report

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1 Report Outlab02: Lagrange Interpolation

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- Data: 01/23/2025

1.1 Code

This is the code that is used to analyze and generate the plots

1.1.1 Imports

[2]:
```
import math
import pathlib

import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec
```

1.1.2 Functions

[3]:
```
def f(x):
    '''Function of interest'''
    return np.exp(x)



def f_n_plus_1(x):
    '''(n + 1)th derivative of f(x)'''
    return np.exp(x)



def M_n_plus_1(x):
    '''Absolute value of f^(n+1)'th largest value over the interval'''
    return np.max(np.abs(f_n_plus_1(x)))

def pi_n_plus_1(x, xi: np.array):
    return np.prod(x[:, np.newaxis] - xi, axis=1)
```

```
def B(x, xi):
    '''Error bound'''
    n = len(xi)
    return M_n_plus_1(x) * np.abs(pi_n_plus_1(x, xi)) / math.factorial(n)
```

1.1.3 Helper Functions

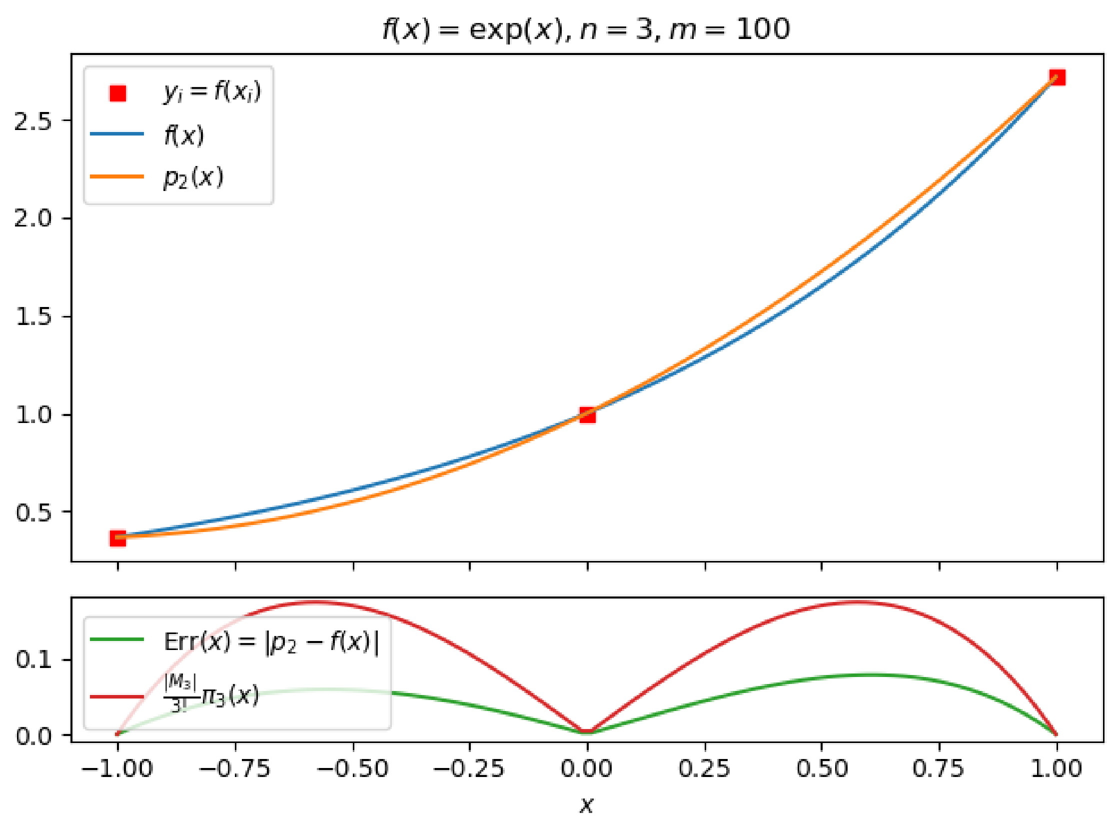
[4]:
```
DATADIR = pathlib.Path(f'./data/')

def get_data(n: int) -> tuple[pd.DataFrame, pd.DataFrame]:
    datadir = DATADIR / f'n{n}'
    dfi = pd.read_csv(datadir / 'input.csv')
    dfo = pd.read_csv(datadir / 'output.csv')

    # Get absolute error
    dfo['|E(x)|'] = np.abs(dfo['E(x)'])

    # Get error bound
    dfo['B(x)'] = B(dfo.x.values, dfi.x.values)

    return dfi, dfo
```

