Theory of Computation

Problem Set 1

Problem 1

Let M be a DFA with n states over alphabet Σ. Two words $x, y \in \Sigma^*$ are called *distinguishable* if there exists a word $z \in \Sigma^*$ such that exactly one of xz and yz is accepted by M. A set $S \subseteq \Sigma^*$ is called *pairwise distinguishable* if every two distinct words in S are distinguishable. What is the maximum possible size $|S|$ of a pairwise distinguishable set S of words for M?

Problem 2

A DFA over alphabet Σ has the following property:

> For every string x, the DFA reaches the same state after reading x and after reading the reverse of x.

Prove or disprove this statement - *The language recognized by the DFA depends only on the length of the input string. That is, if $|x| = |y|$, then either both x and y are accepted or both are rejected.* Also discuss how the choice of Σ affects the truth of the given statement.

Problem 3

An NFA is called *binary-branching* if from each state and each input symbol there are at most two possible distinct next states.

Determine whether the following statement is true or false:

> There exists a binary-branching NFA with n states and a word w such that the number of accepting runs of the NFA on w is at least
>
> $$2^{|w|}.$$

Problem 4

Let M be a deterministic finite automaton with state set Q, where $|Q| = n$. For each word w, define the induced map

$$f_w : Q \to Q, \qquad f_w(q) = \delta(q, w).$$

Prove that there always exists a nonempty subset

$$S \subseteq Q$$

with the following properties:

1. (**Size-invariance**) For every word w,

$$|f_w(S)| = |S|.$$

2. (**Minimality**) No smaller nonempty subset of Q has this property.