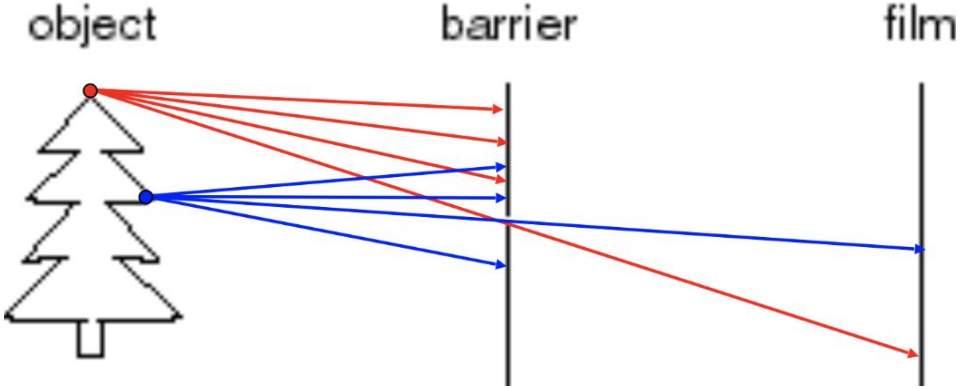
Computer Vision

CSE/ECE 344/544

Pinhole Camera model

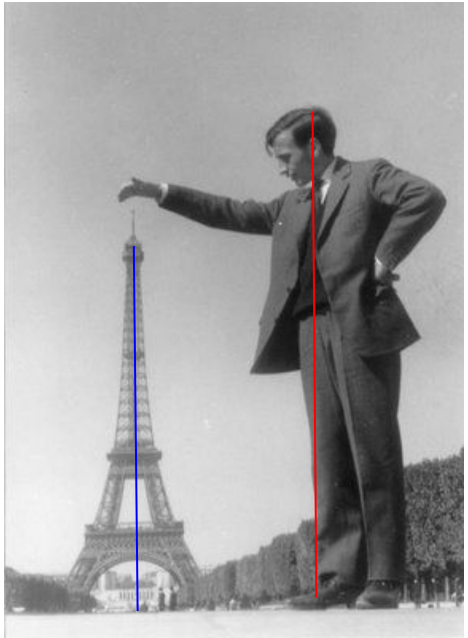
Recall - Pinhole Camera:

- Barrier between world and image plane - Reduce Blurriness
- One point of entry for all light rays (Centre of Projection)
- Image Plane (Film)
- Size of opening (Aperture)



3D Object(world) $\xrightarrow{\text{Perspective Projection}}$ 2D Image(Camera)

Information Loss in Perspective Projection



Loss in Height information

Loss in Angle information

Conversion from World to Image Coordinates

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \Rightarrow \begin{bmatrix} X' \\ Y' \end{bmatrix}$$