[7]:
```
def plot_lagrange(dfi: pd.DataFrame, dfo: pd.DataFrame, n: int) -> None:
    fig = plt.figure(tight_layout=True)

    gs = gridspec.GridSpec(4, 1)

    # Plot the function and the Lagrange Interpolation Polynomial
    ax1 = fig.add_subplot(gs[:3, 0])
    ax1.set_title(fr'$f(x)=\exp(x), n={n}, m=100$')

    ax1.scatter('x', 'y', data=dfi, label='$y_i = f(x_i)$', c='red', marker='s')
    ax1.plot('x', 'f(x)', data=dfo, label='$f(x)$')
    ax1.plot('x', 'L(x)', data=dfo, label=f'$p_{n - 1}(x)$')
    ax1.legend()
    ax1.tick_params(
        axis='x',           # changes apply to the x-axis
        which='both',       # both major and minor ticks are affected
        labelbottom=False
    )


    # Plot Absolute Error and Error Bound
    ax2 = fig.add_subplot(gs[3, 0], sharex=ax1)
    ax2.plot('x', '|E(x)|', data=dfo, label=r'$\text{Err}(x)=|p_{%d} - f(x)|$'␣
 ↪% (n - 1), c='C2')
```

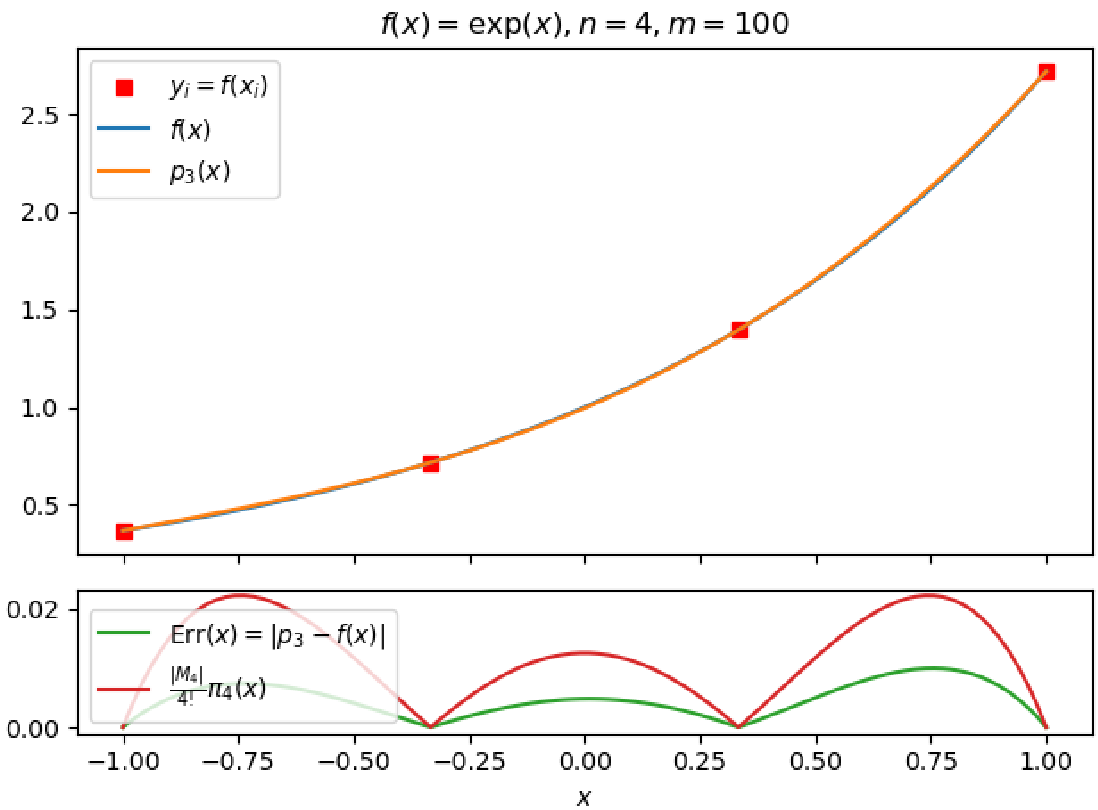
```
    ax2.legend()
    ax2.set_xlabel('$x$')

    ax2.plot('x', 'B(x)', data=dfo, c='C3', label=r'$\frac{|M_{%d}|}{%d!
↪}\pi_{%d}(x)$' % (n, n, n))
    ax2.legend()
    ax2.set_xlabel('$x$')

    plt.savefig(DATADIR / f'n{n}' / 'plot.png')
    plt.show()
```

