Supply Uncertainty in Multi-supplier Framework

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Abstract

In light of uncertainty in a supply chain's logistics network caused by unexpected disruption or various forms of faults (such as shipping damage, missing parts, and misplaced products), this research aims to establish the ideal order amount for a store. Models of mixture distribution describe issues resulting from isolated failures and unavoidable incidents that impair network performance. The uncertainty in the number of good products successfully reaching retail stores poses a challenge in deciding product-order amounts. Because the commonly used ordering plan developed for maximizing expected profits does not allow retailers to address concerns about contingencies; this research proposes two improved procedures with risk-averse characteristics towards low probability and high impact events. Several examples illustrate the impact of a supplier's operation policies and model assumptions on a retailer's product-ordering plan and resulting sales profit.

1 Introduction

In this paper we consider the problem of supply uncertainty for a retailer. We consider that a supplier is either operating normally or has met a contingency. Occurrence of contingency has a very low probability and is marked by a huge impact on supply. In the normal case too, we are likely to get a low supply. Supply chain is global right now. With that come the competitive advantage and also the disadvantage of great uncertainty and many risks for the firm. Among the various activities involved in a company, purchasing is one of the most strategic as it provides opportunities to reduce costs and increase the quality of raw materials. Moreover, efficient purchasing strategies have the potential to affect the competitive advantage of not only a manufacturing facility but of its entire supply chain. It is not difficult to see the impact that suppliers can have on a firm's total cost. In most industries the cost of raw materials and component parts represents the main cost of a product. For instance, in high technology firms, purchased materials and services account for up to 80% of the total product cost.

Suppliers with lower prices are often at a greater distance from the distribution centres and hence the products from overseas suppliers are much susceptible to defects. There can be various reasons for defects which can be broadly categorized as "Supply side risk" and "Catastrophic Risk". Supply side risks are those which are inherent in the system like the quality related risk, transportation risk, product design changes etc. While catastrophic risk would be the ones such as hurricane, terrorist activity, fire etc. It has been studied previously that such defects could reach as high as 20%. In this research both type of defects are considered. Defect rate if not accounted for, leads to a lot of inconveniences such as customer dissatisfaction, damaged reputation, more money in repair and replacing the items. One can choose to keep a high level of inventory to account for the above, but that increases the holding cost. In this regards the constraint of service level and fill rate has been considered and the order quantity for both contingent and non-contingent scenario has been taken into account.

2 Multiplicative model

2.1 Problem Formulation

The retailer sources from k suppliers. Each supplier is providing the identical products with same price. It is assumed that 1. The retailer pays a supplier only for the quantity that he actually delivers (any defective or undelivered products are not paid for) 2. The supplier will not supply quantity more than what has been ordered by the retailer. The retailer is a purchaser here and suppliers are producers with their definite capacity.1

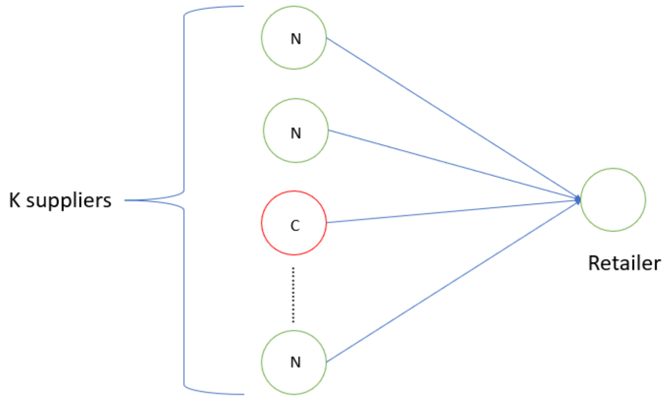

Figure 1: Here C represents a supplier who has met with a contingency and N means normal

