Attachment Cross Section:

OML theory assumes that for every electron energy there exists an electron impact parameter that makes the electron hit the dust particle with a grazing incidence. The trajectory of an electron can be derived as in the classical collision theory except that, due to the finite radius of the dust particle, the electron can be collected if its radial position reaches the radius of the dust particle (r = R). In case of an electron with an energy E and angular momentum J at infinity, and a dust particle with radius R is placed at r = 0, the conservation of energy and angular momentum gives

$$E \;=\; \tfrac{1}{2}m_e r^{*2} + \tfrac{1}{2}\frac{J}{m_e r^2} + (-e)V(r)$$

Where electron impact parameter b is related to J by $J = m_e v_o b$ (v_o is the initial electron velocity). Considering $E \;=\; \tfrac{1}{2}m_e v_o^2$,

$$\frac{E}{E} = \frac{m_e r^{*2}}{2E} \;+\; \frac{\tfrac{1}{2}m_e v_o^2 b^2}{ER^2} + \frac{(-e)V(R)}{E}$$

$$1 = \frac{m_e r^{*2}}{2E} \;+\; \frac{b^2}{R^2} - \frac{eV(R)}{E}$$

$$b \;=\; R\,(1 \;-\; \frac{m_e r^{*2}}{2E} + \frac{eV(R)}{E})^{\frac{1}{2}}$$

The electrons whose impact parameter is b_{coll} with

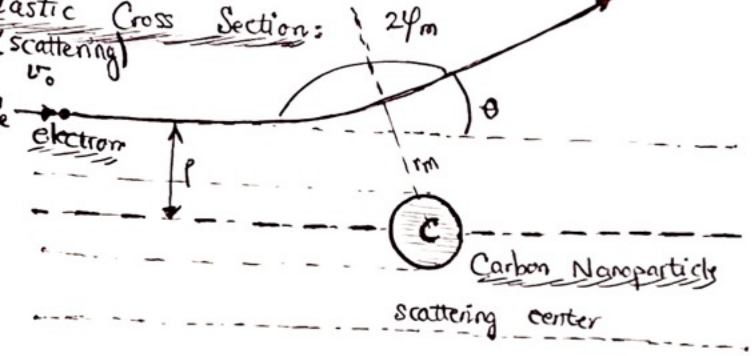
$$b_{coll} = R\,(1 \;+ \frac{eV(R)}{E}\;)^{\frac{1}{2}}$$

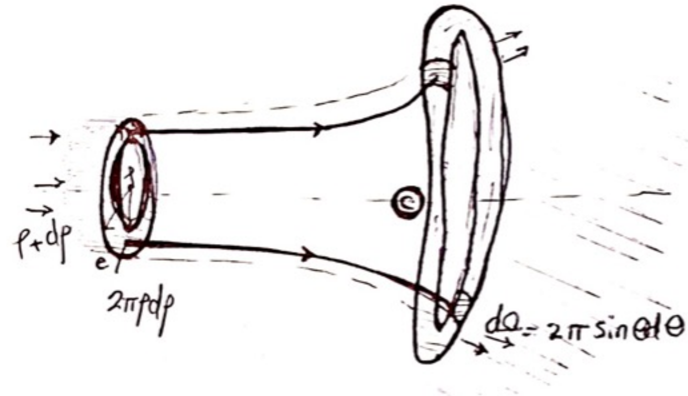
will hit the dust particle with zero radial velocity ($r^* \;=\; 0$). Therefore all electrons of energy E whose impact parameter is less than b_{coll} will hit and be collected by the dust particle. The cross-section for electron collection by the dust particle can be defined as: (Particle potential is negative.)

$$\sigma = \pi\, b_{coll}^{\;2} = \pi\, R^2(1 \;+\; \frac{eV(R)}{E}\;)\,, \qquad\qquad E \;>\; -V(R)$$

Elastic Cross Section: Please read the notes.

Elastic Cross Section: (scattering)





$\rho \sim$ scattering Parameter ; $\theta \sim$ scattering angle

The scattering angle of the electron is dependent on it's own energy and the scattering parameter.

$\left(\begin{array}{l}\text{\# of Particles (electrons) incident} \\ \text{between } \rho \text{ and } \rho+d\rho\end{array}\right) = \left(\begin{array}{l}\text{\# of Particles (electrons)} \\ \text{outgoing between } \theta \\ \text{and } \theta+d\theta\end{array}\right) \Rightarrow$ (Flux conservation requires that for incoming flux Γ:

$$\Gamma 2\pi\rho\, d\rho = -\Gamma I(v,\theta) 2\pi \sin\theta d\theta)$$

$I(v,\theta) \sim$ Proportionality constant (m^2/sr)

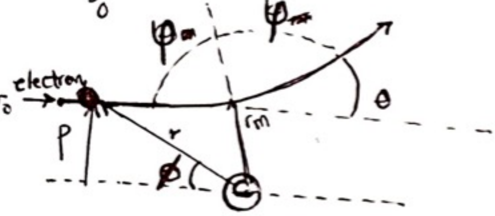
All electrons entering through the differential annulus $2\pi\rho d\rho$ leave through a differential solid angle, $d\Omega = 2\pi \sin\theta d\theta$.

$$I(v,\theta) = \frac{\rho}{\sin\theta}\left|\frac{d\rho}{d\theta}\right|$$

; ($(1-\cos\theta)$ is the fraction of the initial momentum lost by the incident electrons.)

