remap image
- Until convergence

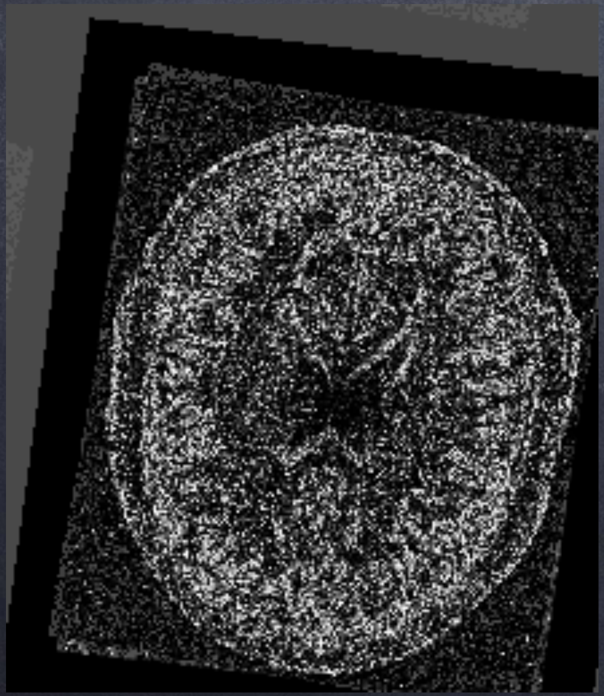
Example



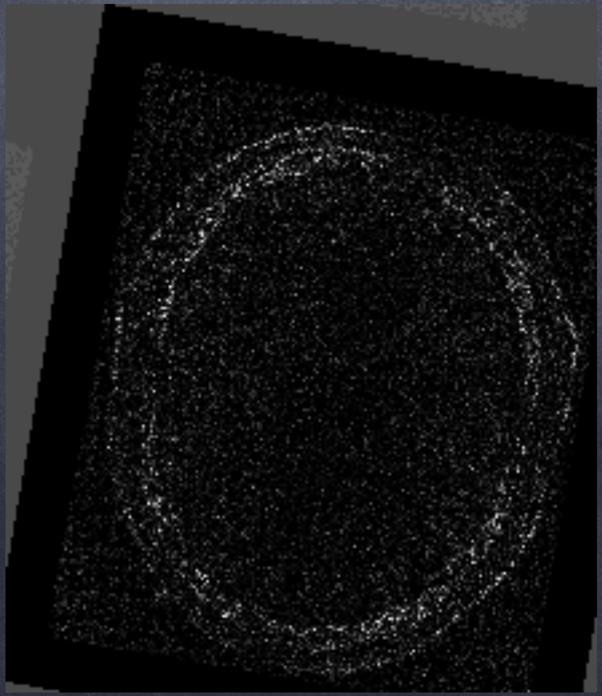
Initial errors

Iteration 100

Iteration 200