1.2 Plots

[8]:
```
n = 3
dfi, dfo = get_data(n)
plot_lagrange(dfi, dfo, n)
```

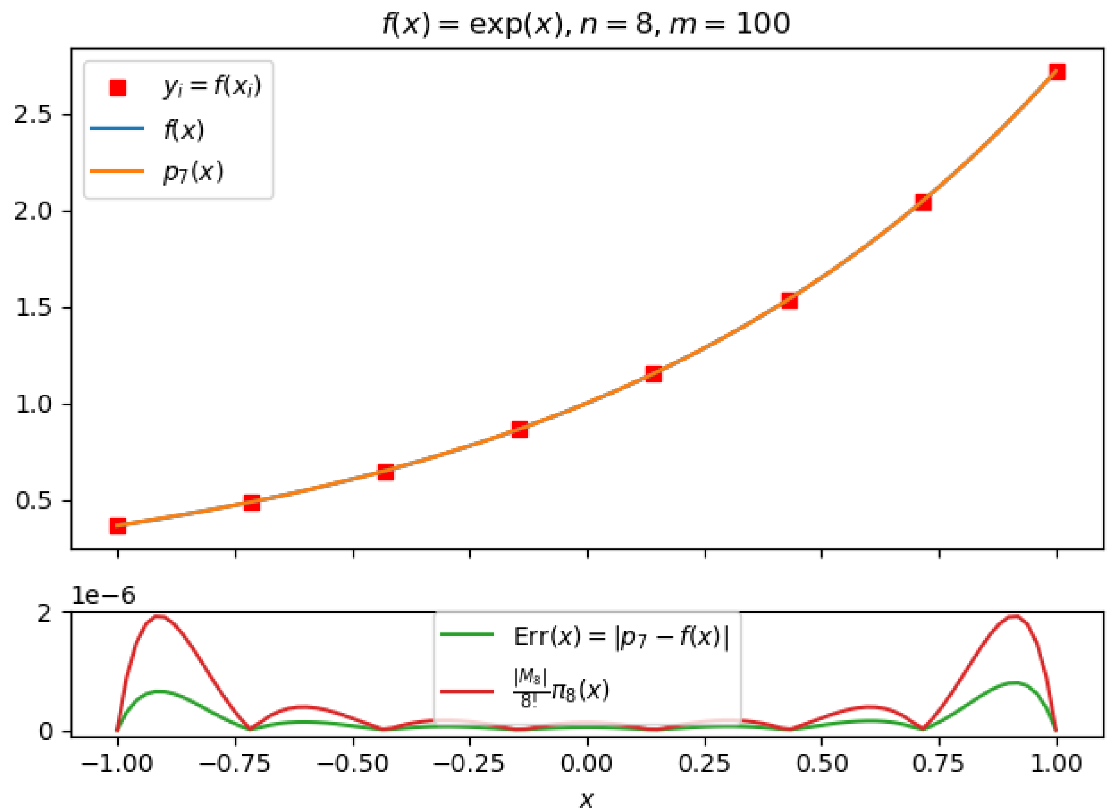


[9]:
```
n = 4
dfi, dfo = get_data(n)
plot_lagrange(dfi, dfo, n)
```



[10]:

```
n = 8
dfi, dfo = get_data(n)
plot_lagrange(dfi, dfo, n)
```



1.3 Notes and Remarks

As we can see from the graphs above as n increases the interpolation error decreases. This can be observed by looking at the lower part of each plot, where the maximum value on he y axis is ≈ 1.5 for $n = 3$, ≈ 0.02 for $n = 4$, and 2×10^{-6} for $n = 8$. The $n = 3$ case is not enough to provide a good approximation for the function. However, just increaesing the number of interpolated points to $n = 4$ already gives a workable approximation.