Let Q_i be the quantity we ask from the i^{th} supplier, Y_i denote the fraction of damaged products delivered by the i^{th} supplier, Y_{c_i} denote the fraction of damaged products delivered by the i^{th} supplier in when the supplier has met with a contingency, Y_{n_i} denote the fraction of damaged products delivered by the i^{th} supplier in the normal scenario. We denote $E[Y_i]$ by μ_i, $Var(Y_i)$ by σ_i^2, $E[Y_{c_i}]$ by μ_{c_i}, $Var(Y_{c_i})$ by $\sigma_{c_i}^2$, $E[Y_{n_i}]$ by μ_{n_i} and $Var(Y_{n_i})$ by $\sigma_{n_i}^2$. Let I_i represent the indicator variable for the contingency for the i^{th} supplier, then

$$Y_i = I_i Y_{c_i} + (1-I_i)Y_{n_i}$$

$$\implies E[Y_i] = E(I_i Y_{c_i}) + E((1-I_i)Y_{n_i})$$

As the probability of occurrence of contingency is independent of the proportion of defective products received in contingent case, we have

$$\implies \mu_i = E(I_i)E(Y_{c_i}) + E(1-I_i)E(Y_{n_i})$$

Let p_i be the probability of the occurrence of contingency for i^{th} supplier, We assume that $p_i = p \ \forall i$

$$\implies \mu_i = p\mu_{c_i} + (1-p)\mu_{n_i}$$

Assumption 1 : Y_{c_i}'s, Y_{n_i}'s and I_i's are iid random variables, which means $Y_i's$ are iid too.

$$\implies \mu_i = p\mu_c + (1-p)\mu_n$$

We can get an expression for TPDP by equating the total quantity received from all suppliers

$$\left(\sum_{i=1}^{k} Q_i\right)(1-Y) = \sum_{i=1}^{k} Q_i(1-Y_i) \implies Y = \frac{\sum_{i=1}^{k} Q_i Y_i}{\sum_{i=1}^{k} Q_i}$$

We assume that we ask every supplier to order exactly the same quantity, $Q_i = Q \ \forall i$

$$\implies Y = \frac{1}{k}\sum_{i=1}^{k} Y_i$$

Calculating mean(μ) and variance(σ^2) of Y : using $E[Y_i] = \mu_i = p\mu_c + (1-p)\mu_n$ (μ_i is independent of i) with the above equation, we have

$$\mu = E[Y] = \frac{1}{k}\sum_{i=1}^{k} E[Y_i] = \frac{1}{k}k(p\mu_c + (1-p)\mu_n) = p\mu_c + (1-p)\mu_n$$

We can see that μ is independent of k i.e. it does not depend on number of suppliers.

$$Var(Y) = \frac{1}{k^2}\sum_{i=1}^{k} Var(Y_i) + \frac{1}{k^2}\sum_{i,j}\sum_{i\neq j} Cov(Y_i, Y_j)$$

Since we assumed that all Y_i's are iid, $\implies Cov(Y_i, Y_j) = 0 \; \forall \; i \neq j \implies$ the second term from above equation vanishes and we are left with

$$Var(Y) = \frac{1}{k^2} \sum_{i=1}^{k} Var(Y_i) = \frac{1}{k^2} k * Var(Y_i) = \frac{Var(Y_i)}{k}$$

Let's first try deriving Var(Y_i) $= \sigma_i^2$

$$\sigma_i^2 = Var(I_i Y_{c_i} + (1-I_i) Y_{N_i}) = Var(I_i Y_{c_i}) + Var(Y_{n_i}(1-I_i)) + 2 * Cov(I_i Y_{c_i}, (1-I_i) Y_{N_i})$$

Solving the first term,

$$Var(I_i Y_{c_i}) = E[I_i^2 Y_{c_i}^2] - E[I_i]^2 E[Y_{c_i}]^2 = E[I_i^2] E[Y_{c_i}^2] - p^2 \mu_{c_i}^2 = p(\mu_{c_i}^2 + \sigma_{c_i}^2) - p^2 \mu_{c_i}^2$$

