Contents

Chapter 1

Divide and Conquer | Set 1 (Introduction)

Like Greedyand Dynamic Programming, Divide and Conquer is an algorithmic paradigm. A typical Divide and Conquer algorithm solves a problem using following three steps.

1. *Divide:* Break the given problem into subproblems of same type.
2. *Conquer:* Recursively solve these subproblems
3. *Combine:* Appropriately combine the answers

Following are some standard algorithms that are Divide and Conquer algorithms.

1) Binary Search is a searching algorithm. In each step, the algorithm compares the input element x with the value of the middle element in array. If the values match, return the index of middle. Otherwise, if x is less than the middle element, then the algorithm recurs for left side of middle element, else recurs for right side of middle element.

2) Quicksort is a sorting algorithm. The algorithm picks a pivot element, rearranges the array elements in such a way that all elements smaller than the picked pivot element move to left side of pivot, and all greater elements move to right side. Finally, the algorithm recursively sorts the subarrays on left and right of pivot element.

3) Merge Sort is also a sorting algorithm. The algorithm divides the array in two halves, recursively sorts them and finally merges the two sorted halves.

4) Closest Pair of Points The problem is to find the closest pair of points in a set of points in x-y plane. The problem can be solved in O(n^2) time by calculating distances of every pair of points and comparing the distances to find the minimum. The Divide and Conquer algorithm solves the problem in O(nLogn) time.

5) Strassen's Algorithm is an efficient algorithm to multiply two matrices. A simple method to multiply two matrices need 3 nested loops and is O(n^3). Strassen's algorithm multiplies two matrices in O(n^2.8974) time.

6) Cooley–Tukey Fast Fourier Transform (FFT) algorithm is the most common algorithm for FFT. It is a divide and conquer algorithm which works in O(nlogn) time.

7) Karatsuba algorithm for fast multiplication it does multiplication of two n-digit numbers in at most $3n^{\log_2 3} \approx 3n^{1.585}$ single-digit multiplications in general (and exactly $n^{\log_2 3}$ when n is a power of 2). It is therefore faster than the classicalalgorithm, which requires n^2 single-digit products. If $n = 2^{10} = 1024$, in particular, the exact counts are $3^{10} = 59{,}049$ and $(2^{10})^2 = 1{,}048{,}576$, respectively.

We will publishing above algorithms in separate posts.

Divide and Conquer (D & C) vs Dynamic Programming (DP)
Both paradigms (D & C and DP) divide the given problem into subproblems and solve subproblems. How to choose one of them for a given problem? Divide and Conquer should be used when same subproblems are not evaluated many times. Otherwise Dynamic Programming or Memoization should be used. For example, Binary Search is a Divide and Conquer algorithm, we never evaluate the same subproblems again. On the other hand, for calculating nth Fibonacci number, Dynamic Programming should be preferred (See thisfor details).

References

Algorithms by Sanjoy Dasgupta, Christos Papadimitriou, Umesh Vazirani

Introduction to Algorithms by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L.
http://en.wikipedia.org/wiki/Karatsuba_algorithm

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

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http://www.geeksforgeeks.org/divide-and-conquer-set-1-find-closest-pair-of-points/

Chapter 2

Write a C program to calculate pow(x,n)

Below solution divides the problem into subproblems of size y/2 and call the subproblems recursively.

```
#include<stdio.h>

/* Function to calculate x raised to the power y */
int power(int x, unsigned int y)
{
    if( y == 0)
        return 1;
    else if (y%2 == 0)
        return power(x, y/2)*power(x, y/2);
    else
        return x*power(x, y/2)*power(x, y/2);

}

/* Program to test function power */
int main()
{
    int x = 2;
    unsigned int y = 3;

    printf("%d", power(x, y));
    getchar();
    return 0;
}
```

Time Complexity: O(n)
Space Complexity: O(1)
Algorithmic Paradigm: Divide and conquer.

Above function can be optimized to O(logn) by calculating power(x, y/2) only once and storing it.

```
/* Function to calculate x raised to the power y in O(logn)*/
int power(int x, unsigned int y)
{
    int temp;
    if( y == 0)
        return 1;
    temp = power(x, y/2);
    if (y%2 == 0)
        return temp*temp;
    else
        return x*temp*temp;
}
```

Time Complexity of optimized solution: O(logn)
Let us extend the pow function to work for negative y and float x.

```
/* Extended version of power function that can work
 for float x and negative y*/
#include<stdio.h>

float power(float x, int y)
{
    float temp;
    if( y == 0)
       return 1;
    temp = power(x, y/2);
    if (y%2 == 0)
        return temp*temp;
    else
    {
        if(y > 0)
            return x*temp*temp;
        else
            return (temp*temp)/x;
    }
}

/* Program to test function power */
int main()
{
    float x = 2;
    int y = -3;
    printf("%f", power(x, y));
    getchar();
    return 0;
}
```

Source

http://www.geeksforgeeks.org/write-a-c-program-to-calculate-powxn/

Category: C/C++ Puzzles Tags: Divide and Conquer

Chapter 3

Median of two sorted arrays

Question: There are 2 sorted arrays A and B of size n each. Write an algorithm to find the median of the array obtained after merging the above 2 arrays(i.e. array of length 2n). The complexity should be O(log(n))

Median: In probability theory and statistics, a median is described as the number separating the higher half of a sample, a population, or a probability distribution, from the lower half.
The median of a finite list of numbers can be found by arranging all the numbers from lowest value to highest value and picking the middle one.

