MTech CSE – 1st Semester
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Question 8

For finding an augmenting path in a flow network, should Breadth-First Search (BFS) or Depth-First Search (DFS) be applied? Justify your answer.

Answer:

Short Answer

Breadth-First Search (BFS) should be used for finding augmenting paths, especially in algorithms such as Edmonds–Karp. While Depth-First Search (DFS) can also find augmenting paths, BFS guarantees better performance and polynomial-time complexity.

Augmenting Paths in Flow Networks

An augmenting path is a path from the source vertex s to the sink vertex t in the residual graph along which additional flow can be pushed.

Augmenting paths are fundamental to:

- the Ford–Fulkerson method,
- the Edmonds–Karp algorithm,
- maximum flow computation.

The strategy used to find augmenting paths directly impacts the efficiency of the algorithm.

Comparison Between DFS and BFS

Depth-First Search (DFS)

- Explores paths deeply before backtracking.
- May select long or inefficient augmenting paths.
- Strongly depends on traversal order.
- Does not guarantee the shortest augmenting path.

Although DFS can find valid augmenting paths, it may lead to poor performance in the worst case.

Breadth-First Search (BFS)

- Explores the residual graph level by level.
- Always finds the shortest augmenting path in terms of number of edges.
- Ensures systematic and predictable progress.
- Limits unnecessary or redundant augmentations.

Why BFS Is Preferred

Shortest Augmenting Path Property

When BFS is applied to the residual graph, the first path found from s to t has the minimum number of edges. Using shorter paths reduces the number of flow augmentations required.

This property is central to the Edmonds–Karp algorithm.

Guaranteed Polynomial-Time Complexity

Using DFS in the Ford–Fulkerson method can result in exponential time complexity in the worst case.

In contrast, BFS ensures that the Edmonds–Karp algorithm runs in polynomial time:

$$O(VE^2),$$

where V is the number of vertices and E is the number of edges.

Avoidance of Inefficient Augmentations

DFS may repeatedly:

- push small amounts of flow,
- undo earlier flow decisions,
- traverse unnecessarily long paths.

BFS avoids these issues by:

- selecting shortest paths,
- ensuring that the distance from source to sink never decreases.

Algorithmic Insight: Edmonds–Karp

The Edmonds–Karp algorithm is a specific implementation of the Ford–Fulkerson method that uses BFS to find augmenting paths.

Key properties include:

- each BFS takes $O(E)$ time,
- the number of augmentations is bounded by $O(VE)$,
- total running time is $O(VE^2)$.

These guarantees are not available when DFS is used.

Can DFS Be Used at All?

Yes, DFS can be used:

- in the basic Ford–Fulkerson algorithm,
- for small or well-structured graphs,
- when worst-case performance is not critical.

However, DFS offers no polynomial-time guarantee and is unsuitable for large or adversarial inputs.

Final Conclusion

Breadth-First Search (BFS) is the preferred choice for finding augmenting paths.

Justification:

- BFS finds shortest augmenting paths,
- guarantees polynomial-time performance,
- avoids inefficient and cyclic augmentations,
- forms the foundation of the Edmonds–Karp algorithm.

Hence, BFS is the correct and theoretically sound method for augmenting path selection in flow networks.