And similarly, The second term in above summation boils down to

$$Var((1-I_i) Y_{n_i}) = E[(1-I_i)^2] E[Y_{n_i}^2] - E[1-I_i]^2 E[Y_{n_i}]^2 = (1-p)(\mu_{n_i}^2 + \sigma_{n_i}^2) - (1-p)^2 \mu_{n_i}^2$$

The third term from the above summation yields,

$$Cov(I_i Y_{c_i}, (1-I_i) Y_{N_i}) = Cov(I_i Y_{c_i}, Y_{N_i}) - Cov(I_i Y_{c_i}, I_i Y_{n_i}) = -Cov(I_i Y_{c_i}, I_i Y_{n_i})$$

The above result follows from the assumption that Y_{N_i} is independent of Y_{c_i}, I_i.

$$Cov(I_i Y_{c_i}, I_i Y_{n_i}) = E[I_i^2 Y_{n_i} Y_{c_i}] - E[I_i Y_{c_i}] E[I_i Y_{n_i}] = E[Y_{n_i}] E[Y_{c_i}] \left(E[I_i^2] - E[I_i]\right) = \mu_{c_i} \mu_{n_i} (p-p) = 0$$

Combining the terms together and using the fact that Y_i's are all identically distributed, we have

$$\sigma^2 = Var(Y) = \frac{Var(Y_i)}{k} = \frac{p(\mu_{c_i}^2 + \sigma_{c_i}^2) - p^2 \mu_{c_i}^2 + (1-p)(\mu_{n_i}^2 + \sigma_{n_i}^2) - (1-p)^2 \mu_{n_i}^2}{k}$$

2.2 Optimization

Let $\hat{Q}$ denote the total quantity received, i.e. $\hat{Q} = Q(1-Y)$, The profit for the retailer,

$$\Pi(Q) = r\left(\min(\hat{Q}, X)\right) - \pi\left(X - \hat{Q}\right)^+ - h\left(\hat{Q} - X\right)^+ - \sum_{i=1}^{k} (c_i Q_i)$$

Where X is the demand from consumers, π is the stock-out cost, h is the inventory holding cost, r is the retail price, c_i is the cost of procurement from the i_{th} supplier, here we assume that $c_i = c \; \forall i$. Using some algebraic manipulations we can show that,

$$\Pi(Q) = (p - c + \pi)\hat{Q} - (p + h + \pi)\left(\hat{Q} - X\right)^+ - \pi X$$

We try to find the expected profit considering the demand to be variable and the occurrence of contingency, which could be effectively captured by the parameter Y

$$E_{X,Y}(\Pi(Q)) = (p - c + \pi) E_{X,Y}(\hat{Q}) - (p + h + \pi) E_{X,Y}\left(\hat{Q} - X\right)^+ - \pi E_{X,Y}(D)$$

$$E_{X,Y}(\hat{Q}) = E_Y(Q(1-Y)) = Q(1 - E_Y(Y)) = Q(1-\mu)$$

For the sake of simplicity, we assume that the demand is a uniform random variable $\mathscr{U}(0, b)$

$$E_{X,Y}\left(\hat{Q} - X\right)^+ = E_Y\left(E_X\left(\hat{Q} - X\right)^+\right) = E_Y\left(\int_0^{min(b,\hat{Q})} (\hat{Q} - x) f_X(x) dx\right)$$

We assume that $\hat{Q} <= Q < b$, which is quite reasonable to assume

$$E_{X,Y}\left(\hat{Q} - X\right)^+ = E_Y\left(\frac{(Q(1-Y))^2}{2b}\right) = \frac{Q^2}{2b}\left((1-\mu)^2 + \sigma^2\right)$$

which leads us to

$$E_{X,Y}(\Pi(Q)) = Q(1-\mu)(p-c+\pi) - \frac{Q^2}{2b}\left((1-\mu)^2+\sigma^2\right)(p+h+\pi) - \pi\left(\frac{b}{2}\right)$$