Iteration 300



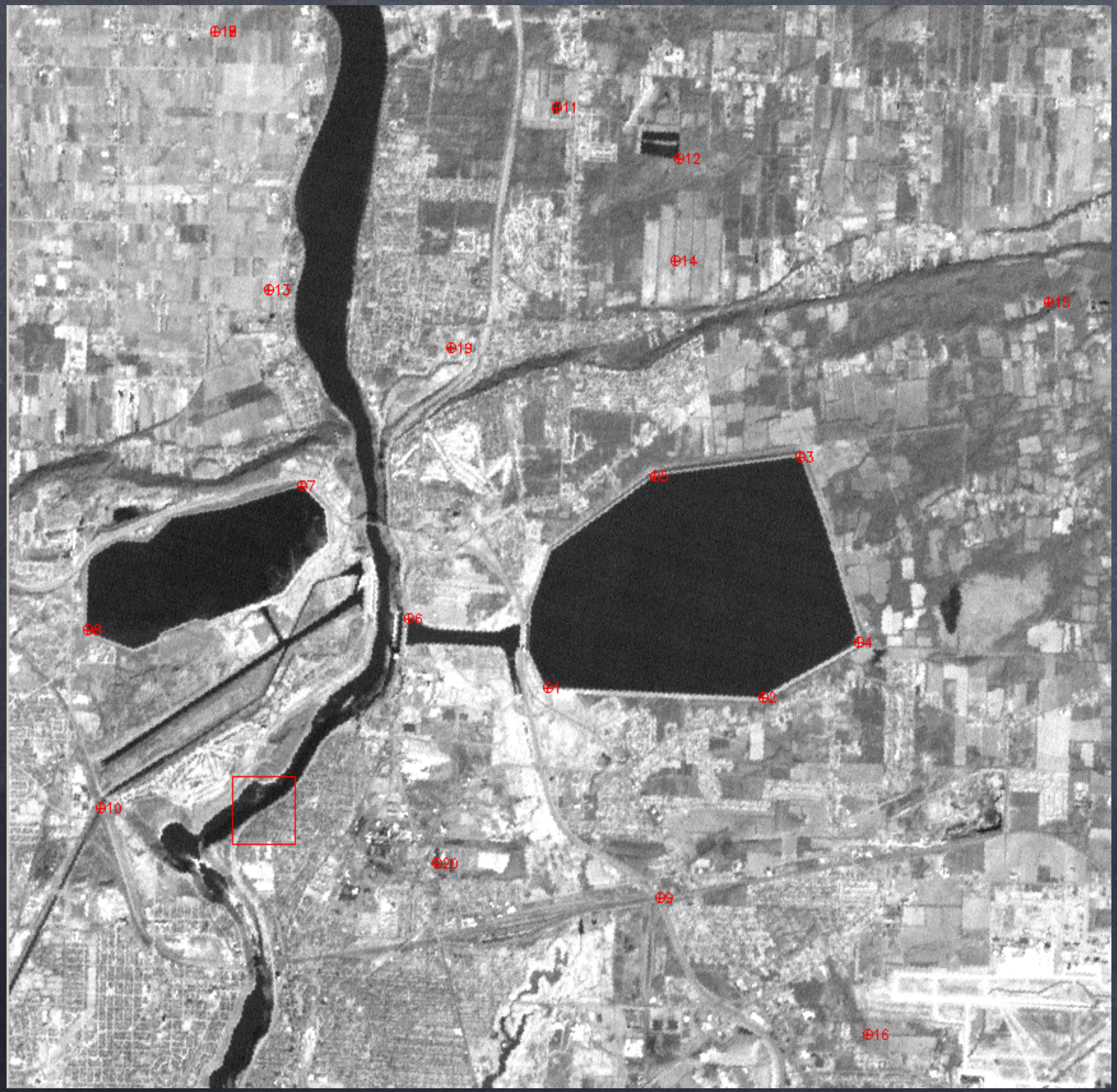
Final: 498 iterations

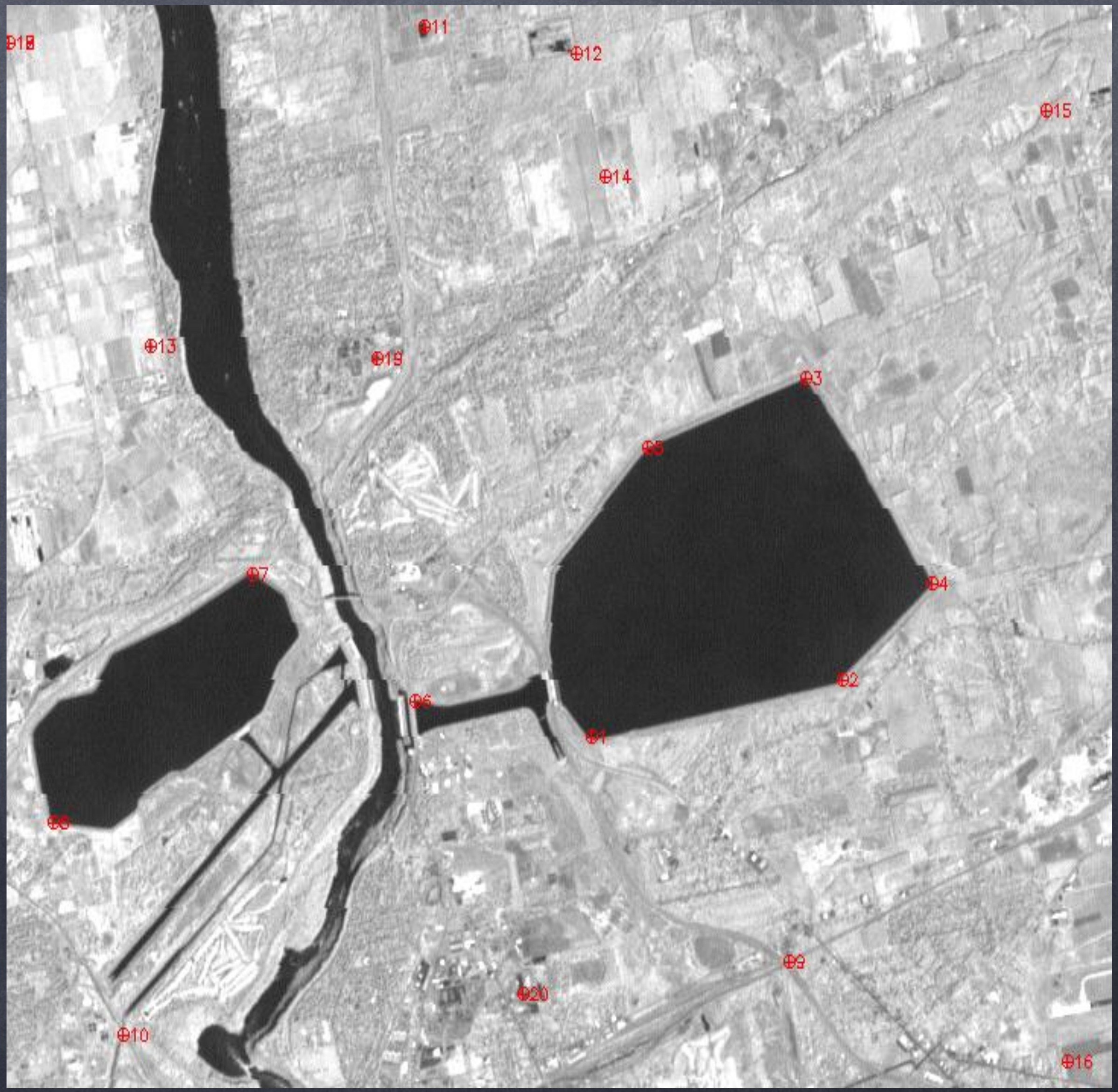
What if we change input

- From intensity to features

Change the input

- Provide coordinates





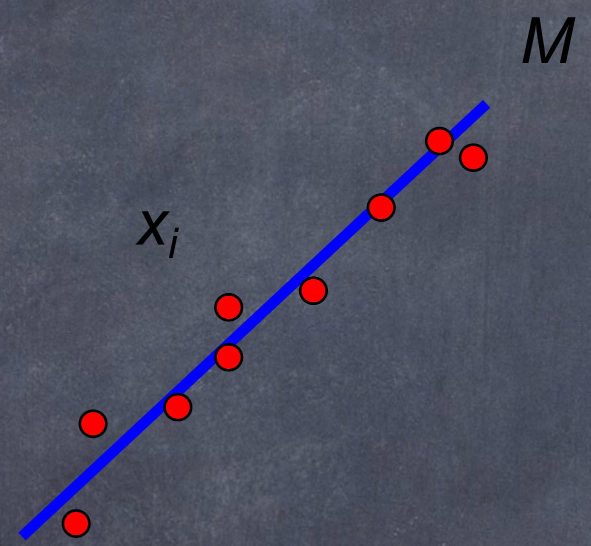
Left: the reference image

Right: the image to be registered

RANSAC based Algorithm

- What is RANSAC?