For getting the median of input array { 12, 11, 15, 10, 20 }, first sort the array. We get { 10, 11, 12, 15, 20 } after sorting. Median is the middle element of the sorted array which is 12.

There are different conventions to take median of an array with even number of elements, one can take the mean of the two middle values, or first middle value, or second middle value.

Let us see different methods to get the median of two sorted arrays of size n each. Since size of the set for which we are looking for median is even (2n), we are taking average of middle two numbers in all below solutions.

Method 1 (Simply count while Merging)
Use merge procedure of merge sort. Keep track of count while comparing elements of two arrays. If count becomes n(For 2n elements), we have reached the median. Take the average of the elements at indexes n-1 and n in the merged array. See the below implementation.

Implementation:

```
#include <stdio.h>

/* This function returns median of ar1[] and ar2[].
   Assumptions in this function:
   Both ar1[] and ar2[] are sorted arrays
   Both have n elements */
int getMedian(int ar1[], int ar2[], int n)
{
    int i = 0;  /* Current index of i/p array ar1[] */
    int j = 0; /* Current index of i/p array ar2[] */
    int count;
    int m1 = -1, m2 = -1;

    /* Since there are 2n elements, median will be average
```

```
     of elements at index n-1 and n in the array obtained after
     merging ar1 and ar2 */
    for (count = 0; count <= n; count++)
    {
        /*Below is to handle case where all elements of ar1[] are
          smaller than smallest(or first) element of ar2[]*/
        if (i == n)
        {
            m1 = m2;
            m2 = ar2[0];
            break;
        }

        /*Below is to handle case where all elements of ar2[] are
          smaller than smallest(or first) element of ar1[]*/
        else if (j == n)
        {
            m1 = m2;
            m2 = ar1[0];
            break;
        }

        if (ar1[i] < ar2[j])
        {
            m1 = m2;  /* Store the prev median */
            m2 = ar1[i];
            i++;
        }
        else
        {
            m1 = m2;  /* Store the prev median */
            m2 = ar2[j];
            j++;
        }
    }

    return (m1 + m2)/2;
}

/* Driver program to test above function */
int main()
{
    int ar1[] = {1, 12, 15, 26, 38};
    int ar2[] = {2, 13, 17, 30, 45};

    int n1 = sizeof(ar1)/sizeof(ar1[0]);
    int n2 = sizeof(ar2)/sizeof(ar2[0]);
    if (n1 == n2)
        printf("Median is %d", getMedian(ar1, ar2, n1));
    else
        printf("Doesn't work for arrays of unequal size");
    getchar();
    return 0;
}
```

Time Complexity: O(n)

Method 2 (By comparing the medians of two arrays)
This method works by first getting medians of the two sorted arrays and then comparing them.

Let ar1 and ar2 be the input arrays.

Algorithm:

```
1) Calculate the medians m1 and m2 of the input arrays ar1[]
   and ar2[] respectively.
2) If m1 and m2 both are equal then we are done.
     return m1 (or m2)
3) If m1 is greater than m2, then median is present in one
   of the below two subarrays.
    a)  From first element of ar1 to m1 (ar1[0...|_n/2_|])
    b)  From m2 to last element of ar2  (ar2[|_n/2_|...n-1])
4) If m2 is greater than m1, then median is present in one
   of the below two subarrays.
   a)  From m1 to last element of ar1  (ar1[|_n/2_|...n-1])
   b)  From first element of ar2 to m2 (ar2[0...|_n/2_|])
5) Repeat the above process until size of both the subarrays
   becomes 2.
6) If size of the two arrays is 2 then use below formula to get
  the median.
    Median = (max(ar1[0], ar2[0]) + min(ar1[1], ar2[1]))/2
```

Example:

```
   ar1[] = {1, 12, 15, 26, 38}
   ar2[] = {2, 13, 17, 30, 45}
```

For above two arrays m1 = 15 and m2 = 17

For the above ar1[] and ar2[], m1 is smaller than m2. So median is present in one of the following two subarrays.

```
   [15, 26, 38] and [2, 13, 17]
```

Let us repeat the process for above two subarrays:

```
    m1 = 26 m2 = 13.
```

m1 is greater than m2. So the subarrays become

```
  [15, 26] and [13, 17]
Now size is 2, so median = (max(ar1[0], ar2[0]) + min(ar1[1], ar2[1]))/2
                        = (max(15, 13) + min(26, 17))/2
                        = (15 + 17)/2
                        = 16
```