The expected profit function is concave and hence, profit maximising quantity is given by

$$\frac{\partial\left(E_{X,Y}(\Pi(Q))\right)}{\partial Q} = 0$$

$$\implies (1-\mu)(p-c+\pi) - \frac{Q}{b}\left((1-\mu)^2+\sigma^2\right)(p+h+\pi) = 0$$

$$\implies Q^* = \frac{(1-\mu)}{(1-\mu)^2+\sigma^2}\frac{b(p-c+\pi)}{p+h+\pi} = \frac{(1-\mu)}{(1-\mu)^2+\sigma^2}Q^o$$

where Q^o is the optimal order quantity in the news-vendor problem. Considering this expression μ is quite low and which leads to a very little impact on optimal order quantity due to contingency. To have a close look at the effects of contingency on the optimal ordering quantity, let's have a look at scenario specific service levels.

2.3 Type I Service level constraint

Type I service level, α is defined as the fraction of instances in which total demand is satisfied,

$$\alpha = E_Y\left[P(X < Q(1-Y))\right] = \frac{Q(1-\mu)}{b} = \frac{Q(1-(p\mu_c+(1-p)\mu_n))}{b}$$

Now, say we want our expected service level to be greater than α_0, we should have

$$\frac{Q(1-\mu)}{b} > \alpha_0 \implies Q > \frac{b\alpha_0}{1-\mu}$$

So in the general case where all the suppliers are identical i.e. they all have same μ which is independent of the number of suppliers, therefore the optimal order quantity and the profit do not depend on the number of suppliers.

2.4 Scenario specific Type 1 Service level constraints

We consider the impact of contingency by considering various scenarios, scenario in which no supplier is down, one is down, two are down..., all k are down. Let $Y^{(j)}$ represent the TPDP in the case when j of the suppliers are down and S_j represent the set of suppliers which are down

$$Y^{(j)} = \sum_i \frac{Y_i}{k} = \frac{\sum_{i\in S_j} Y_{c_i} + \sum_{i\notin S_j} Y_{n_i}}{k}$$

$$E[Y^{(j)}] = \mu^{(j)} = \frac{j\mu_c + (k-j)\mu_n}{k} = \mu_n + \frac{j(\mu_c-\mu_n)}{k}$$

$$Var[Y^{(j)}] = \sigma^{2(j)} = \frac{j\sigma_c^2 + (k-j)\sigma_n^2}{k} = \sigma_n^2 + \frac{j(\sigma_c^2-\sigma_n^2)}{k}$$

As derived earlier, for a particular demand distribution and quantity, the service level only depends on the expected proportion of defective products. Say we want type I service level to not go below α_j in the scenario when j of the suppliers are down, then we would have the following k+1 constraints,