(Total scattering cross section obtained by integrating I over the solid angle) $\sim 2\pi\int_0^\pi I(v,\theta)\sin\theta d\theta \rightarrow \sigma = 2\pi\int_0^\pi (1-\cos\theta) I(v,\theta) \sin\theta d\theta = 2\pi\int_0^\pi (1-\cos\theta)\frac{\rho}{\sin\theta}\sin\theta\left|\frac{d\rho}{d\theta}\right| d\theta$

$$\sigma = 2\pi\int_{\rho_{min}}^{\rho_{max}} (1-\cos\theta)\rho\, d\rho$$



• Let's assume a Yukawa potential for the interaction between charged particles (electron and dust grain).

$$U(r) = \frac{-U_0}{r}\exp\left(\frac{-r}{\lambda}\right)$$ for repulsion $U_0 < 0$ and $r \lll \lambda$ ($\exp(\frac{-r}{\lambda}) \simeq 1$)

Therefore $U(r) = \frac{U_0}{r}$

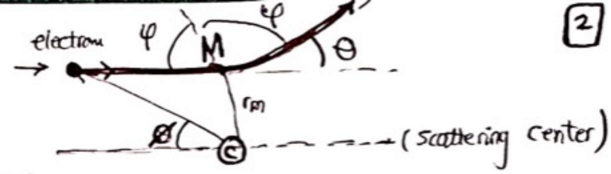
Conservation of Energy: $\frac{1}{2}m_T(\dot r^2 + r^2\dot\phi^2) + U(r) = \frac{1}{2}m_T v_0^2$

Dust particle is relatively stationary with respect to the nanoparticle as almost the kinetic energy is coming from the electrons ($m_T = m_e \sim$ (reduced mass $\simeq$ Electron Mass) \$ $v_0 \sim$ Electron Velocity)

Conservation of angular Momentum: $r^2\dot\phi = v_0\rho \Rightarrow \dot\phi = \frac{v_0\rho}{r^2}$

Combined conservation of energy and angular momentum: $\frac{1}{2}m_e v_0^2 = \epsilon$

$$\frac{1}{2}m_e\left(\dot r^2 + r^2\left(\frac{v_0\rho}{r^2}\right)^2\right) + U(r) = \frac{1}{2}m_e v_0^2 \Longrightarrow \dot r^2 + \frac{v_0^2\rho^2}{r^2} + \frac{2U(r)}{m_e} = v_0^2 \Rightarrow \dot r^2 = v_0^2\left(1 - \frac{\rho^2}{r^2} - \frac{U(r)}{\frac{1}{2}m_e v_0^2}\right)$$



$$\dot r = \pm v_0\sqrt{1-\frac{\rho^2}{r^2}-\frac{U(r)}{\epsilon}} \quad ; \quad \frac{1}{2}m_e v_0^2 = \epsilon \quad ; \quad \dot\phi = \frac{v_0 b}{r^2}$$

$$\frac{\dot r}{\dot\phi} = \frac{dr/dt}{d\phi/dt} = \frac{dr}{d\phi} = \pm\frac{r^2 v_0}{v_0 \rho}\sqrt{1-\frac{\rho^2}{r^2}-\frac{U(r)}{\epsilon}} \Rightarrow d\phi = \pm\frac{\rho\left(\frac{dr}{r^2}\right)}{\sqrt{1-\frac{\rho^2}{r^2}-\frac{U(r)}{\epsilon}}} = \varphi$$

(+) applies past point M of closest approach; (−) applies before M. At M, the distance r_m follows from $\frac{dr}{d\theta} = 0$.

We derived the following Equations so far which would help us determine elastic cross sec.