Implementation:

```
#include<stdio.h>

int max(int, int); /* to get maximum of two integers */
int min(int, int); /* to get minimum of two integeres */
int median(int [], int); /* to get median of a sorted array */

/* This function returns median of ar1[] and ar2[].
   Assumptions in this function:
   Both ar1[] and ar2[] are sorted arrays
   Both have n elements */
int getMedian(int ar1[], int ar2[], int n)
{
    int m1; /* For median of ar1 */
    int m2; /* For median of ar2 */

    /* return -1  for invalid input */
    if (n <= 0)
        return -1;

    if (n == 1)
        return (ar1[0] + ar2[0])/2;

    if (n == 2)
        return (max(ar1[0], ar2[0]) + min(ar1[1], ar2[1])) / 2;

    m1 = median(ar1, n); /* get the median of the first array */
    m2 = median(ar2, n); /* get the median of the second array */

    /* If medians are equal then return either m1 or m2 */
    if (m1 == m2)
        return m1;

     /* if m1 < m2 then median must exist in ar1[m1....] and ar2[....m2] */
    if (m1 < m2)
    {
        if (n % 2 == 0)
            return getMedian(ar1 + n/2 - 1, ar2, n - n/2 +1);
        else
            return getMedian(ar1 + n/2, ar2, n - n/2);
    }
```

```
    /* if m1 > m2 then median must exist in ar1[....m1] and ar2[m2...] */
    else
    {
        if (n % 2 == 0)
            return getMedian(ar2 + n/2 - 1, ar1, n - n/2 + 1);
        else
            return getMedian(ar2 + n/2, ar1, n - n/2);
    }
}

/* Function to get median of a sorted array */
int median(int arr[], int n)
{
    if (n%2 == 0)
        return (arr[n/2] + arr[n/2-1])/2;
    else
        return arr[n/2];
}

/* Driver program to test above function */
int main()
{
    int ar1[] = {1, 2, 3, 6};
    int ar2[] = {4, 6, 8, 10};
    int n1 = sizeof(ar1)/sizeof(ar1[0]);
    int n2 = sizeof(ar2)/sizeof(ar2[0]);
    if (n1 == n2)
      printf("Median is %d", getMedian(ar1, ar2, n1));
    else
     printf("Doesn't work for arrays of unequal size");

    getchar();
    return 0;
}

/* Utility functions */
int max(int x, int y)
{
    return x > y? x : y;
}

int min(int x, int y)
{
    return x > y? y : x;
}
```

Time Complexity: O(logn)
Algorithmic Paradigm: Divide and Conquer

Method 3 (By doing binary search for the median):

The basic idea is that if you are given two arrays ar1[] and ar2[] and know the length of each, you can check whether an element ar1[i] is the median in constant time. Suppose that the median is ar1[i]. Since the array is sorted, it is greater than exactly i values in array ar1[]. Then if it is the median, it is also greater than exactly j = n – i – 1 elements in ar2[].
It requires constant time to check if ar2[j] 1) Get the middle element of ar1[] using array indexes left and right. Let index of the middle element be i. 2) Calculate the corresponding index j of ar2[] j = n – i – 1 3) If ar1[i] >= ar2[j] and ar1[i]

Example:

```
ar1[] = {1, 5, 7, 10, 13}
ar2[] = {11, 15, 23, 30, 45}
```

Middle element of ar1[] is 7. Let us compare 7 with 23 and 30, since 7 smaller than both 23 and 30, move to right in ar1[]. Do binary search in {10, 13}, this step will pick 10. Now compare 10 with 15 and 23. Since 10 is smaller than both 15 and 23, again move to right. Only 13 is there in right side now. Since 13 is greater than 11 and smaller than 15, terminate here. We have got the median as 12 (average of 11 and 13)

Implementation:

```
#include<stdio.h>

int getMedianRec(int ar1[], int ar2[], int left, int right, int n);

/* This function returns median of ar1[] and ar2[].
   Assumptions in this function:
   Both ar1[] and ar2[] are sorted arrays
   Both have n elements */
int getMedian(int ar1[], int ar2[], int n)
{
    return getMedianRec(ar1, ar2, 0, n-1, n);
}

/* A recursive function to get the median of ar1[] and ar2[]
   using binary search */
int getMedianRec(int ar1[], int ar2[], int left, int right, int n)
{
    int i, j;

    /* We have reached at the end (left or right) of ar1[] */
    if (left > right)
        return getMedianRec(ar2, ar1, 0, n-1, n);

    i = (left + right)/2;
    j = n - i - 1;  /* Index of ar2[] */

    /* Recursion terminates here.*/
    if (ar1[i] > ar2[j] && (j == n-1 || ar1[i] <= ar2[j+1]))
    {
        /* ar1[i] is decided as median 2, now select the median 1
           (element just before ar1[i] in merged array) to get the
           average of both*/
```

```
        if (i == 0 || ar2[j] > ar1[i-1])
            return (ar1[i] + ar2[j])/2;
        else
            return (ar1[i] + ar1[i-1])/2;
    }

    /*Search in left half of ar1[]*/
    else if (ar1[i] > ar2[j] && j != n-1 && ar1[i] > ar2[j+1])
        return getMedianRec(ar1, ar2, left, i-1, n);

    /*Search in right half of ar1[]*/
    else /* ar1[i] is smaller than both ar2[j] and ar2[j+1]*/
        return getMedianRec(ar1, ar2, i+1, right, n);
}

/* Driver program to test above function */
int main()
{
    int ar1[] = {1, 12, 15, 26, 38};
    int ar2[] = {2, 13, 17, 30, 45};
    int n1 = sizeof(ar1)/sizeof(ar1[0]);
    int n2 = sizeof(ar2)/sizeof(ar2[0]);
    if (n1 == n2)
        printf("Median is %d", getMedian(ar1, ar2, n1));
    else
        printf("Doesn't work for arrays of unequal size");

    getchar();
    return 0;
}
```