Curve Fitting

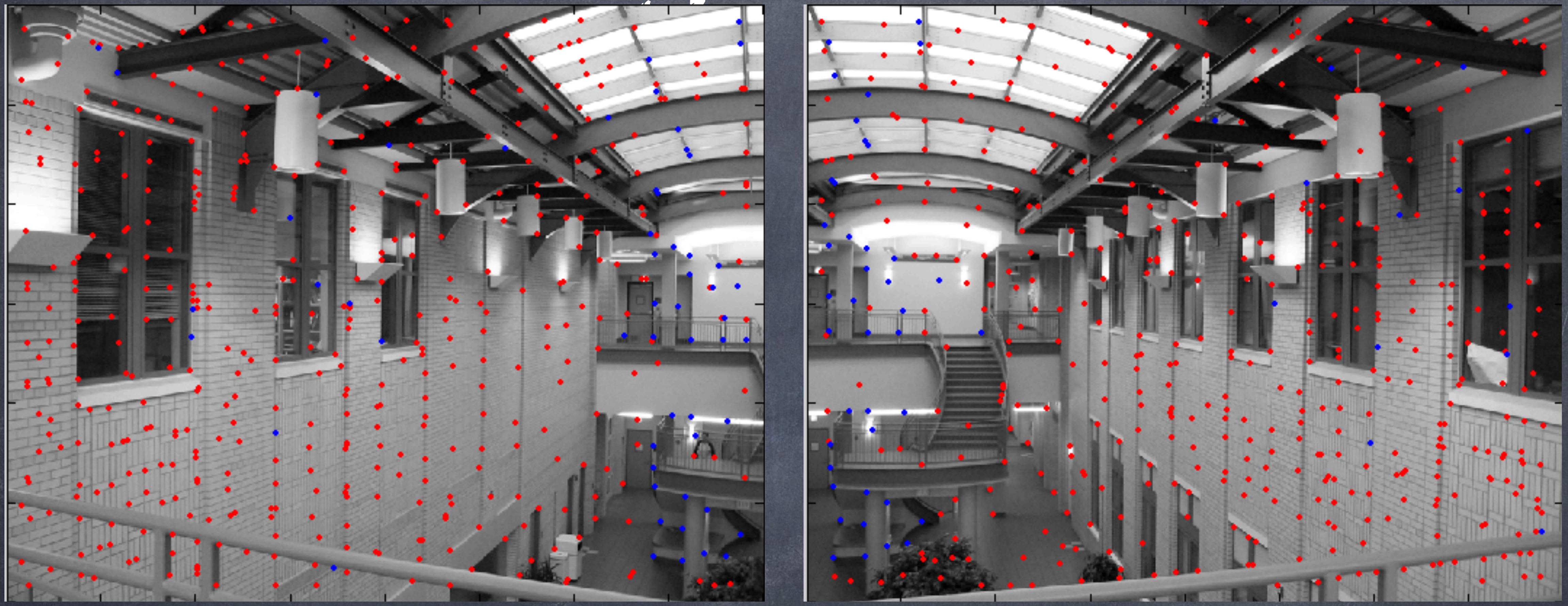


Find model *M* that minimizes

$$\sum_i \mathrm{residual}(x_i, M)$$

- fitting a model to features in one image

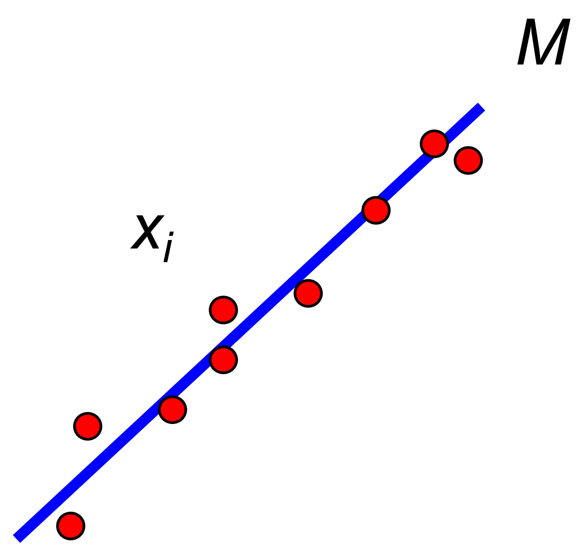
Recall Image Stitching Discussion



Can we now compute M from the blue points?

