PAGE NO.:
DATE: / /

Assignment 3CO24 102103660

Predictive Analysis Using statistics

Parameter Estimation

1. $(X_1, X_2 \ldots)$, be a random sample of size n taken from a Normal Population, mean $= \theta_1$ and variance $= \theta_2$. Find MLE of these 2 Parameters.

Function will be. [Normal Population distribution]

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\left(\left(\frac{x_i - \mu}{2\sigma^2}\right)^2\right)}$$

Take natural log on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(X_i - \mu)^2}{2\sigma^2} - \frac{1}{2} \ln(2\pi\sigma^2) \right)$$

To find the MLE, differentiate log-likelihood fn w.r.t θ_1 and θ_2

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1} \left(\frac{X_i - \mu}{\sigma^2} \right) = 0$$

This implies,

$$\sum_{i=1}^{n} X_i - n\mu = 0$$

$$\theta_1 / \mu = \frac{1}{n} \sum_{i=1}^{n} X_i$$

PAGE NO.:
DATE: / /

For θ_2,

$$\frac{d}{d\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(X_i-\theta_1)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2} \right) = 0$$

=>

$$\sum_{i=1}^{n} \frac{(X_i-\theta_1)^2}{\theta_2^2} - \frac{n}{\theta_2} = 0$$

$$\frac{1}{\theta_2^2} \sum_{i=1}^{n} (X_i-\theta_1)^2 = \frac{n}{\theta_2}$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (X_i-\theta_i)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (X_i-\theta_1)^2$$

Sample Variance

---

(2). $X_1, X_2 \dots X_n$ be a random sample from $B(m, \theta)$ distribution, where $\theta \in \Theta = (0,1)$ is unknown and 'm' is a known +ve integer. Compute θ using MLE.

We are given Binomial distribution

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking natural log,

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left[\ln\binom{m}{x_i} + x_i \ln(\theta) + (m-x_i)\ln(1-\theta) \right]$$

DIFF w.r.t θ

$$\frac{\partial}{\partial\theta}\ln L(\theta) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta}\right) = 0$$

Solving for θ

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\sum_{i=1}^{n} x_i(1-\theta) = \sum_{i=1}^{n} (m-x_i)\theta$$

$$\theta \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\boxed{\theta = \frac{1}{m} \sum_{i=1}^{n} x_i}$$

MLE of θ is sample mean of observations.