Time Complexity: O(logn)
Algorithmic Paradigm: Divide and Conquer

The above solutions can be optimized for the cases when all elements of one array are smaller than all elements of other array. For example, in method 3, we can change the getMedian() function to following so that these cases can be handled in O(1) time. Thanks to nutcracker for suggesting this optimization.

```
/* This function returns median of ar1[] and ar2[].
   Assumptions in this function:
   Both ar1[] and ar2[] are sorted arrays
   Both have n elements */
int getMedian(int ar1[], int ar2[], int n)
{
   // If all elements of array 1 are smaller then
   // median is average of last element of ar1 and
   // first element of ar2
   if (ar1[n-1] < ar2[0])
     return (ar1[n-1]+ar2[0])/2;

   // If all elements of array 1 are smaller then
   // median is average of first element of ar1 and
```

```
   // last element of ar2
   if (ar2[n-1] < ar1[0])
     return (ar2[n-1]+ar1[0])/2;

   return getMedianRec(ar1, ar2, 0, n-1, n);
}
```

References:
http://en.wikipedia.org/wiki/Median

http://ocw.alfaisal.edu/NR/rdonlyres/Electrical-Engineering-and-Computer-Science/6-046JFall-2005/30C68118-E436-4FE3-8C79-6BAFBB07D935/0/ps9sol.pdf ds3etph5wn

Asked by Snehal

Please write comments if you find the above codes/algorithms incorrect, or find other ways to solve the same problem.

Source

http://www.geeksforgeeks.org/median-of-two-sorted-arrays/

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Chapter 4

Count Inversions in an array

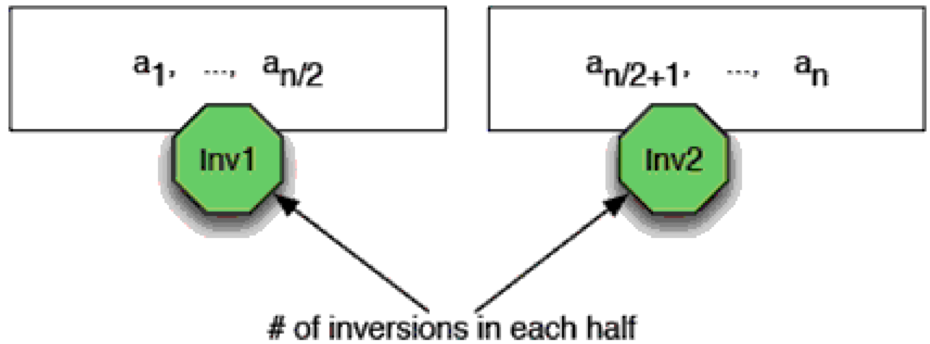
Inversion Count for an array indicates – how far (or close) the array is from being sorted. If array is already sorted then inversion count is 0. If array is sorted in reverse order that inversion count is the maximum. Formally speaking, two elements a[i] and a[j] form an inversion if a[i] > a[j] and i Example:
The sequence 2, 4, 1, 3, 5 has three inversions (2, 1), (4, 1), (4, 3).

METHOD 1 (Simple)
For each element, count number of elements which are on right side of it and are smaller than it.

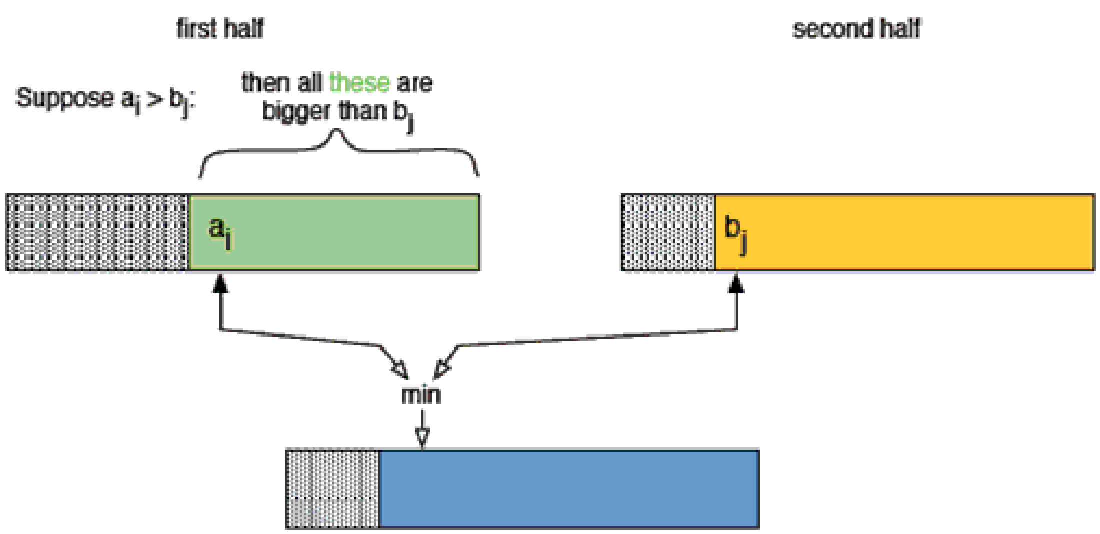
```
int getInvCount(int arr[], int n)
{
  int inv_count = 0;
  int i, j;

  for(i = 0; i < n - 1; i++)
    for(j = i+1; j < n; j++)
      if(arr[i] > arr[j])
        inv_count++;

  return inv_count;
}

/* Driver progra to test above functions */
int main(int argv, char** args)
{
  int arr[] = {1, 20, 6, 4, 5};
  printf(" Number of inversions are %d \n", getInvCount(arr, 5));
  getchar();
  return 0;
}
```

Time Complexity: O(n^2)