$$Q > \frac{b\alpha_0}{1-\mu^{(0)}}$$

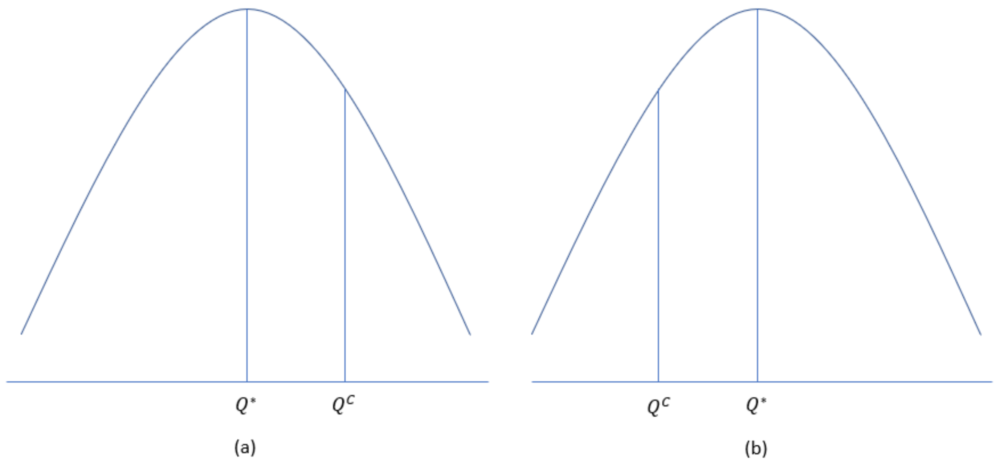

Figure 2: **Here $Q^C = \frac{b\alpha_0}{1-\mu}$, in case (a) the minimum order quantity that satisfies the constrain (Q^C) is greater than the order quantity giving the most profit (Q^*), so it is best to choose Q^C as optimal order quantity as we cannot reduce it and increasing it will lead to less profit. In (b) Q^* should be chosen as we can increase the order quantity form Q^C to Q^* without violating constraint and increasing profit. Hence the optimal order quantity $Q = max(Q^C, Q^*)$.**

$$Q > \frac{b\alpha_1}{1-\mu^{(1)}}$$
$$\vdots$$
$$Q > \frac{b\alpha_k}{1-\mu^{(k)}}$$

Let $Q^C = \max(\frac{b\alpha_i}{1-\mu^{(i)}})\ \forall i \in [0,k]$ by the conclusion we draw from figure 2 we infer that the optimal order quantity $= \max(Q^C, Q^*)$. We assume, $\mu_c > \mu_n$ and $\alpha_0 > \alpha_1 > \cdots > \alpha_k$ which are reasonable to assume. Since we know

$$\mu^{(j)} = \mu_n + \frac{j(\mu_c - \mu_n)}{k}$$

$\mu^{(j)}$'s decrease with an increase in k. Now, suppose we increase k to k_2, then we will have some additional constraints on the optimal order quantity (refer to fig 3) and $\mu^{(j)}$'s for $j \in [0,k]$ decrease, leading to less stringent constraints on the constrained order quantity for $j \in [0,k]$. But, the additional constraints may be more strict than earlier ones and hence we may need higher optimal order quantity. If we ensure, that none of the additional constraints is higher than any of the constraints we earlier had(with k suppliers), we can say that, as all constraints with $j \in [0,k]$ have become less strict, the retailer need not order higher, and the retailer gains at least as much as what he gained with less suppliers.

This suggests that optimal order quantity can either decrease, stay the same, or even increase, with an increase in k(the number of suppliers). Moreover, the profit for the retailer can also increase, stay constant or decrease due to an increase in the number of suppliers.

2.5 Implicit Service level Type 1

$$f(Q) = f(Q^*) = \frac{(1-\mu)^2}{(1-\mu)^2 + \sigma^2}\left(\frac{Q^o}{b}\right)$$

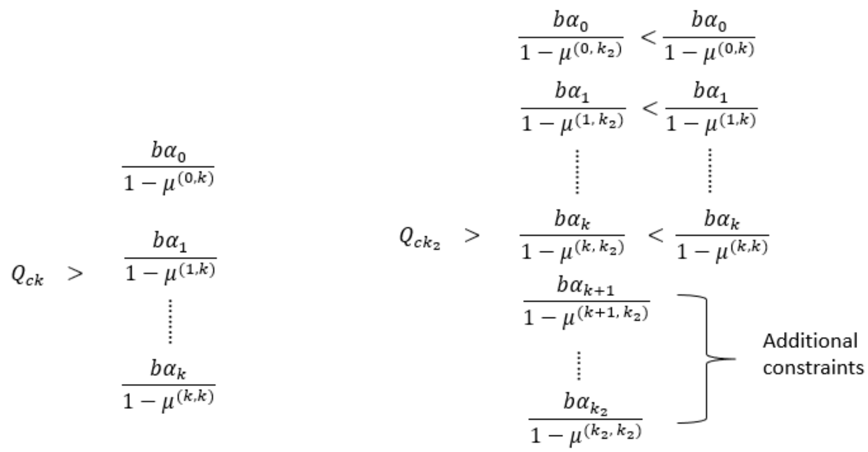

Figure 3:

2.6 Type II Service level constraint

The Type II service level (β) is defined as the fraction of demand which is satisfied

$$\beta = \frac{E_{X,Y}[\hat{Q} - (\hat{Q} - X)^+]}{E_{X,Y}[X]} = \frac{Q(1-\mu) - E_{X,Y}[\hat{Q} - X)^+]}{\frac{b}{2}}$$

$$\beta = \frac{E_{X,Y}[\hat{Q} - (\hat{Q} - X)^+]}{E_{X,Y}[X]} = \frac{Q(1-\mu) - \frac{Q^2}{2b}\left((1-\mu)^2 + \sigma^2\right)}{\frac{b}{2}}$$

Say we want service level $> \beta_0$

$$\frac{Q(1-\mu) - \frac{Q^2}{2b}\left((1-\mu)^2 + \sigma^2\right)}{\frac{b}{2}} > \beta_0$$

$$\implies \frac{Q^2\left((1-\mu)^2 + \sigma^2\right)}{b^2} - \frac{2 * Q(1-\mu)}{b} + \beta_0 < 0$$

The quadratic equation corresponding to the above inequality has both roots positive say Q_1 and Q_2 and $Q_2 < Q_1$, we want $Q_2 < Q < Q_1$ for the above inequality to be satisfied, where Q_1 and Q_2 are given by

$$Q_1 = \frac{2b\left((1-\mu) + \sqrt{(1-\mu)^2(1-\beta_0) - \beta_0\sigma^2}\right)}{(1-\mu)^2 + \sigma^2}, Q_2 = \frac{2b\left((1-\mu) - \sqrt{(1-\mu)^2(1-\beta_0) - \beta_0\sigma^2}\right)}{(1-\mu)^2 + \sigma^2}$$

We can see that σ^2 is dependent on k(number of suppliers), it decreases as k increases. If we increase k then σ decreases and hence Q_1 increases for sure, but we cannot say anything for sure about Q_2, it may increase or decrease.

But if we assume that $(1-\mu) >> \sigma$ along with $2/3 < \beta < \frac{(1-\mu)^2}{(1-\mu)^2+\sigma^2}$, then Q_2 also increases slightly with an increase in the number of suppliers. Three different cases arise in this constrained optimization problem(refer to fig 4).We try to find the impact of increase in the number of suppliers on the profit and optimal order quantity. Going ahead with the above assumptions, with an increase in k, Q_1 and Q_2 both increase. As an consequence, In the case(iii), we are sure to get more optimal order quantity and higher profit. In the case(ii), we would get more optimal order quantity but lesser profit. And in the case(i), depending on the increase in Q_2, we may either, (a) Need to order more quantity and would have less profit or (b) Need to order the same quantity and have the same profit.

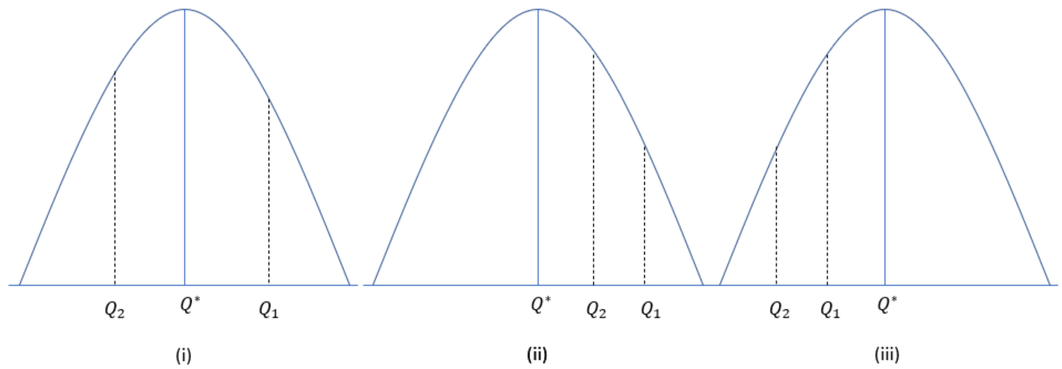

Figure 4: **The figure above shows different cases which arise in the constraint optimization problem.**