1. $\theta + 2\varphi = \pi \Rightarrow \theta = \pi - 2\varphi(\rho)$
2. $U(r) = \frac{U_0}{r}$ for repulsion
3. $\varphi(\rho) = \int_{r_m}^{\infty} \frac{\pm\rho\,(dr/r^2)}{\sqrt{1-\frac{\rho^2}{r^2}-\frac{U(r)}{\epsilon}}}$
4. $2\pi\int_{\rho_{min}}^{\rho_{max}} (1-\cos\theta)\,\rho\, d\rho$

- $$\varphi(\rho) = \int_{r_m}^{\infty}\frac{+\rho\,(dr/r^2)}{\sqrt{1-\frac{\rho^2}{r^2}-\frac{U(r)}{\epsilon}}} \xrightarrow[du = -\frac{dr}{r^2}\ ;\ U(r)=\frac{U_0}{r}]{\text{Let } u = -\frac{1}{r}\ \&\ 2\rho_0 = \frac{U_0}{\epsilon}} \int_{u_m=\frac{1}{r_m}}^{u=\frac{1}{\infty}=0}\frac{-\rho\, du}{\sqrt{1-\rho^2u^2-2\rho_0 u}} = \int_{u_m}^{0}\frac{-\rho\, du}{\sqrt{1-\left(\rho^2u^2+2\rho_0 u+\left(\frac{\rho_0}{\rho}\right)^2-\left(\frac{\rho_0}{\rho}\right)^2\right)}}$$

$\int_{u_m}^{0} - : \int_{0}^{u_m}$

$$\Rightarrow \int_{u_m}^{0}\frac{-\rho\, du}{\sqrt{\left(1+\left(\frac{\rho_0}{\rho}\right)^2\right)-\left(\rho u+\frac{\rho_0}{\rho}\right)^2}} = \sin^{-1}\left(\frac{\rho u + \left(\frac{\rho_0}{\rho}\right)}{\sqrt{1+\left(\frac{\rho_0}{\rho}\right)^2}}\right)\Bigg|_{0}^{u_m}$$

$$\int\frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right)+C$$

- $u_m = \frac{1}{r_m} \rightarrow$ at r_m $\frac{dr}{d\theta} = 0 = \frac{r^2}{\rho}\sqrt{1-\frac{\rho^2}{r^2}-\frac{U(r)}{\epsilon}} = 0 \Rightarrow \sqrt{1-\frac{\rho^2}{r^2}-\frac{U(r)}{\epsilon}} = 0 \Rightarrow 1-\frac{\rho^2}{r^2}-\frac{U_0}{r\epsilon} = 0$

$$1-\frac{\rho^2}{r^2}-2\rho_0\left(\frac{1}{r}\right) \xrightarrow{u=\frac{1}{r}} -1+\rho^2u^2+2\rho_0 u = 0 \Rightarrow \rho^2u^2+2\rho_0 u+\left(\frac{\rho_0}{\rho}\right)^2-\left(\frac{\rho_0}{\rho}\right)^2 = 1$$

$$\left(\rho u+\left(\frac{\rho_0}{\rho}\right)\right)^2 = 1+\left(\frac{\rho_0}{\rho}\right)^2 \Rightarrow \rho u+\left(\frac{\rho_0}{\rho}\right) = \pm\sqrt{1+\left(\frac{\rho_0}{\rho}\right)^2} \xrightarrow[\text{For the largest root}]{\text{We are looking}} \rho u = \left(-\frac{\rho_0}{\rho}\right)+\sqrt{1+\left(\frac{\rho_0}{\rho}\right)^2}$$

$$u_m = \left[\left(-\frac{\rho_0}{\rho}\right)+\sqrt{1+\left(\frac{\rho_0}{\rho}\right)^2}\right]\left(\frac{1}{\rho}\right)$$

At u_m:

$$\sin^{-1}\left(\frac{\left(\frac{\cancel{\rho}}{\cancel{\rho}}\right)\left[\left(\cancel{-\frac{\rho_0}{\rho}}\right)+\sqrt{1+\left(\frac{\rho_0}{\rho}\right)^2}\right]+\left(\cancel{\frac{\rho_0}{\rho}}\right)}{\sqrt{1+\left(\frac{\rho_0}{\rho}\right)^2}}\right) = \sin^{-1}(1) = \pi/2$$

Therefore: $\sin^{-1}\left(\frac{\rho u+(\rho_0/\rho)}{\sqrt{1+(\rho_0/\rho)^2}}\right)\Bigg|_{0}^{u_m} = \pi/2 - \sin^{-1}\left(\frac{(\rho_0/\rho)}{\sqrt{1+(\rho_0/\rho)^2}}\right) = \varphi(\rho)$

$$\bullet\ 1-\cos\theta = 1-\cos(\pi-2\varphi) = 1-\cos\left(\pi - 2\left(\tfrac{\pi}{2} - \sin^{-1}\left(\frac{(\rho_0/\rho)}{\sqrt{1+\left(\frac{\rho_0}{\rho}\right)^2}}\right)\right)\right) \Rightarrow$$