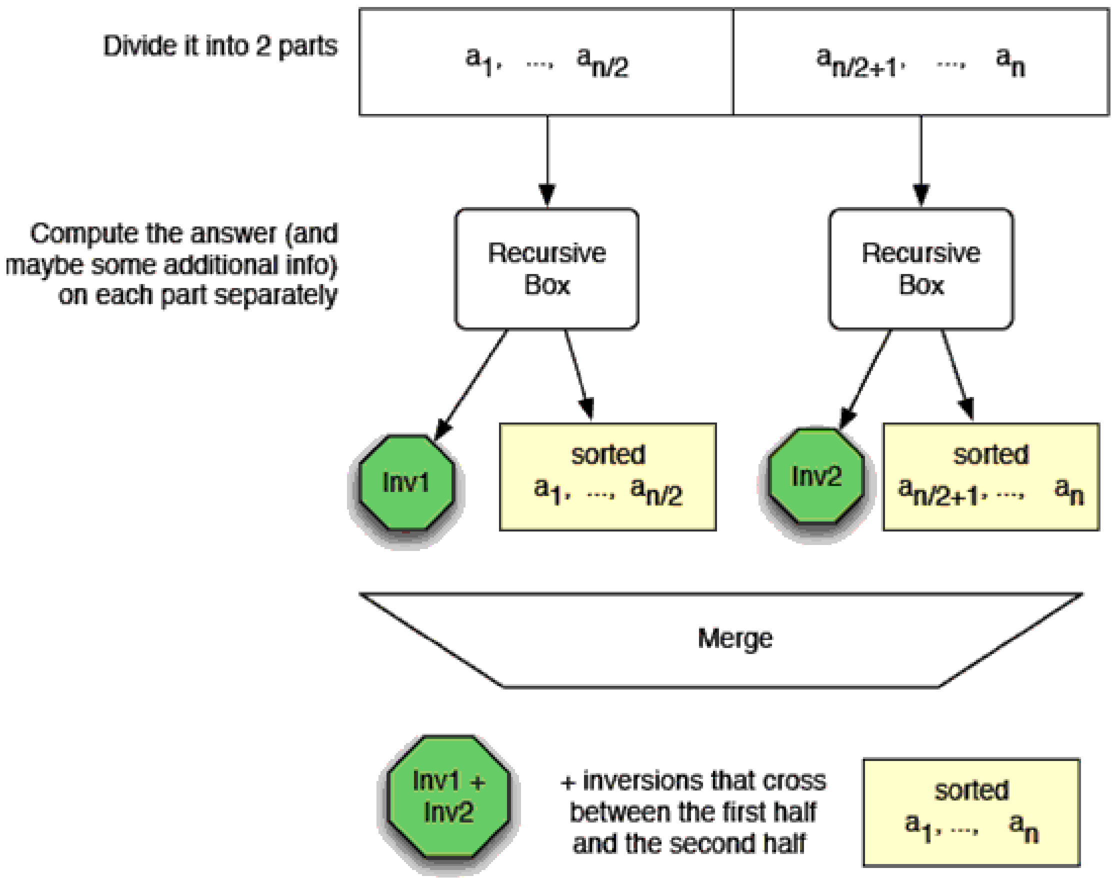
METHOD 2(Enhance Merge Sort)
Suppose we know the number of inversions in the left half and right half of the array (let be inv1 and inv2), what kinds of inversions are not accounted for in Inv1 + Inv2? The answer is – the inversions we have to count during the merge step. Therefore, to get number of inversions, we need to add number of inversions in left subarray, right subarray and merge().



How to get number of inversions in merge()?
In merge process, let i is used for indexing left sub-array and j for right sub-array. At any step in merge(), if a[i] is greater than a[j], then there are (mid – i) inversions. because left and right subarrays are sorted, so all the remaining elements in left-subarray (a[i+1], a[i+2] … a[mid]) will be greater than a[j]



The complete picture:



Implementation:

```
#include <stdio.h>
#include <stdlib.h>

int  _mergeSort(int arr[], int temp[], int left, int right);
int merge(int arr[], int temp[], int left, int mid, int right);

/* This function sorts the input array and returns the
   number of inversions in the array */
int mergeSort(int arr[], int array_size)
{
    int *temp = (int *)malloc(sizeof(int)*array_size);
    return _mergeSort(arr, temp, 0, array_size - 1);
}

/* An auxiliary recursive function that sorts the input array and
  returns the number of inversions in the array. */
int _mergeSort(int arr[], int temp[], int left, int right)
{
  int mid, inv_count = 0;
```

```
  if (right > left)
  {
    /* Divide the array into two parts and call _mergeSortAndCountInv()
       for each of the parts */
    mid = (right + left)/2;

    /* Inversion count will be sum of inversions in left-part, right-part
      and number of inversions in merging */
    inv_count  = _mergeSort(arr, temp, left, mid);
    inv_count += _mergeSort(arr, temp, mid+1, right);

    /*Merge the two parts*/
    inv_count += merge(arr, temp, left, mid+1, right);
  }
  return inv_count;
}

/* This funt merges two sorted arrays and returns inversion count in
   the arrays.*/
int merge(int arr[], int temp[], int left, int mid, int right)
{
  int i, j, k;
  int inv_count = 0;

  i = left; /* i is index for left subarray*/
  j = mid;  /* i is index for right subarray*/
  k = left; /* i is index for resultant merged subarray*/
  while ((i <= mid - 1) && (j <= right))
  {
    if (arr[i] <= arr[j])
    {
      temp[k++] = arr[i++];
    }
    else
    {
      temp[k++] = arr[j++];

     /*this is tricky -- see above explanation/diagram for merge()*/
      inv_count = inv_count + (mid - i);
    }
  }

  /* Copy the remaining elements of left subarray
   (if there are any) to temp*/
  while (i <= mid - 1)
    temp[k++] = arr[i++];

  /* Copy the remaining elements of right subarray
   (if there are any) to temp*/
  while (j <= right)
    temp[k++] = arr[j++];

  /*Copy back the merged elements to original array*/
  for (i=left; i <= right; i++)
```

```
    arr[i] = temp[i];

  return inv_count;
}

/* Driver progra to test above functions */
int main(int argv, char** args)
{
  int arr[] = {1, 20, 6, 4, 5};
  printf(" Number of inversions are %d \n", mergeSort(arr, 5));
  getchar();
  return 0;
}
```