2.7 Scenario specific Type II Service level constraint

When we consider those $k+1$ scenarios again, and want our type 2 service level to be greater than β_j in the case when j of the suppliers are down, then we will have to consider the following k constraints again

$$\frac{Q(1-\mu^{(0)}) - \frac{Q^2}{2b}\left((1-\mu^{(0)})^2 + (\sigma^{(0)})^2\right)}{\frac{b}{2}} > \beta_0$$

$$\frac{Q(1-\mu^{(1)}) - \frac{Q^2}{2b}\left((1-\mu^{(1)})^2 + (\sigma^{(1)})^2\right)}{\frac{b}{2}} > \beta_1$$

$$\vdots$$

$$\frac{Q(1-\mu^{(k)}) - \frac{Q^2}{2b}\left((1-\mu^{(k)})^2 + (\sigma^{(k)})^2\right)}{\frac{b}{2}} > \beta_k$$

3 Additive Modelling

So far, we had discussed the effect of various parameters in general scenario and scenario specific service levels, when the quantity we get was $\hat{Q} = Q(1-Y)$ where Y was some random variable. Using this multiplicative model we were not able to derive much insights on the optimal order quantity and the profit. We hope using additive model will make things simple enough to derive some useful insights. Intuitively, the number of defective products we get when we order 100 units will be different than the number of defective products we get when we order 1000 units. Hence, this model may not be of much use if the quantity we ask for, viz. Q, is itself highly variable.

3.1 Problem Formulation

The quantity we get from the ith supplier, $\hat{Q}_i = Q_i - Z_i$ where $Z_i = I_{C_i} Z_{C_i} + (1 - I_{C_i}) Z_{N_i}$, and I_{C_i} is the indicator variable, which takes value 1 when there is contingent scenario for the ith supplier, and is zero otherwise. Z_{C_i}, Z_{N_i} are random variables which denote the defect in quantity supplied by the ith supplier in the contingent scenario and normal scenario respectively. We assume that all Z_{C_i}'s, Z_{N_i}'s and I_{C_i}'s are iid random variables, which effectively means that all suppliers are identical. Let $E[Z_{C_i}] = \mu_C$, $E[Z_{N_i}] = \mu_N$, $E[I_{C_i}] = p$ and $Var[Z_{C_i}] = \sigma_C^2$, $Var[Z_{N_i}] = \sigma_N^2$. Let $Q = \sum_i Q_i$ and $Z = \sum_i Z_i$. Now, the total quantity we receive is $\hat{Q} = \sum_i Q_i - \sum_i Z_i = Q - Z$.

Let's derive the quantities $E[Z] = \mu$ and $Var[Z] = \sigma^2$, which will come in handy later.

$$\mu = E[Z] = E[\sum_i Z_i] = \sum_i E[Z_i] = \sum_i (p\mu_c + (1-p)\mu_n) = k(p\mu_c + (1-p)\mu_n)$$

$$\sigma^2 = Var(Z) = Var(\sum_i Z_i) = \sum_i Var(Z_i) = \sum_i Var(I_{c_i} Z_{c_i} + (1-I_{c_i})Z_{n_i})$$

Using the result from $Var(Y_i)$, we get

$$Var(Z) = \sum_i Var(I_{C_i} Z_{C_i} + (1-I_{C_i})Z_{N_i}) = k\left(p(\mu_c^2+\sigma_c^2) - p^2\mu_c^2 + (1-p)(\mu_n^2+\sigma_n^2) - (1-p)^2\mu_n^2\right)$$

Profit function of the retailer in this case is given by,

$$\Pi(Q) = (r-c+\pi)\hat{Q} - (r+h+\pi)(\hat{Q}-X)^+ - X\pi$$

we seek to maximize the expected profit, which is given by

$$E[\Pi(Q)] = (r-c+\pi)E[\hat{Q}] - (r+h+\pi)E[(\hat{Q}-X)^+] - E[X]\pi$$

where

$$E[\hat{Q}] = E[\sum_i Q_i - \sum_i Z_i] = Q - \mu$$

For the sake of simplicity, we assume that the demand is a uniform random variable $\mathscr{U}(0,b)$

$$E_{X,Z}\left(\hat{Q}-X\right)^+ = E_Z\left(E_X\left(\hat{Q}-X\right)^+\right) = E_Z\left(\int_0^{min(b,\hat{Q})} (\hat{Q}-x)f_X(x)dx\right)$$