$$1-\cos\left(\pi-\pi+2\sin^{-1}\left(\frac{(\rho_0/\rho)}{\sqrt{1+\left(\frac{\rho_0}{\rho}\right)^2}}\right)\right) = 1-\cos\left(2\sin^{-1}\left(\frac{(\rho_0/\rho)}{\sqrt{1+\left(\frac{\rho_0}{\rho}\right)^2}}\right)\right) \quad \therefore \quad \cos 2x = 1-2\sin^2(x)$$

$$\bullet\ 1-\cos\theta = 1-\left[1-2\left[\sin\left(\sin^{-1}\left(\frac{(\rho_0/\rho)}{\sqrt{1+(\rho_0/\rho)^2}}\right)\right)\right]^2\right] = 1-\left[1-2\left(\frac{(\rho_0/\rho)^2}{1+(\rho_0/\rho)^2}\right)\right] = \frac{+2(\rho_0/\rho)^2}{1+(\rho_0/\rho)^2}$$

$$4.\ 2\pi\int_{\rho_{min}}^{\rho_{max}}(1-\cos\theta)\rho\,d\rho = 2\pi\int_{\rho_{min}}^{\rho_{max}}\frac{2(\rho_0/\rho)^2\rho\,d\rho}{1+(\rho_0/\rho)^2}\times\frac{(\rho/\rho_0)^2}{(\rho/\rho_0)^2} = 2\pi\int_{\rho_{min}}^{\rho_{max}}\frac{(\rho_0^2)\left(\frac{1}{\rho_0^2}\right)2\rho\,d\rho}{(\rho/\rho_0)^2+1} = 2\pi\rho_0^2\int_{\rho_{min}}^{\rho_{max}}\ldots$$

$$2\pi\rho_0^2\ln\left[1+(\rho/\rho_0)^2\right]\Bigg|_{\rho_{min}}^{\rho_{max}} = 2\pi\rho_0^2\left[\ln\left(1+\left(\frac{\rho_{max}}{\rho_0}\right)^2\right)-\ln\left(1+\left(\frac{\rho_{min}}{\rho_0}\right)^2\right)\right] = 2\pi\rho_0^2\ln\left[\frac{1+\left(\frac{\rho_{max}}{\rho_0}\right)^2}{1+\left(\frac{\rho_{min}}{\rho_0}\right)^2}\right]$$

$$\sigma = 2\pi\rho_0^2\ln\left[\frac{\rho_0^2+\rho_{max}^2}{\rho_0^2+\rho_{min}^2}\right]$$

We consider the upper bound for the scattering parameter to be the linearized Debye length. Based on all the papers that I read λ_{LD} is defined as $\sqrt{\left(\frac{1}{\lambda_{De}}\right)^2+\left(\frac{1}{\lambda_{Di}}\right)^2}$ where $\lambda_{De}=\sqrt{\frac{\epsilon_0 k_B T_e}{e^2 n_e}}$ & $\lambda_{Di}=\sqrt{\frac{\epsilon_0 k_B T_i}{e^2 n_i}}$.

Based on our discussion before we defined this parameter differently as:

$$\lambda_{LD} = \left[\frac{1}{\lambda_{De}}+\frac{1}{\lambda_{Di}}+\frac{1}{\lambda_p}\right]^{-1} \quad \text{where} \quad \lambda_p = \sqrt{\frac{\epsilon_0 k_B T_p}{k e^2 n_p}} \ ; \ k=\frac{n_i-n_e}{n_p} \text{ and } T_p = 300\,K.$$

The lower bound for the scattering parameter is $\rho_c = R\left(1+\frac{eV(r)}{E}\right)^{1/2}$. Therefore for attachment (collection) cross section ($\rho_{min}=0$ and $\rho_{max}=\rho_c$) and for elastic (scattering) cross section ($\rho_{min}=\rho_c$ & $\rho_{max}=\lambda_{LD}$)

As discussed before, we set ρ_{min} to 0 which make much of a difference from when it was $\rho_{min}=\rho_c$.

$\rho_0 = \frac{U_0}{2E}$; We defined the particle potential as $V_p = \frac{ke}{4\pi\epsilon_0}\left(\frac{1}{r}\right) = \frac{U_0}{R} \xrightarrow{\text{So}} V_p R = U_0$

$$\rho_0^2 = \frac{V_p^2R^2}{4E} = \left(\frac{V_pR}{2E}\right)^2 \ ; \ \rho_{min} = \rho_c = R\left(1+\frac{V_p}{E}\right)^{1/2} \ ; \ \rho_{max} = \lambda_{LD}$$

$$\sigma = 2\pi\left(\frac{V_pR}{2E}\right)^2\ln\left[\frac{\left(\frac{V_pR}{2E}\right)^2+\lambda_{LD}^2}{\left(\frac{V_pR}{2E}\right)^2+R^2\left(1+\frac{V_p}{E}\right)}\right]$$