Note that above code modifies (or sorts) the input array. If we want to count only inversions then we need to create a copy of original array and call mergeSort() on copy.

Time Complexity: O(nlogn)
Algorithmic Paradigm: Divide and Conquer

References:

http://www.cs.umd.edu/class/fall2009/cmsc451/lectures/Lec08-inversions.pdf

http://www.cp.eng.chula.ac.th/~piak/teaching/algo/algo2008/count-inv.htm

Please write comments if you find any bug in the above program/algorithm or other ways to solve the same problem.

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http://www.geeksforgeeks.org/counting-inversions/

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Chapter 5

Divide and Conquer | Set 2 (Closest Pair of Points)

We are given an array of n points in the plane, and the problem is to find out the closest pair of points in the array. This problem arises in a number of applications. For example, in air-traffic control, you may want to monitor planes that come too close together, since this may indicate a possible collision. Recall the following formula for distance between two points p and q.

$$\|pq\| = \sqrt{(p_x - q_x)^2 + (p_y - q_y)^2}.$$

The Brute force solution is O(nˆ2), compute the distance between each pair and return the smallest. We can calculate the smallest distance in O(nLogn) time using Divide and Conquer strategy. In this post, a O(n x (Logn)ˆ2) approach is discussed. We will be discussing a O(nLogn) approach in a separate post.

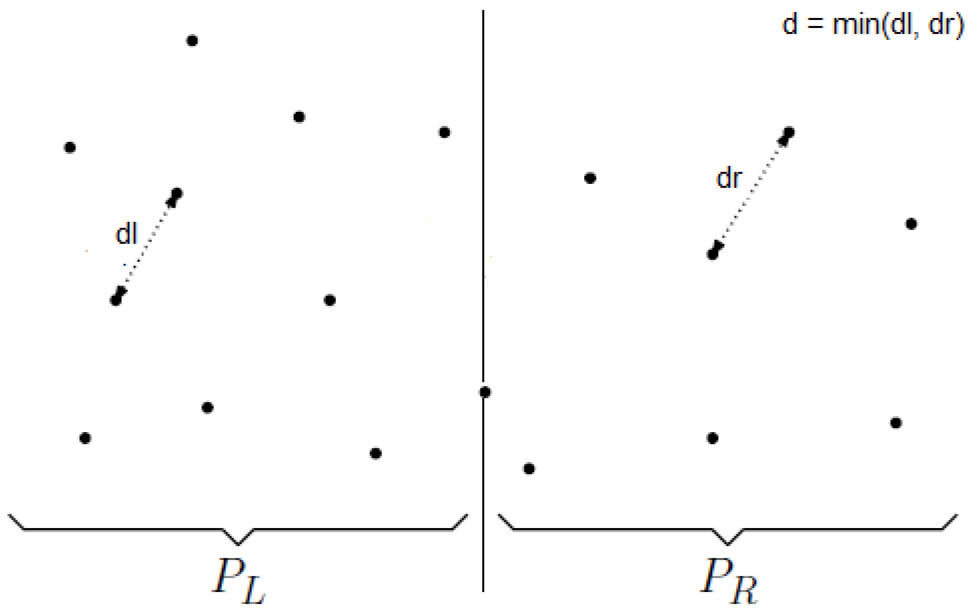
Algorithm
Following are the detailed steps of a O(n (Logn)ˆ2) algortihm.
Input: An array of n points *P[]*
Output: The smallest distance between two points in the given array.

As a pre-processing step, input array is sorted according to x coordinates.

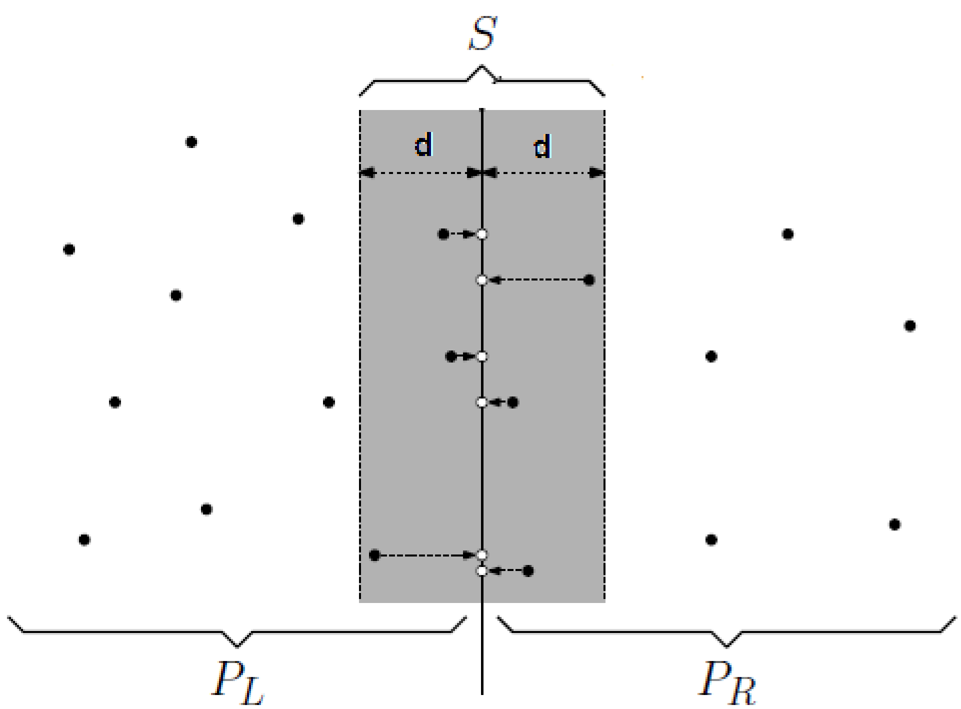
1) Find the middle point in the sorted array, we can take *P[n/2]* as middle point.