We assume that $\hat{Q} <= Q < b$, which is a reasonable assumption

$$E_{X,Z}\left(\hat{Q}-X\right)^+ = E_Z\left(\frac{(Q-Z)^2}{2b}\right) = \frac{Q^2+\mu^2+\sigma^2-2Q\mu}{2b}$$

Hence the expected profit function takes the form,

$$E[\Pi(Q)] = (r-c+\pi)(Q-\mu) - (r+h+\pi)\left(\frac{Q^2+\mu^2+\sigma^2-2Q\mu}{2b}\right) - \frac{b}{2}\pi$$

3.2 Optimization and Introduction to supplier selection

The profit maximizing quantity is given by

$$Q^* = \frac{b(r-c+\pi)}{r+h+\pi} + \mu = Q^o + k(p\mu_c + (1-p)\mu_n)$$

Lets derive the useful quantities then we shall move ahead with the supplier selection quantities.

3.3 Type 1 service level constraint

$$\alpha = E_Y\left[P(X < Q-Z)\right] = \frac{Q-\mu}{b}$$

Say we want a service level greater than α_0, i.e..

$$\frac{Q-\mu}{b} > \alpha_0 \implies Q > b\alpha_0 + k(p\mu_c + (1-p)\mu_n)$$

Here, referring to fig 2 we can observe again that the constrained optimal order quantity either increases or stays constant with an increase in the number of suppliers, and so the profit either reduces or stays the same.

3.4 Type 2 service level constraint

$$\beta = \frac{E[\hat{Q}-(\hat{Q}-x)^+]}{E[x]} = \frac{(Q-\mu)-(\frac{Q^2+\mu^2+\sigma^2-2Q\mu}{2b})}{\frac{b}{2}} = (-1)*\frac{Q^2+\mu^2+\sigma^2-2b\mu-Q(2b+2\mu)}{b^2}$$

Now, suppose we want a fill rate $\geq \beta$ i.e..

$$\implies \frac{Q^2+\mu^2+\sigma^2-2b\mu-Q(2b+2\mu)}{b^2}+\beta \leq 0$$

$$Q^2-2Q(b+\mu)+\mu^2-2b\mu+\beta b^2+\sigma^2 \leq 0$$

$$Q^2-2Q(b+\mu)+\mu^2-2b\mu+b^2+(\beta-1)b^2+\sigma^2 \leq 0$$

$$Q^2-2Q(b+\mu)+(\mu-b)^2+\sigma^2-(1-\beta)b^2 \leq 0$$

Under this constraint,

$$Q^- \leq Q \leq Q^+$$

where Q^+ and Q^- are given by,

$$Q^+ = (b+\mu)+\sqrt{4(b\mu)+(1-\beta)b^2-\sigma^2} \quad \text{and} \quad Q^- = (b+\mu)-\sqrt{4(b\mu)+(1-\beta)b^2-\sigma^2}$$

We assume that $4b\mu > \sigma^2$, hence the discriminant will be positive and the roots will be real. If we put

$$\mu = k(p\mu_c+(1-p)\mu_n) = k\mu_i$$

and

$$\sigma^2 = k\left(p(\mu_c^2+\sigma_c^2)-p^2\mu_c^2+(1-p)(\mu_n^2+\sigma_n^2)-(1-p)^2\mu_n^2\right) = k\sigma_i^2$$

in the expressions of Q_1 and Q_2 we have a conclusion similar to what we got in multiple supplier model fill rate constraint, i.e.. If we increase the number of suppliers (i.e.. k) then Q^+ will increase for sure, but we are not sure about Q^-.