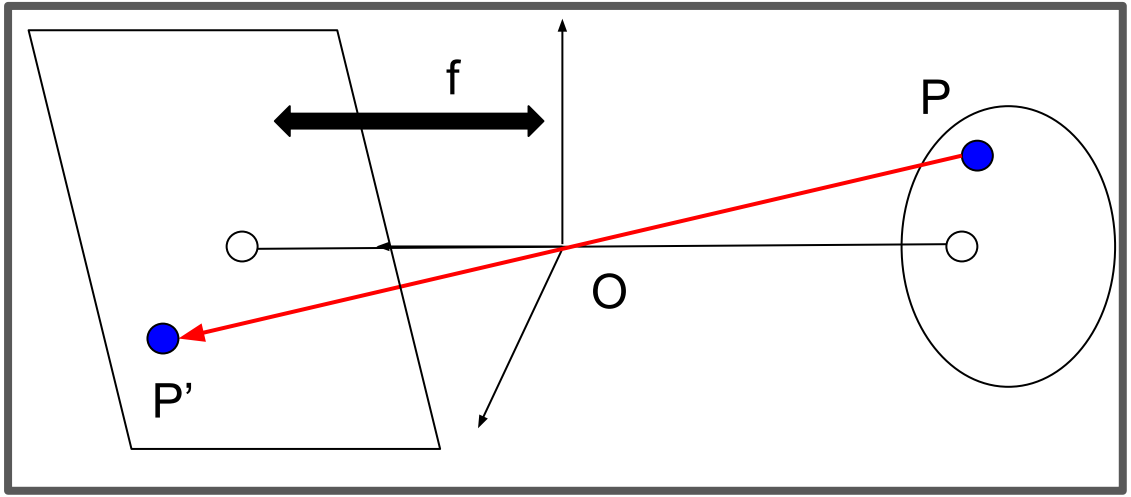
World 3D Coordinates

Camera 2D Coordinates

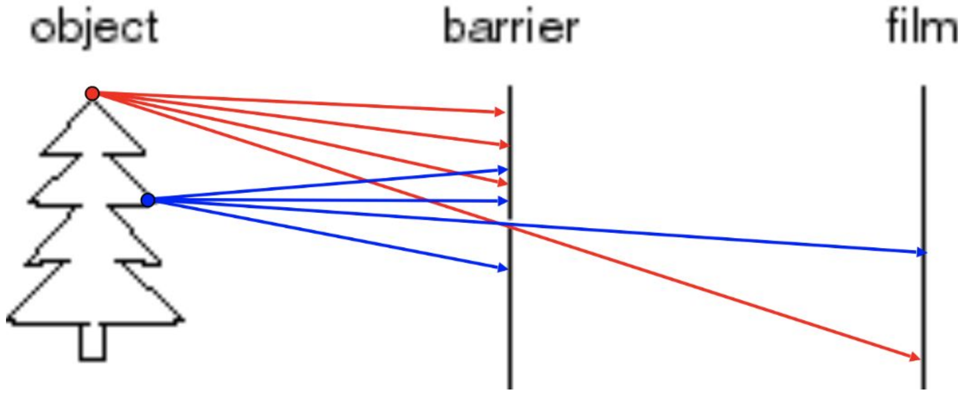
$X' = f\, X/Z$

$Y' = f\, Y/Z$

These can be derived using concepts of similar triangles



3D World to 2D Image?



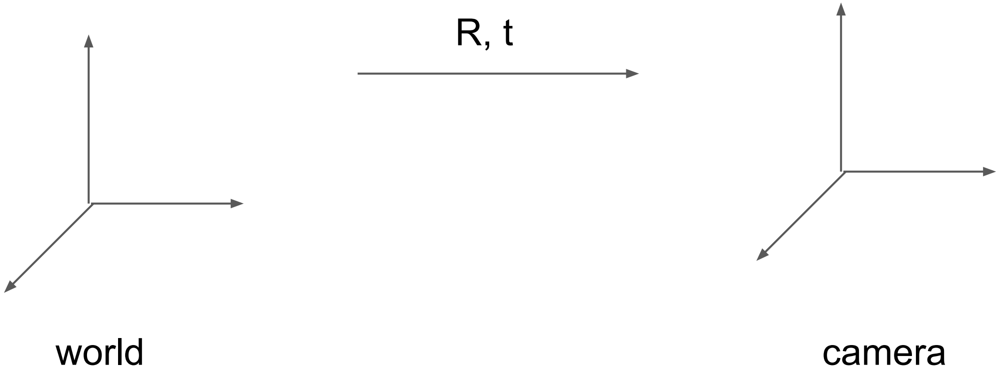
3D Object(world) → Perspective Projection → 2D Image(Camera)