2) Divide the given array in two halves. The first subarray contains points from P[0] to P[n/2]. The second subarray contains points from P[n/2+1] to P[n-1].

3) Recursively find the smallest distances in both subarrays. Let the distances be dl and dr. Find the minimum of dl and dr. Let the minimum be d.



4) From above 3 steps, we have an upper bound d of minimum distance. Now we need to consider the pairs such that one point in pair is from left half and other is from right half. Consider the vertical line passing through passing through P[n/2] and find all points whose x coordinate is closer than d to the middle vertical line. Build an array strip[] of all such points.



5) Sort the array strip[] according to y coordinates. This step is O(nLogn). It can be optimized to O(n) by recursively sorting and merging.

6) Find the smallest distance in strip[]. This is tricky. From first look, it seems to be a O(n^2) step, but it is actually O(n). It can be proved geometrically that for every point in strip, we only need to check at most

7 points after it (note that strip is sorted according to Y coordinate). See thisfor more analysis.

7) Finally return the minimum of d and distance calculated in above step (step 6)

Implementation
Following is C/C++ implementation of the above algorithm.

```
// A divide and conquer program in C/C++ to find the smallest distance from a
// given set of points.

#include <stdio.h>
#include <float.h>
#include <stdlib.h>
#include <math.h>

// A structure to represent a Point in 2D plane
struct Point
{
    int x, y;
};

/* Following two functions are needed for library function qsort().
   Refer: http://www.cplusplus.com/reference/clibrary/cstdlib/qsort/ */

// Needed to sort array of points according to X coordinate
int compareX(const void* a, const void* b)
{
    Point *p1 = (Point *)a,  *p2 = (Point *)b;
    return (p1->x - p2->x);
}
// Needed to sort array of points according to Y coordinate
int compareY(const void* a, const void* b)
{
    Point *p1 = (Point *)a,   *p2 = (Point *)b;
    return (p1->y - p2->y);
}

// A utility function to find the distance between two points
float dist(Point p1, Point p2)
{
    return sqrt( (p1.x - p2.x)*(p1.x - p2.x) +
                 (p1.y - p2.y)*(p1.y - p2.y)
               );
}

// A Brute Force method to return the smallest distance between two points
// in P[] of size n
float bruteForce(Point P[], int n)
{
    float min = FLT_MAX;
    for (int i = 0; i < n; ++i)
        for (int j = i+1; j < n; ++j)
            if (dist(P[i], P[j]) < min)
                min = dist(P[i], P[j]);
```

```
    return min;
}

// A utility function to find minimum of two float values
float min(float x, float y)
{
    return (x < y)? x : y;
}


// A utility function to find the distance beween the closest points of
// strip of given size. All points in strip[] are sorted accordint to
// y coordinate. They all have an upper bound on minimum distance as d.
// Note that this method seems to be a O(n^2) method, but it's a O(n)
// method as the inner loop runs at most 6 times
float stripClosest(Point strip[], int size, float d)
{
    float min = d;  // Initialize the minimum distance as d

    qsort(strip, size, sizeof(Point), compareY);

    // Pick all points one by one and try the next points till the difference
    // between y coordinates is smaller than d.
    // This is a proven fact that this loop runs at most 6 times
    for (int i = 0; i < size; ++i)
        for (int j = i+1; j < size && (strip[j].y - strip[i].y) < min; ++j)
            if (dist(strip[i],strip[j]) < min)
                min = dist(strip[i], strip[j]);

    return min;
}

// A recursive function to find the smallest distance. The array P contains
// all points sorted according to x coordinate
float closestUtil(Point P[], int n)
{
    // If there are 2 or 3 points, then use brute force
    if (n <= 3)
        return bruteForce(P, n);

    // Find the middle point
    int mid = n/2;
    Point midPoint = P[mid];

    // Consider the vertical line passing through the middle point
    // calculate the smallest distance dl on left of middle point and
    // dr on right side
    float dl = closestUtil(P, mid);
    float dr = closestUtil(P + mid, n-mid);

    // Find the smaller of two distances
    float d = min(dl, dr);

    // Build an array strip[] that contains points close (closer than d)
```

```
    // to the line passing through the middle point
    Point strip[n];
    int j = 0;
    for (int i = 0; i < n; i++)
        if (abs(P[i].x - midPoint.x) < d)
            strip[j] = P[i], j++;

    // Find the closest points in strip.  Return the minimum of d and closest
    // distance is strip[]
    return min(d, stripClosest(strip, j, d) );
}

// The main functin that finds the smallest distance
// This method mainly uses closestUtil()
float closest(Point P[], int n)
{
    qsort(P, n, sizeof(Point), compareX);

    // Use recursive function closestUtil() to find the smallest distance
    return closestUtil(P, n);
}

// Driver program to test above functions
int main()
{
    Point P[] = {{2, 3}, {12, 30}, {40, 50}, {5, 1}, {12, 10}, {3, 4}};
    int n = sizeof(P) / sizeof(P[0]);
    printf("The smallest distance is %f ", closest(P, n));
    return 0;
}
```