Alignment as fitting

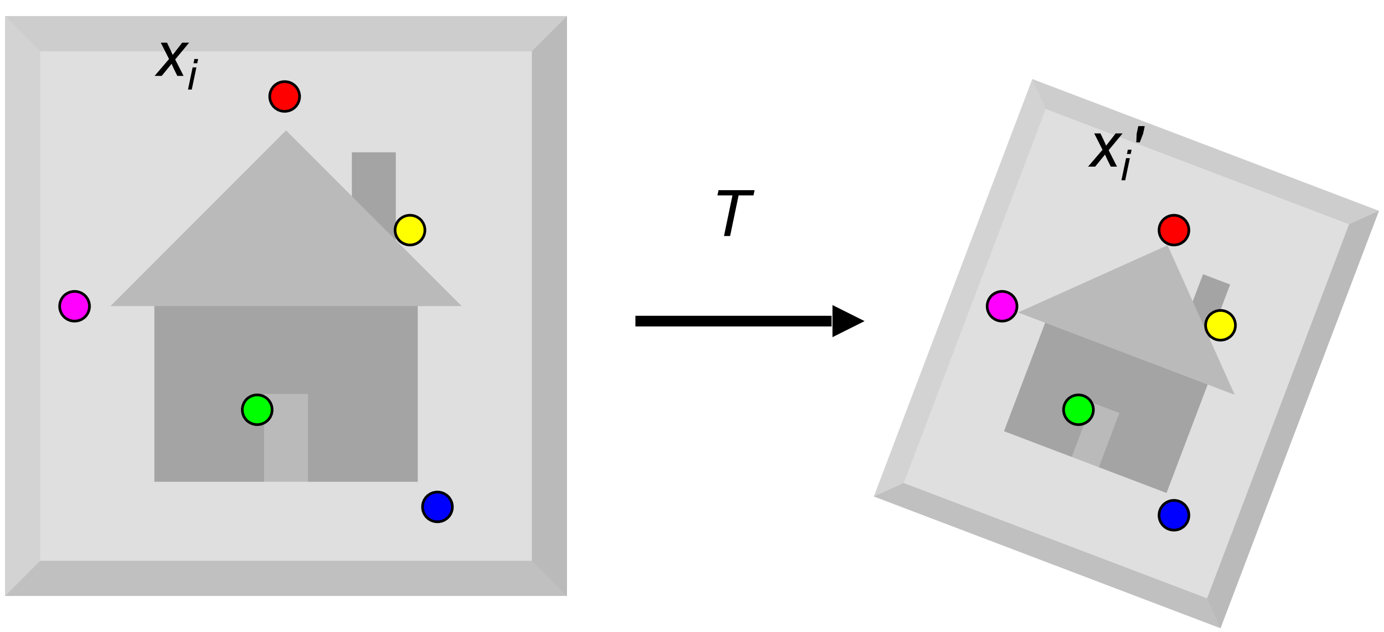
- Previous lectures: fitting a model to features in one image



Find model M that minimizes

$$\sum_i \mathrm{residual}(x_i, M)$$

- Alignment: fitting a model to a transformation between pairs of features (*matches*) in two images



Find transformation T that minimizes

$$\sum_i \mathrm{residual}(T(x_i), x_i')$$

RANSAC in general

- RANSAC = Random Sample Consensus
- an algorithm for robust fitting of models in the presence of many data outliers
- Compare to robust statistics

- Given N data points x_i, assume that mjority of them are generated from a model with parameters Θ, try to recover Θ.

RANSAC algorithm

Run k times:

(1) draw n samples randomly

(2) fit parameters Θ with these n samples

(3) for each of other N-n points, calculate its distance to the fitted model, count the number of inlier points, c

Output Θ with the largest c

RANSAC for estimating homography

- RANSAC loop:

1. Select four feature pairs (at random)
2. Compute homography M (exact)
3. Compute *inliers* where $SSD(p_i', \mathbf{M}\, p_i) < \varepsilon$
4. Record the largest set of inliers so far
5. Re-compute least-squares H estimate on the largest set of the inliers

There are many registration algorithms

- There is a full course on image registration at RPI :-)
- There are books on image registration

Next Topic

- Quality Assessment

What is Image Quality Assessment?



What are some subjective qualities of images?

- Professional or “snapshot”?
- Aesthetically pleasing, or not?
- Photorealistic or not?
- “Original” or not?
- “Familiar” or not?

How to measure it?

- Basic Features:
- Mean pixel intensity
- Contrast
- Color distribution (compared with dist. Metric)
- Mean color saturation and Hue variance
- All of the above, but restricted to the center of the image
- Texture variations
- Edge densities

Types of Methods

- Full-Reference (FR) QA Methods
- Non-Reference (NR) QA Methods
- Reduced-Reference (RR) QA Methods

Reference based QA

- MSE

$$MSE = \frac{\sum_{j=1}^{N}\left(\sum_{i=1}^{M}(X_{i,j}-Y_{i,j})^2\right)}{M\,N}$$

$$RMS = \sqrt{\frac{\sum_{j=1}^{N}\left(\sum_{i=1}^{M}(X_{i,j}-Y_{i,j})^2\right)}{M\,N}}$$

- PSNR

$$PSNR = 20\log\left(\frac{255}{MSE}\right)$$

- SSIM, UIQI … etc (Alan Bovik: http://live.ece.utexas.edu/)

Original bird

Sinusoidal error
(MSE = 12.34)

Image offset 1 pixel
(MSE = 230.7)

More in next lecture

- Exam - 2
- Written assignment to help better prepare for Exam-2 (no need to submit but please solve the questions)