Image formation model

$$\underset{\text{image}}{\begin{pmatrix}\text{2D}\\ \text{point}\\ \text{(3x1)}\end{pmatrix}} = \underset{\substack{\text{(3x3) Camera to}\\ \text{pixel coordinate}}}{\begin{pmatrix}\text{Camera Intrinsic}\\ \text{parameters}\\ \\ \end{pmatrix}} \underset{\text{[I 0]}}{\begin{pmatrix}\text{Perspective}\\ \text{projection matrix}\\ \text{(3x4)}\\ \end{pmatrix}} \underset{\substack{\text{External matrix}\\ \text{(R and t transformation)}}}{\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}} \underset{\text{world}}{\begin{pmatrix}\text{3D}\\ \text{point}\\ \text{(4x1)}\\ \end{pmatrix}}$$

External Matrix

Frame of reference different for camera and world



Rotation matrix

$R = R_x * R_y * R_z$

$$
\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) \\ 0 & \sin(a) & \cos(a) \end{pmatrix}
\begin{pmatrix} \cos(b) & 0 & \sin(b) \\ 0 & 1 & 0 \\ -\sin(b) & 0 & \cos(b) \end{pmatrix}
\begin{pmatrix} \cos(c) & -\sin(c) & 0 \\ \sin(c) & \cos(a) & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

$R_x(a)$ $R_y(b)$ $R_z(c)$

Final matrix

$$E = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

Perspective projection matrix

$$\underbrace{\begin{pmatrix}\text{2D}\\ \text{point}\\ \text{(3x1)}\end{pmatrix}}_{\text{image}} = \underbrace{\begin{pmatrix}\text{Camera Intrinsic}\\ \text{parameters}\end{pmatrix}}_{\substack{\text{(3x3) Camera to}\\ \text{pixel coordinate}}} \underbrace{\boxed{\begin{pmatrix}\text{Perspective}\\ \text{projection matrix}\\ \text{(3x4)}\end{pmatrix}}}_{\text{[I 0]}} \underbrace{\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}}_{\substack{\text{External matrix}\\ \text{(R and t transformation)}}} \underbrace{\begin{pmatrix}\text{3D}\\ \text{point}\\ \text{(4x1)}\end{pmatrix}}_{\text{world}}$$

Perspective Projection Transformation

Given X in 3D homogeneous and X' in 2D homogeneous

$$\underset{(3x1)}{X'} = \underset{(3x4)}{M}\underset{(4x1)}{X}$$

$$X' = (x_1, y_1, z_1)$$
$$(x, y) = (x_1/z_1, y_1/z_1)$$

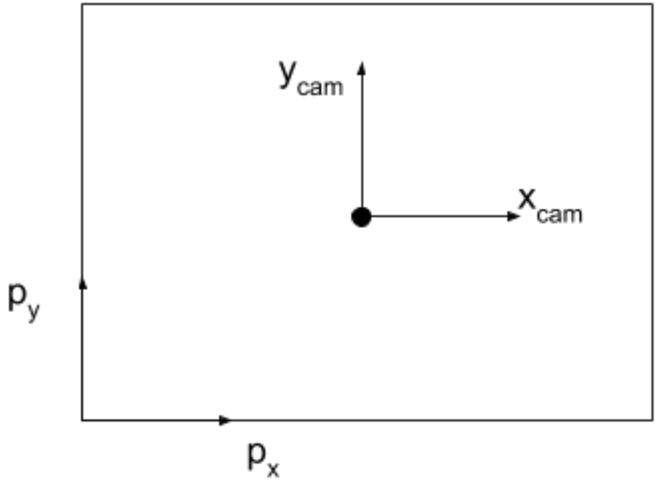
Camera Matrix

$$\underset{\text{image}}{\begin{pmatrix}\text{2D}\\\text{point}\\\text{(3x1)}\end{pmatrix}} = \underset{\substack{\text{(3x3) Camera to}\\\text{pixel coordinate}}}{\boxed{\begin{pmatrix}\text{Camera Intrinsic}\\\text{parameters}\end{pmatrix}}} \underset{\text{[I 0]}}{\begin{pmatrix}\text{Perspective}\\\text{projection matrix}\\\text{(3x4)}\end{pmatrix}} \underset{\substack{\text{External matrix}\\\text{(R and t transformation)}}}{\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}} \underset{\text{world}}{\begin{pmatrix}\text{3D}\\\text{point}\\\text{(4x1)}\end{pmatrix}}$$