Output:

```
 The smallest distance is 1.414214
```

Time Complexity Let Time complexity of above algorithm be T(n). Let us assume that we use a O(nLogn) sorting algorithm. The above algorithm divides all points in two sets and recursively calls for two sets. After dividing, it finds the strip in O(n) time, sorts the strip in O(nLogn) time and finally finds the closest points in strip in O(n) time. So T(n) can expressed as follows
T(n) = 2T(n/2) + O(n) + O(nLogn) + O(n)
T(n) = 2T(n/2) + O(nLogn)
T(n) = T(n x Logn x Logn)

Notes
1) Time complexity can be improved to O(nLogn) by optimizing step 5 of the above algorithm. We will soon be discussing the optimized solution in a separate post.
2) The code finds smallest distance. It can be easily modified to find the points with smallest distance.
3) The code uses quick sort which can be O(n^2) in worst case. To have the upper bound as O(n (Logn)^2), a O(nLogn) sorting algorithm like merge sort or heap sort can be used

References:
http://www.cs.umd.edu/class/fall2013/cmsc451/Lects/lect10.pdf
http://www.youtube.com/watch?v=vS4Zn1a9KUc
http://www.youtube.com/watch?v=T3T7T8Ym20M
http://en.wikipedia.org/wiki/Closest_pair_of_points_problem

Source

http://www.geeksforgeeks.org/closest-pair-of-points/

Category: Misc Tags: Closest Pair of Points, Divide and Conquer

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Chapter 6

Divide and Conquer | Set 5 (Strassen's Matrix Multiplication)

Given two square matrices A and B of size n x n each, find their multiplication matrix.

Naive Method
Following is a simple way to multiply two matrices.

```
void multiply(int A[][N], int B[][N], int C[][N])
{
    for (int i = 0; i < N; i++)
    {
        for (int j = 0; j < N; j++)
        {
            C[i][j] = 0;
            for (int k = 0; k < N; k++)
            {
                C[i][j] += A[i][k]*B[k][j];
            }
        }
    }
}
```

Time Complexity of above method is $O(N^3)$.

Divide and Conquer
Following is simple Divide and Conquer method to multiply two square matrices.
1) Divide matrices A and B in 4 sub-matrices of size N/2 x N/2 as shown in the below diagram.
2) Calculate following values recursively. ae + bg, af + bh, ce + dg and cf + dh.

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}}_{C}$$

A, B and C are square metrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2

In the above method, we do 8 multiplications for matrices of size N/2 x N/2 and 4 additions. Addition of two matrices takes $O(N^2)$ time. So the time complexity can be written as

```
T(N) = 8T(N/2) + O(N2)

From Master's Theorem, time complexity of above method is O(N3)
which is unfortunately same as the above naive method.
```

Simple Divide and Conquer also leads to $O(N^3)$, can there be a better way?
In the above divide and conquer method, the main component for high time complexity is 8 recursive calls. The idea of **Strassen's method** is to reduce the number of recursive calls to 7. Strassen's method is similar to above simple divide and conquer method in the sense that this method also divide matrices to sub-matrices of size N/2 x N/2 as shown in the above diagram, but in Strassen's method, the four sub-matrices of result are calculated using following formulae.

$$\begin{aligned} p1 &= a(f - h) & p2 &= (a + b)h \\ p3 &= (c + d)e & p4 &= d(g - e) \\ p5 &= (a + d)(e + h) & p6 &= (b - d)(g + h) \\ p7 &= (a - c)(e + f) \end{aligned}$$

The A x B can be calculated using above seven multiplications.
Following are values of four sub-matrices of result C

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} p5 + p4 - p2 + p6 & p1 + p2 \\ p3 + p4 & p1 + p5 - p3 - p7 \end{bmatrix}}_{C}$$

A, B and C are square metrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2
p1, p2, p3, p4, p5, p6 and p7 are submatrices of size N/2 x N/2

Time Complexity of Strassen's Method
Addition and Subtraction of two matrices takes $O(N^2)$ time. So time complexity can be written as

```
T(N) = 7T(N/2) +  O(N2)

From Master's Theorem, time complexity of above method is
O(NLog7) which is approximately O(N2.8074)
```

Generally Strassen's Method is not preferred for practical applications for following reasons.
1) The constants used in Strassen's method are high and for a typical application Naive method works better.
2) For Sparse matrices, there are better methods especially designed for them.
3) The submatrices in recursion take extra space.
4) Because of the limited precision of computer arithmetic on noninteger values, larger errors accumulate in Strassen's algorithm than in Naive Method (Source: CLRS Book)

References:

Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest
https://www.youtube.com/watch?v=LOLebQ8nKHA
https://www.youtube.com/watch?v=QXY4RskLQcI

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http://www.geeksforgeeks.org/strassens-matrix-multiplication/