Principal point offset

(X, Y, Z) -> (f*X/Z, f*Y/Z)

With offset
(X, Y, Z) -> (f*X/Z + Px, f*Y/Z + Py)



Q. Given (X, Y, Z), derive the transformation matrix for the camera coordinates

Camera Intrinsic matrix

$$\begin{pmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{pmatrix}$$

Camera matrix in pixels

$$\begin{pmatrix} m_x & 0 & 0 \\ 0 & m_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \alpha_x & 0 & \beta_x \\ 0 & \alpha_y & \beta_y \\ 0 & 0 & 1 \end{pmatrix}$$

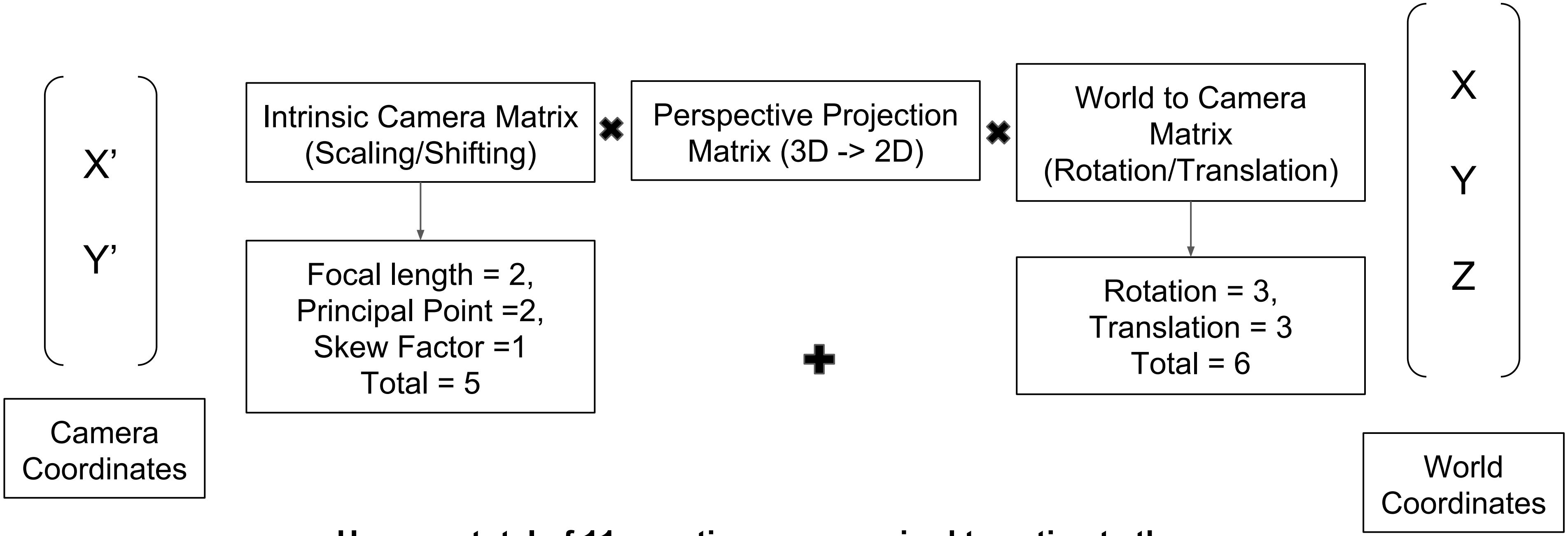
pixels/m m pixels

If the axes are not orthogonal

$$\begin{pmatrix} \alpha_x & \boxed{s} & \beta_x \\ 0 & \alpha_y & \beta_y \\ 0 & 0 & 1 \end{pmatrix}$$

Degrees of Freedom

There are 11 degrees of freedom in the matrices.



Hence, a total of 11 equations are required to estimate the parameters.